Economics 421 **Fall 2015**

Topics in Macroeconomics **Thorsten Koeppl**

Assignment 1

(Due: Tuesday, October 6 – Drop Box by 4pm)

1. Each period a generation with size N_t is born, where the population grows (shrinks) at a constant rate $n > 1$ ($n < 1$). All generations have an endowment of y of the single consumption good when young and no endowment when old. Preferences for each generation t are given by

$$u(c_t(t), c_t(t+1)) = \sqrt{c_t(t)} + \sqrt{c_t(t+1)},$$

with preferences for the initial old given by $u(c_{-1}(0)) = \sqrt{c_{-1}(0)}$.

(a) Derive the set of feasible allocations for this economy in terms of the population growth rate n. Draw a diagram properly labelled showing the feasible set.

Set now $n = 2$, i.e. the population doubles each generation.

(b) Show that the stationary allocation $(c_1, c_2) = (\frac{3}{4}y, \frac{1}{2}y)$ is **not** Pareto optimal. [Hint: Find a feasible allocation that pareto-dominates this allocation.]

(c) Let $n = 2$ and set $y = 1$. Find **all** stationary Pareto optimal allocations for this economy.

(d) Find a stationary transfer scheme (τ_1, τ_2) that achieves the best allocation for all generations except the initial old.

Suppose now that the young have access to a storage technology that yields a gross return $r > 0$; i.e., if 1 unit of the good is invested today, it yields r units tomorrow.

(e) In the absence of transfers, find the optimal storage of resources by each generation. For which values of r do all generations except the initial old prefer storage over the transfer scheme that you have found in part (d)?

(f) Suppose the transfer scheme lasts only for T periods. How does your anwser to part (e) change, if this is publicly announced? What if noone anticipates that the scheme ends at T? Explain your answer.

2. Suppose the population size is constant over time and normalized to 1, i.e. $N_t = 1$ for all t. The initial old have an endowment of M units of money. All members of the other generations have an endowment of y of the single consumption good when young and no endowment when old. Preferences for each generation t are given by

$$u(c_t(t), c_t(t+1)) = \ln c_t(t) + \beta \ln c_t(t+1),$$

with preferences for the initial old given by $u(c_{-1}(0)) = \ln c_{-1}(0)$ and $\beta \in (0, 1]$.

(a) Define a stationary (monetary) equilibrium with perfect foresight.

(b) Derive the savings function (or real balances) $\frac{m_t}{p_t}$ in terms of y and β for a stationary equilibrium.

(c) Find the stationary equilibrium.

(d) Suppose now that the endowment of money for the initial old doubles to $2M$. What are the effects on equilibrium prices and equilibrium allocations? Interpret your results.

3. Suppose the population stays constant at $N_t = 2$ for all t and the initial old generation has a total endowment of M units of money. All other generations have a total endowment of Y of the single consumption good when young and no endowment when old. Preferences for each generation t are given by

$$u(c_t(t), c_t(t+1)) = \sqrt{c_t(t)} + \beta\sqrt{c_t(t+1)},$$

with preferences for the initial old given by $u(c_{-1}(0)) = \sqrt{c_{-1}(0)}$ and $\beta \in (0, 1]$.

(a) What are the individual endowments of m_{-1} for the old generation and y for the young generation?

(b) Find a stationary equilibrium with perfect foresight.

Suppose now that people in generation t expect the price level to change by $(a-1)$ in the next period; i.e., $p^e_{t+1} = ap_t$ for all t, where $a \in (0, \infty)$.

(c) Find generation t's consumption plan $(c_t(t), c^e_t(t+1))$ as a function of prices p_t and expected prices p^e_{t+1}. [Hint: Use the MRS and the budget constraint given p^e_{t+1}.]

(d) Find generation t's consumption when old in terms of prices p_{t+1}.

(e) Find the temporary equilibrium. [Hint: Use market clearing in period t to solve for p_t.]

(f) Compare consumption, the savings function and welfare in the temporary and perfect foresight equilibrium for the cases $a = 1$, $a > 1$ and $a < 1$. Interpret your results by looking at the differences in expectations about future prices.