Economics 421 **Fall 2015**

Topics in Macroeconomics **Thorsten Koeppl**

Assignment 2

(Due: Tuesday, October 27: Drop Box until 3:00 pm)

1. Consider the following version of the OG environment where the population size grows over time at rate $n > 1$ with the size of the initial old generation being normalized to $N_{-1} = 1$. The initial old have an endowment of $M_0 = 1$ units of money. All other generations have an endowment of $y = 1$ of the single consumption good when young and no endowment when old. Preferences for each generation t are given by

$$u(c_t(t), c_t(t+1)) = \ln c_t(t) + \ln c_t(t+1),$$

with preferences for the initial old given by $u(c_{-1}(0)) = \ln c_{-1}(0)$.

The money stock grows over time at a given rate μ. Assume that every period the new money created is spread equally among the members of the old generation.

(a) Find the stationary perfect foresight monetary equilibrium, i.e., the stationary allocation, (c_1, c_2), the initial price p_0 and price sequence p_t in terms of μ and n.

(b) Suppose now the rate of increase in the money stock doubles and money grows at rate 2μ. Does the stationary equilibrium (i.e., the allocation and the price sequence) change? Compare your answer to a case where money still grows at rate μ, but only the initial stock of money doubles, i.e. $M_0 = 2$.

(c) Suppose the central bank targets inflation and chooses μ such as to keep prices fixed. Calculate the utility of each generation under this monetary growth rule. Compare it to the "optimal monetary growth rule" $\mu = 1$ that maximizes utility of every generation except the initial old. Interpret your answer.

(d) Set now $n = 1$, i.e. the population stays constant across time. The central bank announces that it will run a money growth rule with $\mu = 2$. The initial old generation hires a long-lived shaman that organizes intergenerational transfers. The shaman maximizes the profits he obtains from the transfer scheme each period. These are given by

$$\Pi = N_t(y - c_1 - c_2).$$

Find the transfer scheme a self-interested shaman will offer. Is the initial old generation better off with this schedule? [Hint: The shaman will maximize his profits subject to the constraint that the consumption profile (c_1, c_2) he offers to each generation yields exactly the same utility as the perfect foresight equilibrium with money and $\mu = 2$. His profits are simply total endowment minus consumption for the young and the old.]

2. Consider an OG environment with preferences equal to

$$u(c_t(t), c_t(t+1)) = \sqrt{c_t(t)} + \sqrt{c_t(t+1)},$$

for all generations except the initial old whose preferences are given by $u(c_{-1}(0)) = \sqrt{c_{-1}(0)}$. Assume that the population stays constant over time, i.e. $n = 1$ and assume that the size of population is given by $N_t = N_{-1} = 2$. Each member of a generation has an endowment of 1 when young and none when old.

A government has to finance the building of an ever-increasing number of pagodas. The only way for the government to finance these pagodas is by printing money and purchasing goods each period on a competitive market for price p_t and turning them 1-1 into pagodas denoted by G. Let $\mu \in [1, \infty)$ be the money growth rate.

(a) Find the stationary perfect foresight equilibrium, allocation (c_1, c_2) and prices p_t, in terms of the money growth rate μ.

(b) Plot government expenditures G as a function of μ. [Hint: Use a well labelled Excel spreadsheet to do so.]

(c) Find the growth rate of money μ that maximizes the number of pagodas a government can finance.

(d) Suppose now that households expect that prices stay constant. Find the temporary equilibrium as a function of the money growth rate μ.

(e) Plot again the number of pagodas the government can finance. How does it compare to your answer in part (c)?

3. Consider an OG environment with preferences equal to

$$u(c_t(t), c_t(t+1)) = \ln c_t(t) + \ln c_t(t+1),$$

for all generations except the initial old whose preferences are given by $u(c_{-1}(0)) = \ln c_{-1}(0)$. Assume that the population doubles over time, i.e. $n = 2$ and normalize $N_{-1} = 1$. Each member of a generation has an endowment of $y_1 = 2$ when young and $y_2 = 1$ when old. There is an initial stock of debt equal to b_{-1} owned by the initial old generation.

(a) Write down the budget constraint for the government using the interest rate $r(t)$ on debt it issues in period t.

(b) Set up the household's decision problem and find the FOC in terms of the interest rate $r(t)$ on debt.

(c) Find a stationary (in consumption) perfect foresight equilibrium. What is the initial amount of debt b_{-1}^{SS}, the sequence of debt levels and the sequence of interest rates associated with this equilibrium?

(d) Find an equivalent lump-sum tax scheme that yields the same stationary perfect foresight equilibrium.

(e) Use an excel program to plot the evolution of debt and interest rates when $b_{-1} = 0.99b_{-1}^{SS}$. Does such a policy enhance welfare for all generations? [Hint: Use the iterative procedure described in class.]

(f) Is a debt policy feasible with $b_{-1} = 1.01b_{-1}^{SS}$? Explain your answer. [Hint: You may again use the iterative procedure described in class.]