Economics 421 **Fall 2015**

Topics in Public Economics **Thorsten Koeppl**

Assignment 4

(Due: Friday, December 4 – Drop Box until 3:00 pm)

1. Consider an OG environment where population is fixed over time and given by $N_t = 2$ for all t. People have one unit of labour as an endowment. Preferences are described by $\log c_t(t) + \beta \log c_t(t+1)$. Production depends on *aggregate* consumption $C_1(t)$ by the young according to the production function

$$Y_t = C_1(t)^{\omega} N$$

where N is labour input. Note that aggregate consumption of the young is given by

$$C_1(t) = N_t c_t(t) = 2c_t(t),$$

and that people supply their labour inelastically so that $N = N_t = 2$ for all t. Finally, assume that people can store resources at a rate of return $r = 1$.

   (a) Characterize the optimal stationary allocation a social planner would choose.

   (b) Find the competitive equilibrium in this economy.

   (c) Set $\beta = 1$ and $\omega = 1/2$. Compare the optimal allocation with the competitive equilibrium allocation. In particular, how do (implied) savings differ across the allocations?

Suppose now a government can levy a tax on savings τ. The revenue from this tax is given as a lump-sum transfer T to the young generation.

   (d) Find the tax τ that achieves the optimal allocation as an equilibrium allocation. [Hint: The tax simply equates the FOC of the household's problem in the competitive equilibrium with the FOC condition of the planner's problem.]

   (e) Suppose now $\beta = 2$. Does the tax have to increase or decrease the after tax return on savings? Explain your answer.

(f) Suppose now $\omega = 1/4$. Does the tax have to increase or decrease the after tax return on savings? Explain your answer.

2. Consider an overlapping generations economy where people can invest in the education of their children. Their preferences are given by

$$\ln c_t(t) + \beta \ln c_t(t+1) + \gamma \ln e(t)$$

where $e(t)$ is investment of generation t in education. Education investment is productive and yields human capital next period according to

$$h(t+1) = Be(t)^{\theta} h(t)^{1-\theta}.$$

with $h(0)$ being the initial endowment of human capital for generation 0. People have a total effective endowment of labour equal to $h(t)$ which they supply at a wage rate $w(t)$. Their total income is given by $w(t)h(t)$.

Production takes place according to the production function given by

$$F(K_t, L_t) = AK_t^{\alpha} L_t^{1-\alpha}$$

where $L_t = h(t)$. The per-capita capital stock is now expressed as $k_t = K_t/L_t$.

(a) Find the law of motion for human capital in terms of wages $w(t)$.

(b) Find the law of motion for the per-capita capital stock.

(c) Find the steady state per-capita capital stock.

(d) What is the long-run growth rate of human capital?

Suppose now that there are two households, academics a and non-academics b. The former has an initial endowment of human capital given by $h_a(0) = 2$, while the latter has an endowment equal to $h_b(0) = 1$. For parameters, set $\beta = 1$, $A = 1$ and $B = 1$.

(e) Find the steady state per-capita capital stock and wage rate. [Hint: You can use directly the law of motion you found earlier, simply distinguishing between $s_a(t+1)$ and $s_b(t+1)$ taking into account that $L_{t+1} = h_a(t+1) + h_b(t+1)$.]

(f) How is long-run inequality in human capital and consumption related to the initial endowment?

Suppose the economy starts out in steady state you have found above, but one redistributes income in the first period only with an income tax according to $\bar{w}h_a(0)(1-\tau_a)$ and $\bar{w}h_b(0)(1+\tau_b)$.

(g) When is such a policy feasible?

(h) Can such a policy achieve the same level of income, consumption and human capital across households in steady state? If so, what is level of taxes (τ_a, τ_b)?

(i) Briefly discuss how your answer would change if $\gamma_a > \gamma_b$?