ECON 421

Investment and Growth

Fall 2015

Production

Aggregate production function:

$$Y(t) = A(t)K(t)^{\alpha}N_D(t)^{1-\alpha}$$

- A – TFP
- K – capital
- N_D – labour input

In per-capita terms:

$$y(t) = A(t)k(t)^{\alpha}$$

Capital:

- fully depreciates every period
- needs one period to come online (investment)

Young:

- one unit of labour
- inelastically supplied
- wage $w(t)$
- consume and invest into capital goods

Old:

- cannot work
- sell capital to finance consumption

Three markets:

- Labor market
- Goods market
- Rental market for capital

Firms

Representative firm runs production to maximize profits.

Firm hires workers and rents capital from the old.

Perfect competition implies that firm takes prices as given:

$$\max_{K(t),N_D(t)} A(t)K(t)^{\alpha}N_D(t)^{1-\alpha} - r(t)K(t) - w(t)N^s(t)$$

Factors are paid their marginal product (zero profits):

$$\begin{aligned} w(t) &= (1-\alpha)A(t)k(t)^{\alpha} \\ r(t) &= \alpha A(t)k(t)^{\alpha-1} \end{aligned}$$

Wages and interest rates depend on the marginal product of per capita capital.

Young's problem:

$$\max_{c_t(t),c_t^e(t+1),k(t)} u(c_t(t)) + \beta u(c_t^e(t+1))$$

$$\text{subject to}$$

$$c_t(t) + s(t+1) = w(t)$$

$$c_t^e(t+1) = r^e(t+1)s(t+1)$$

When old:

$$c_t(t+1) = r(t+1)s(t+1)$$

With perfect foresight ($r^e(t+1) = r(t+1)$), the household's choice is just governed by an intertemporal budget constraint:

$$c_t(t) + \frac{c_t(t+1)}{r(t+1)} = w(t)$$

Equilibrium

Definition: A perfect foresight equilibrium is given by an allocation $\{c_t(t), c_t(t+1), s(t+1), k(t)\}_{t=0}^{\infty}$ and prices $\{(w(t), r(t+1)\}_{t=0}^{\infty}$ such that

(i) households taking prices as given choose $(c_t(t), c_t(t+1), s(t+1))$ to maximize utility

(ii) firms taking prices as given choose $K(t)$ and $N_D(t)$ to maximize profits

(iii) markets clear, i.e.

$$\begin{aligned} Y(t) &= N_t c_t(t) + N_{t-1} c_{t-1}(t) + N_t s(t+1) \\ K(t) &= N_{t-1} s(t) \\ N_D(t) &= N_t \end{aligned}$$

Growth

How can we get growth here?

- savings show up in tomorrow's capital stock
- savings depend on wage earnings
- wage earnings increase with the total capital stock
- transition to a steady state over time

Dynamic system with capital as a state variable with convergence to a steady state.

Exogenous sources of growth:

- population $N_t = nN_{t-1}$
- technology: $A(t) = (1 + \gamma)A(t - 1)$

Law of Motion for Capital

Savings-function in period t is given by

$$s(t+1) = w(t) - c_t(t)$$

or

$$s(t+1) = s\left(w(t), r(t+1)\right)$$

which depends on $k(t)$.

Capital stock in $t+1$:

$$k(t+1) = \frac{K(t+1)}{N_{t+1}} = \frac{N_t s(t+1)}{N_{t+1}} = \frac{1}{n} s\left(w(t), r(t+1)\right)$$

There will be a transition to a steady state where

$$\bar{k} = \frac{1}{n} s\left(w(\bar{k}), r(\bar{k})\right)$$

Example:

Take log utility.

Savings are a constant fraction of wage earnings.

$$c_t(t) = \frac{1}{1+\beta} w(t)$$

Hence:

$$\begin{aligned} k(t+1) &= \frac{1}{n} s(t+1) \\ &= \frac{1}{n} \frac{\beta}{1+\beta} w(t) \\ &= \frac{1}{n} \frac{\beta}{1+\beta} (1-\alpha) A(t) k(t)^{\alpha} \end{aligned}$$

We obtain a first-order difference equation for capital accumulation over time.