ECON 421

Limits on Public Debt

Fall 2015

Does the Timing of Debt and Taxes matter?

Consider a feasible gov't policy $(b(t), \tau_1(t), \tau_2(t))$

$$\frac{1}{n}b(t-1) = \frac{b(t)}{1+r(t)} + \tau_1(t) + \frac{1}{n}\tau_2(t)$$

where $r(t)$ are equilibrium interest rates.

Change the debt level and alter taxes for generation t according to

- $\hat{b}(t) \neq b(t)$
- $\hat{\tau}_1(t) = \tau_1(t) + \frac{b(t)-\hat{b}(t)}{1+r(t)}$
- $\hat{\tau}_2(t+1) = \tau_2(t+1) - \Big(b(t) - \hat{b}(t)\Big)$

The interest rate $r(t)$ is the one associated with equilibrium under the original feasible gov't policy.

Given the original interest rate $r(t)$, the new policy is feasible since

$$\begin{aligned}\frac{1}{n}b(t-1) &= \frac{\hat{b}(t)}{1+r(t)} + \tau_1(t) + \frac{b(t)-\hat{b}(t)}{1+r(t)} + \frac{1}{n}\tau_2(t) \\ \frac{1}{n}\hat{b}(t) &= \frac{b(t+1)}{1+r(t+1)} + \tau_1(t+1) + \frac{1}{n}\left(\tau_2(t+1) - b(t) + \hat{b}(t)\right)\end{aligned}$$

Neither the FONC, nor the NPV of taxes changes for generation t.

The equilibrium thus remains unchanged with the policy change.

Ricardian Equivalence:

The timing of taxes does not matter, **unless** the time path of debt is altered so that there is redistribution between different generations.

Gov't Intertemporal Budget Constraint

Gov't Spending must equal revenue.

$$G_t + (1 + r_t)B_{t-1} = T_t + \frac{M_t - M_{t-1}}{p_t} + B_t$$

In per capital terms:

$$\begin{aligned} N_t g_t + (1 + r_t)N_{t-1}b_{t-1} &= N_t \tau_t + \frac{N_t m_t}{p_t}\left(1 - \frac{1}{\mu}\right) + N_t b_t \\ g_t + \left(\frac{1 + r_t(b_{t-1})}{n}\right) b_{t-1} &= \tau_t + (y - c_1(\mu_t))\left(1 - \frac{1}{\mu_t}\right) + b_t \end{aligned}$$

where

- $\frac{\partial r_t}{\partial b_{t-1}} > 0$
- $\frac{\partial c_1}{\partial \mu_t} > 0$

Consider this constraint every period and substitute out debt.

$$g_0 + \left(\frac{1+r_0}{n}\right) b_{-1} = \tau_0 + (y - c_1)\left(1 - \frac{1}{\mu_0}\right) + b_0$$

$$g_0 + \left(\frac{1+r_0}{n}\right) b_{-1} = \tau_0 + (y - c_1)\left(1 - \frac{1}{\mu_0}\right) + \left(\tau_1 + (y - c_1)\left(1 - \frac{1}{\mu_1}\right) + b_1 - g_1\right)\left(\frac{n}{1+r_1}\right)$$

$$b_{-1}\left(\frac{1+r}{n}\right) + g\left(1 + \left(\frac{n}{1+r}\right) + \left(\frac{n}{1+r}\right)^2 + \ldots\right) = \left(\tau + (y - c_1)\left(1 - \frac{1}{\mu}\right)\right)\left(1 + \left(\frac{n}{1+r}\right) + \left(\frac{n}{1+r}\right)^2 + \ldots\right) + \lim_{t\to\infty}\left(\frac{n}{1+r}\right)^t b_t$$

where we have assumed that all variables except debt stay constant.

We need to impose a No-Ponzi Game Condition

$$\lim_{t\to\infty}\left(\frac{n}{1+r}\right)^t b_t \leq 0$$

which holds for $n < 1 + r$ as long as b_t is bounded.

Intertemporal budget constraint then becomes

$$g \leq \tau + (y - c_1)\left(1 - \frac{1}{\mu}\right) + b_{-1}\left(1 - \frac{1+r}{n}\right)$$

Note that the last term is negative:

- if $b_{-1} < 0$, the government has initial assets
- if $b_{-1} > 0$, the government has initial debt

Some Unpleasant Arithmetic

Scenario 1: Independent Central Bank

- μ is fixed
- Suppose g increases permanently. It must be financed by taxes.

Scenario 2: Permanent Budget Deficit

- $\Delta = g - \tau$
- Need to finance it via $\mu > 0$.

Scenario 3: Temporary Spending Increase

- $g_1 = (1 + \Delta)g$
- Promise to never increase taxes or inflation is not credible if $1 + r > n$.

Debt Crises and Primary Surpluses

What spending restraints are necessary to avoid gov't default?

Normalize the gov't budget constraint by GDP

$$\begin{aligned}\frac{B_{t+1}}{Y_{t+1}} &= \frac{B_t}{Y_t}\left(\frac{1+r}{1+\gamma}\right) - \frac{T-G}{Y_t(1+\gamma)} \\ b_{t+1} &= b_t\left(\frac{1+r}{1+\gamma}\right) - \frac{s}{1+\gamma}\end{aligned}$$

where

- $b_{t+1} = \frac{B_{t+1}}{Y_{t+1}}$ is the future Debt-to-GDP ratio
- s is today's primary surplus
- γ is the growth rate of GDP

We can iterate backwards to obtain a solution to this first-order difference equation

$$b_t = \left(\frac{1+r}{1+\gamma}\right)^t b_0 + \frac{s}{1+\gamma}\sum_{j=0}^{t-1}\left(\frac{1+r}{1+\gamma}\right)^j$$

or

$$b_t - \frac{s}{r-\gamma} = \left(\frac{1+r}{1+\gamma}\right)^t \left(b_0 - \frac{s}{r-\gamma}\right).$$

What matters is

- growth rate γ relative to the real interest rate r
- primary surplus s relative to initial debt position b_0

If $r > \gamma$, one needs $s \geq b_0(r-\gamma)$ to stabilize debt.

One needs to require a primary surplus that is increasing in (i) the initial debt and (ii) interest rates net of growth.

Some remarks:

1) Increases in interest rates can lead to unsustainable debt levels.

2) Recessions tend to worsen debt problems.

3) Primary surpluses are limited by political concerns and limits on tax revenue.

4) There can be feedback effects from primary surpluses to the growth rate.

5) The interest rate on external, non-domestically denominated debt depends on exchange rate movements.