Economics 421 **Fall 2015**

Topics in Macroeconomics **Thorsten Koeppl**

Assignment 3

(Due: Friday, November 20: Drop Box until 3:00 pm)

1. Consider the neoclassical growth model with OG where the population size grows over time at rate n and where the size of the initial old generation is normalized to $N_{-1} = 1$. Each generation has an endowment of one unit of labor when young and none when old. The initial old have an endowment of capital given by k_0. Preferences for each generation t are given by

$$u(c_t(t), c_t(t+1)) = \ln c_t(t) + \beta \ln c_t(t+1),$$

with preferences for the initial old given by $u(c_{-1}(0)) = \ln c_{-1}(0)$. The aggregate production function in this economy is given by

$$Y(t) = AN_t^{1-\alpha} K(t)^{\alpha}$$

with full depreciation of capital each period.

(a) Derive the law of motion for the per capita capital stock, i.e. derive $k(t+1)$ as a function of parameters and $k(t)$.

(b) What is the steady state per capita capital stock and the steady state interest rate?

(c) Find the golden rule per capita capital stock and consumption allocation.

(d) At what levels of parameters is there over- and underaccumulation in the steady state? [Hint: Your condition will only depend on α and β.]

2. Consider again the environment of Question 1 except for now there is also a government that roles over a constant level of per capita debt

$$\frac{B(t)}{N_{t-1}} = b(t) = b$$

and levies a lump-sum tax or transfer on the young. The government, however, does not consume anything ($G(t) = 0$) or tax the old ($\tau_2 = 0$).

(a) Write down the government budget constraint in per capita terms for any arbitrary period t and for the steady state.

(b) Households can now save in two assets, capital $K(t+1)$ and bonds $B(t+1)$. In equilibrium aggregate savings have to equal total assets. Write down the relationship between aggregate savings and total assets in per capital terms for any arbitrary period t and for the steady state. [Hint: Derive the demand for savings as a function of total first period income, $w(t) - \tau_1(t)$.]

(c) Find the new steady state per capita level for capital as a function of per capita debt $b(t)$. For what values of debt does the economy achieve the golden rule capital stock k_{GR}? [Hint: Use the results of part (e) and (f) to obtain two equations in the three unknowns (k, b, τ_1).]

3. Consider an economy where there are investors and entrepreneurs. Investors have an endowment of y and entrepreneurs own a technology. The technology requires an input of capital k to produce output according to the production function

$$f(k) = 2\sqrt{k}$$

Investors and entrepreneurs trade capital in a competitive market when young at a gross interest rate R. Investors also have an opportunity to store goods across periods at a gross interest rate δ where

$$\delta \leq \frac{1}{\sqrt{y}}$$

Investors and entrepreneur maximize their consumption (i.e. they are risk-neutral and have linear utility in consumption).

(a) Derive the supply of capital by investors as a function of R.

(b) Derive the demand of capital by entrepreneurs as a function of R.

(c) Find the equilibrium interest rate R and the consumption of entrepreneurs and investors in equilibrium.

Suppose now that entrepreneurs can only pledge a fraction ρ of their returns $f(k)$ as payment for capital. This implies that they face the borrowing constraint $Rk \leq \rho f(k)$.

(d) Set up the maximization problem for entrepreneurs taking into account the borrowing constraint.

(e) Derive the demand for capital by entrepreneurs as a function of R. For which values of ρ will the borrowing constraint be binding? [Hint: Solve the problem for a binding and a non-binding borrowing constraint. Then find the values of ρ for which it must be the case that $f'(k) > R$.]

Set now $y = 1/4$ and $\delta = 1$.

(f) What is the equilibrium interest rate for (i) $\rho = 1/2$, (ii) $\rho = 1/4$ and $\rho = 1/8$? Plot the interest rate, output and consumption for investors and entrepreneurs for $\rho \in [0, 1/2]$. [Hint: Don't forget that we $R \geq \delta$ for positive investment. For low values of ρ you will get that investment will decline and storage is being used.]

(g) Suppose now $\rho = 1/8$. Consider a government or central bank that can lower the return on storage to $\delta < 1$. Find the value of δ such that in equilibrium $k = 1/4$. What is the equilibrium interest rate R for this case? Compare output and the consumption level of investors and entrepreneurs to your results in part (f).

<u>Bonus:</u> Show that a decrease in δ will increase output whenever the borrowing constraint is binding.