ECON 442

Sustainability of Pensions

Winter 2015

Sustainability

When are PAYG systems optimal?

- savings externality that leads to overaccumulation of capital
- behavioral biases that lead to too little savings

Evidence: Unfunded PAYG systems have lower (private) savings rates.

When are PAYG systems feasible?

- Productivity growth matters, as it increases returns on capital.
- Population growth matters, as it determines what transfers across generations are feasible.

Evidence: Slower productivity and population growth.

Analysis

Model:

- log utility
- $f(k_t) = Ak_t^\alpha$
- PAYG system $\tau(t) = a(t)/n$

Household takes PAYG as given:

$$\begin{aligned} \frac{c_t(t+1)}{\beta c_t(t)} &= r(t+1) \\ c_t(t) + \frac{c_t(t+1)}{r(t+1)} &= w(t) - \tau(t) + \frac{a(t+1)}{r(t+1)} \end{aligned}$$

Savings:

$$s(t+1) = w(t) - \tau(t) - c_t(t)$$

Steady State

Current consumption anticipates pensions

$$c_t(t) = \frac{1}{1+\beta}\left(w(t) - \tau(t) + \frac{a(t+1)}{r(t+1)}\right)$$

The savings function is given by

$$s(t+1) = \frac{\beta}{1+\beta}(w(t) - \tau(t)) - \frac{1}{1+\beta}\frac{a(t+1)}{r(t+1)}$$

Capital accumulates according to

$$\begin{aligned} nk(t+1) &= s(t+1) = \frac{\beta}{1+\beta}(w(t) - \tau(t)) - \frac{1}{1+\beta}\frac{a(t+1)}{r(t+1)} \\ &= \frac{\beta}{1+\beta}((1-\alpha)Ak(t)^{\alpha} - \tau(t)) - \frac{n\tau(t+1)}{(1+\beta)\alpha Ak(t+1)^{\alpha-1}} \end{aligned}$$

In steady state with a fixed PAYG system we obtain

$$n\bar{k} = \frac{\beta}{1+\beta}((1-\alpha)A\bar{k}^{\alpha} - \tau) - \frac{\tau}{(1+\beta)\alpha A\bar{k}^{\alpha-1}}$$

Each PAYG scheme is associated with a steady state

$$\tau(k) = \frac{\beta(1-\alpha)Ak^{\alpha} - (1+\beta)nk}{\beta + \frac{n}{\alpha A k^{\alpha-1}}}$$

For $\tau = 0$, we obtain our earlier result that $\bar{k} = \left(\frac{1}{n}\frac{\beta}{1+\beta}(1-\alpha)A\right)^{\frac{1}{1-\alpha}}$.

There is a maximal τ that is consistent with a steady state.

We interpret any $\tau \in (0, \bar{\tau})$ as a **sustainable PAYG system**.

Graph – Sustainability

ADD

Population Growth Slows

Consider a fall in n.

This implies that the $\tau(k)$ graph shifts upwards.

- Suppose we want to maintain the same steady state capital stock.

  $\Longrightarrow$ We need to increase the per capita tax τ on the young immediately.

- Suppose we want to keep the tax τ constant.

  $\Longrightarrow$ The per-capita capital stock will increase.
  - Pensions a must fall.
  - Effect on consumption and savings are ambiguous.

Productivity Slowdown

Consider a fall in A.

This implies that the graph $\tau(k)$ shifts downward.

- Suppose we want to maintain the same per-capita capital stock.

  $\Longrightarrow$ We need to decrease the tax τ (and, hence, pensions) on the young immediately.

- When we keep the tax constant, we will converge to a lower per-capita capital stock.

- For a sufficiently large drop in A, the original social security scheme might become infeasible.