# ECON 442

# The OG Model

Queen's University

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# Public Economics

- Why is a government needed?
- How should government policies be structured?
- What are the problems with implementing policies?

First question is key.

- public goods and externalities
- other market failures (information)
- redistribution (normative?)
- behavioral problems (“paternalistic” or “pathological”)

# This Course

For the most part we will rely on the **Overlapping Generations Model**.

This model has a fundamental structure where government matters for transferring resources intertemporally.

Two basic questions:

**1)** What is the role of debt and fiscal policy in the short- and long-run?

**2)** Is there a role for redistributing wealth and/or income intergenerationally?

# The OG Model

Time $t = 0, 1, 2, \ldots$

$N_t$ people are born of generation $t$.

Alive for two periods, $t$ and $t+1$.

| | $t=0$ | $t=1$ | $t=2$ | $t=3$ |
|---|---|---|---|---|
| Gen. ? | $N_{-1}$ | | | |
| Gen. 0 | $N_0$ | $N_0$ | | |
| Gen. 1 | | $N_1$ | $N_1$ | |
| Gen. 2 | | | $N_2$ | $N_2$ |
| Gen. 3 | | | | $N_3$ |

Endowments:

- $y_t(t)$ when young
- $y_t(t+1)$ when old

Consumption:

- $c_t(t)$ when young
- $c_t(t+1)$ when old

Preferences:

- $u(c_t(t), c_t(t+1))$

Initial generation:

- $N_{-1}$ people with $y_{-1}(0)$ endowment
- $u(c_{-1}(0))$ preferences

Assumptions:

- $N_t = n^t N_0$
- either $u(c_t(t)) + \beta u(c_t(t+1))$, where $\beta \in (0, 1]$ ...
- ... or $\lambda u(c_t(t)) + (1-\lambda) u(c_t(t+1))$, where $\lambda \in [0, 1]$
- $u' > 0$ and $u'' < 0$

Hence: incentive to smooth consumption over time

But: cannot necessarily achieve it

**Question:**

**Is this a matter of technology or market failure?**

# Pareto Optimal Allocations

**Definition:** An allocation is Pareto-optimal if (i) it is feasible and (ii) there is no other allocation that makes everybody at least well off and someone better off (pareto-dominates).

Allocation: $c = (c_1(0), (c_0(0), c_0(1)), (c_1(1), c_1(2)), \dots)$

Stationary Allocation:

- independent of time $t$
- same across all generations
- $c_t(t) = c_1$ and $c_t(t+1) = c_2$ for all $t$

Feasible Allocation:

$$N_t c_t(t) + N_{t-1} c_{t-1}(t) \leq N_t y_t(t) + N_{t-1} y_{t-1}(t) \text{ for all } t$$

With stationarity:

$$c_1 + \frac{c_2}{n} = y_1 + \frac{y_2}{n}$$

Allocation $\tilde{c}$ dominates allocation $c$ if and only if

$$\begin{aligned} u(\tilde{c}_t(t)) + \beta u(\tilde{c}_t(t+1)) &\geq u(c_t(t)) + \beta u(c_t(t+1)) \text{ for all } t \\ u(\tilde{c}_t(t)) + \beta u(\tilde{c}_t(t+1)) &> u(c_t(t)) + \beta u(c_t(t+1)) \text{ for some } t \end{aligned}$$

Note: initial old generation matters too!

$$\tilde{c}_{-1}(0) \geq c_{-1}(0)$$

# The Set of Feasible Allocation

$$\begin{aligned} N_t c_t(t) + N_{t-1} c_{t-1}(t) &\leq N_t y_t(t) + N_{t-1} y_{t-1}(t) \equiv Y_t \\ N_{t+1} c_{t+1}(t+1) + N_t c_t(t+1) &\leq N_{t+1} y_{t+1}(t+1) + N_t y_t(t+1) \equiv Y_{t+1} \end{aligned}$$

With stationary endowments we have

$$Y_{t+1} = nY_t$$

The set of per-capita (gen. $t$) feasible allocations is then constant across time

$$c_t(t) + \frac{c_{t-1}(t)}{n} = \frac{Y_t}{N_t} \equiv y$$

With stationary allocations, the frontier of the set of feasible allocations is given by

$$c_2 = n\left(y - c_1\right)$$

# Indifference Curves

Initial old generation only likes period 2 consumption.

For other generations, fix utility level $u(c_t(t), c_t(t+1)) = \bar{u}$.

Slope of indifference curves are given by

$$u'_1 dc_t(t) + u'_2 dc_t(t+1) = 0$$

$$\frac{dc_t(t+1)}{dc_t(t)} = -\frac{u'_1}{u'_2}$$

For time-separable utility function and stationarity this yields

$$\frac{dc_2}{dc_1} = -\frac{u'(c_1)}{\beta u'(c_2)}$$

# Conditions for Pareto-Optimality

(i) Allocation needs to be on the boundary of the feasible set.

(ii) MRS $\geq$ MRT

Here:

$$\begin{aligned} c_t(t+1) &= n\,(y_t - c_t(t)) \\ -\frac{u'(c_t(t))}{\beta u'(c_t(t+1))} &\leq -n \end{aligned}$$

SEE GRAPH

# How can we compute one stationary PO allocation?

$$\max_{c_1, c_2} u(c_1) + \beta u(c_2)$$
subject to
$$c_1 + \frac{c_2}{n} = y$$

Lagrangian/FOC:

$$\begin{aligned} u'(c_1) - \lambda &= 0 \\ \beta u'(c_2) - \frac{\lambda}{n} &= 0 \\ c_1 + \frac{c_2}{n} = y \end{aligned}$$

Hence:

$$\begin{aligned} \frac{u'(c_1)}{\beta u'(c_2)} &= n \\ c_1 &= n\,(y - c_2) \end{aligned}$$