ECON 421

Long-run Inequality

Fall 2015

Pareto Distribution

CDF:

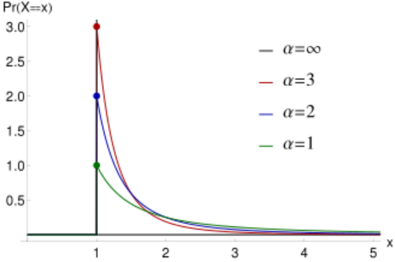
$$F(z) = 1 - \left(\frac{1}{z}\right)^{\alpha}$$

where α is the shape parameter

Tail:

$$\mathcal{P}(x \geq z) = z^{-\alpha}$$

Higher α means thinner tail (inequality $\uparrow$)



A First Pass

Suppose we have that age is exponentially distributed

$$\mathcal{P}[\text{Age} > x] = e^{-\delta x}$$

and income grows exponentially with time

$$y = e^{\mu x}.$$

Inequality of income is given by a **Pareto distribution**.

$$\mathcal{P}[\text{Income} > y] = \mathcal{P}[\text{Age} > x(y)] = \mathcal{P}[\text{Age} > \frac{1}{\mu}\ln y]$$
$$= e^{-\frac{\delta}{\mu}\ln y} = y^{-\frac{\delta}{\mu}}$$

Pareto coefficient $\eta = \frac{\mu}{\delta}(< 1)$ measures inequality.

The top $a\%$ of $b\%$ get a fraction of income for all of b equal to

$$S(a, b) = \left(\frac{a}{b}\right)^{\eta - 1}$$

A (Partial) Theory of Wealth Inequality

Population evolves according to

$$\dot{N}_t = B_t - \delta N_t$$

where B_t is births and δ is the death rate.

Hence:

$$n + \delta = \frac{B_t}{N_t} = b$$

where b is the birth rate.

This implies a stationary distribution

$$\mathcal{P}[\text{Age} > x] = e^{-(n+\delta)x}$$

that depends on the birth rate b.

We assume that people

- earn rate of return r
- consume a fraction α
- pay a fraction τ as deadweight (!) taxes

of their wealth a.

This implies that

$$\dot{a} = (r - \tau - \alpha)a$$

Wealth at t of a person born in $t - x$ (age x) is given by

$$a_t(x) = a_{t-x}(0)e^{(r-\tau-\alpha)x}$$

Assuming that newborns receive identical inheritances and that per capita wealth w_t grows at rate g, we have that

$$a_{t-x}(0) = \frac{\delta}{\delta + n}\frac{W_{t-x}}{N_{t-x}} = \bar{a}w_t e^{-gx}$$

The wealth distribution at time t is given by

$$\begin{aligned} \mathcal{P}[\text{Wealth} > a] &= \mathcal{P}[\text{Age} > x(a)] \\ &= e^{-(n+\delta)x(a_t)} \\ &= \left(\frac{a}{\bar{a}w_t}\right)^{-\frac{n+\delta}{r-g-\tau-\alpha}} \end{aligned}$$

where $n + \delta$ is the birth rate in the economy.

Pareto coefficient is given by

$$\eta = \frac{r - g - \tau - \alpha}{n + \delta}$$

Conclusion:

The Pareto coefficient (inequality) rises with $r - g$ and falls with n.

General Equilibrium

Production function is given by

$$Y_t = A_t K_t$$

where accumulates according to

$$\dot{K}_t = Y_t - C_t - T_t - \delta K_t = Y_t - (\alpha + \tau + \delta) K_t$$

Hence, in a balanced growth path ($g_Y = g_K$) we have with $r = A - \delta$

$$g_K = A - (\alpha + \tau + \delta) = r - \tau - \alpha.$$

The per capita capital stock and output then grow at rate

$$g = \frac{\dot{k}_t}{k_t} = \frac{\dot{K}_t}{N_t k_t} - \frac{K_t}{N_t k_t} n = g_K - n$$

Hence: $r - g - \tau - \alpha = n$.

Conclusion:

The Pareto coefficient is given by $\eta = \frac{n}{n+\delta}$ and is independent of taxes and $r - g$.

How can we interpret this result?

Recall that in the OG environment we have

$$r - \tau - (n + \tilde{\gamma}) = r - \tau - g = \text{const.}$$

where the constant depends on technology and preferences.

1) We have $r > g$, but this doesn't matter for inequality at all.

2) Wealth taxes and equal inheritances do not matter fundamentally for inequality. Of course, existing distortions could have an influence.

3) Population growth has different effects depending on partial vs. general equilibrium view.