ECON 421

Money and Equilibrium

Winter 2015

Introducing Money

Endowments:

- y units of goods when young
- no endowment of goods when old

Need to transfer wealth into the future implies the need for an asset.

Money:

- useless object
- initial old have m_{-1} units per capita
- fixed aggregate supply: $M_{-1} = m_{-1}N_{-1}$

Aggregate demand for money given by

$$M_t = m_t N_t$$

Monetary Exchange

Suppose the old have acquired m_t units of money when young.

Generation t's problem when old:

$$\max_{c_t(t+1)} c_t(t+1)$$
subject to
$$p(t+1)c_t(t+1) \leq m_t$$

Solution:

$$c_t(t+1) = \frac{m_t}{p(t+1)}$$

Generation t's problem when young:

$$\max_{c_t(t), c_t^e(t+1), m_t} u(c_t(t)) + \beta u(c_t^e(t+1))$$

subject to

$$p(t)c_t(t) + m_t \leq p(t)y$$
$$p^e(t+1)c_t^e(t+1) \leq m_t$$

Take current prices and expected (forecast) future prices as given.

Money allows one to save for the future.

Return on savings depends on $p^e(t+1)/p(t)$ (expected inflation).

Savings Function

FOC when young:

$$\begin{aligned}
\frac{\beta u'(c_t^e(t+1)}{u'(c_t(t))} &= \frac{p^e(t+1)}{p(t)} \equiv \pi^e(t) \\
\frac{m_t}{p(t)} &= y - c_t(t) \\
c_t^e(t+1) &= \frac{m_t}{p^e(t+1)}
\end{aligned}$$

Savings-function:

$$\frac{m_t}{p(t)} = s\left(\frac{p^e(t+1)}{p(t)}, y\right)$$

What matters is wealth (y) and the **expected** inflation $(\pi^e(t))$.

Role of Expectations

Two different equilibrium concepts (imply different savings behavior):

- **temporary** equilibrium

$$p^e(t+1) \text{ given}$$

- **perfect foresight** equilibrium

$$p^e(t+1) = p(t+1) \text{ and } c_t^e(t+1) = c_t(t+1).$$

Key difference:

- temporary equilibria have exogenous dynamics
- perfect foresight equilibria have endogenous dynamics

This is linked to the so-called “Lucas”–critique.

Perfect Foresight Equilibrium

Definition: A perfect foresight equilibrium is given by a sequence of price $\{p(t)\}_{t=0}^{\infty}$ and allocations $(c_{-1}(0), \{(c_t(t), c_t(t+1))\}_{t=0}^{\infty})$ and money demands $\{m_t\}_{t=0}^{\infty}$ such that

(i) taking prices and expectations ($p^e(t+1) = p(t+1)$) as given, $(c_t(t), c_t(t+1))$ solves generation t's problem

(ii) markets clear

$$\begin{aligned} N_t c_t(t) + N_{t-1} c_{t-1}(t) &= N_t y \\ N_{t-1} m_{t-1} = M_{t-1} &= N_t m_t \end{aligned}$$

Remark: Perfect foresight ensures a life-time budget constraint for the household of the form

$$c_t(t) + \frac{p(t+1)}{p(t)} c_t(t+1) = y$$

How do we find a perfect foresight equilibrium?

Step 1: Solve the HH problem.

$$\begin{aligned} \frac{p(t+1)}{p(t)} &= \frac{\beta u'(c_t(t+1)}{u'(c_t(t))} \\ y &= c_t(t) + \frac{p(t+1)}{p(t)} c_t(t+1) \end{aligned}$$

Step 2: Use market clearing to find prices.

$$\frac{M_{t+1}}{M_t} = \frac{N_{t+1}}{N_t} \frac{p(t+1)}{p(t)} \frac{y - c_{t+1}(t+1)}{y - c_t(t)}$$

Stationarity and a constant money supply ($M_t = M_{t+1}$) implies

$$\begin{aligned} \frac{1}{n} &= \pi \\ \frac{\beta u'(c_2)}{u'(c_1)} &= \pi \\ c_1 + \frac{1}{n} c_2 &= y \end{aligned}$$

Result:

Perfect Foresight equilibrium is Pareto-optimal with a constant money supply.

Why? Slope of the budget set is $-n = -\frac{1}{\pi}$.

Temporary Equilibrium

Definition: A temporary equilibrium **for given expectations** $\{p^e(t+1)\}_{t=0}^{\infty}$ is given by a sequence of price $\{p(t)\}_{t=0}^{\infty}$ and allocations $(c_{-1}(0), \{(c_t(t), c_t(t+1))\}_{t=0}^{\infty})$ and money demands $\{m_t\}_{t=0}^{\infty}$ such that

(i) taking prices and expectations as given, $(c_t(t), c_t^e(t+1))$ solves generation t's problem

(ii) $c_t(t+1) = \frac{m_t}{p(t+1)}$ for all t

(iii) markets clear

$$\begin{aligned} N_t c_t(t) + N_{t-1} c_{t-1}(t) &= N_t y \\ N_{t-1} m_{t-1} M_{t-1} &= N_t m_t \end{aligned}$$

What's different in a temporary equilibrium?

Two state variables (given) in period t:

- m_{t-1}
- $p^e(t+1)$ (exogenous!)

Demand for goods in t are given as a function of $p(t)$:

$$\begin{aligned} c_{t-1}(t) &= \frac{m_{t-1}}{p(t)} \\ \frac{p^e(t+1)}{p(t)} &= \frac{\beta u'(c_t^e(t+1))}{u'(c_t(t))} \\ y &= c_t(t) + \frac{p^e(t+1)}{p(t)} c_t^e(t+1) \end{aligned}$$

Price $p(t)$ needs to clear market:

$$c_t(t) + \frac{1}{n} c_{t-1}(t) = y$$