ECON 421

Inequality II

Insurance vs. Incentives

Fall 2015

Is correcting outcomes feasible?

Suppose that inequality in outcomes is purely an outcome of luck.

Ideally, we would like to insure against such outcomes.

Problem 1:

We need **enforcement** of the insurance scheme ex-post as people have an incentive to leave.

Problem 2:

We need **information** which people got lucky to implement the insurance scheme.

These problems might preclude us from running a (perfect) insurance scheme.

A Model of Heterogenous Productivity

Two people:

- endowment of labour $n_i \in [0, 2]$
- can produce output $y_i = \theta_i n_i$
- heterogeneity $\theta_h > \theta_\ell$
- value consumption and costs of production

Social planner problem:

$$\max \ln c_h - \frac{y_h}{\theta_h} + \ln c_\ell - \frac{y_\ell}{\theta_\ell}$$

subject to

$$c_h + c_\ell = y_h + y_\ell$$

$$y_i \leq 2\theta_i \text{ for } i \in \{h, \ell\}$$

Solution

Planner would like to ensure people against consumption risk.

$$c_h = c_\ell = c.$$

Planner would like to produce efficiently.

$$y_h > y_\ell = 0.$$

Problem:

$$\max 2\ln c - \frac{y_h}{\theta_h}$$

$$\text{subject to}$$

$$2c = y_h$$

Solution:

$$c = \theta_h$$

$$y_h = 2n_h = 2\theta_h$$

Limited Enforcement

Suppose each of the two people can be a high productivity guy with probability 1/2.

Then both would prefer the planner's allocation vs. autarky from an ex-ante point of view.

Why?

$$\frac{1}{2}\Big[(\ln\theta_h - 2) + \ln\theta_h\Big] > \frac{1}{2}\Big[(\ln\theta_h - 1) + (\ln\theta_\ell - 1)\Big]$$

However, ex post the high productivity guy has a strong incentive to stay by himself.

$$\ln\theta_h - 1 > \ln\theta_\ell - 2$$

Conclusion:

One needs to enforce the insurance scheme ex post.

Limited Information

Suppose the planner can only observe output y_i (but not θ_i or n_i).

He offers a contract (c_i, y_i) that needs to extract the underlying private information of the people:

$$\max \ln c_h - \frac{y_h}{\theta_h} + \ln c_\ell - \frac{y_\ell}{\theta_\ell}$$

subject to

$$c_h + c_\ell = y_h + y_\ell$$

$$\ln c_h - \frac{y_h}{\theta_h} \geq \ln c_\ell - \frac{y_\ell}{\theta_h}$$

$$\ln c_\ell - \frac{y_\ell}{\theta_\ell} \geq \ln c_h - \frac{y_h}{\theta_\ell}$$

The last constraints are incentive compatibility constraints that require people to reveal their type which is underlying information.

Rewriting the two constraints, we have

$$\theta_h \ln\left(\frac{c_h}{c_\ell}\right) \geq y_h - y_\ell \geq \theta_\ell \ln\left(\frac{c_h}{c_\ell}\right)$$

with only the first constraint binding.

The **efficient allocation** $y_h = 2\theta_h$, $y_\ell = 0$ and $c_h = c_\ell = \theta_h$ is **not feasible** anymore, since the high type would have an incentive to lie.

We need to increase $\frac{c_h}{c_\ell}$ or decrease $y_h - y_\ell$ or both.

Treating both agents the same

$$y_h = y_\ell = \frac{1}{2}\frac{\theta_h\theta_\ell}{\theta_\ell + \theta_h} = c$$

is feasible, but **not efficient**.

Why?

Shift in output by ϵ from high to low type, keep c_ℓ constant and increase c_h to give the high type the same utility as before.

Claim: We can generate extra resources.

Change in output is given by $y_h - y_\ell = 2\epsilon$.

Change in c_h required to keep utility constant is given by

$$\theta_h \frac{\partial \ln(c_h/c_\ell)}{\partial c_h}\Big|_{c_h=c} dc_h - d(y_h - y_\ell) = 0$$

or

$$dc_h = 2\epsilon \frac{c}{\theta_h} = \epsilon \frac{\theta_\ell}{\theta_h + \theta_\ell} < \epsilon$$

Hence, by increasing output y_h we can generate more resources which we can give to both types in a lump-sum fashion.

Optimal Insurance with Incentives

FOC:

$$\frac{c_\ell}{c_h} = \frac{1 - \mu\theta_h}{1 + \mu\theta_h}$$

$$\frac{1}{c_h}(1 + \mu\theta_h) = \frac{1}{\theta_h} + \mu$$

where μ is the multiplier on the high type's incentive constraint.

$$c_h = \theta_h > c_\ell = \theta_h \left(\frac{3\theta_\ell - \theta_h}{\theta_\ell + \theta_h} \right)$$

$$y_h > y_\ell \geq 0$$

$$y_h + y_\ell = 4\frac{\theta_h\theta_\ell}{\theta_h + \theta_\ell} < 2\theta_h$$

Conclusion:

The necessity to provide incentives reduces total production by the high type and introduces inequality in consumption. This is **constrained efficient**.