ECON 421

Aggregate Demand

Fall 2015

Demand Externality

Population is fixed at $N = 1$ in each generation with the young generation having 1 unit of labour.

Production function:

$$Y = \left(AC_1^{\omega}\right) N$$

where

- C_1 is **aggregate** consumption of the young generation
- aggregate demand externality
- $\omega \in (0, 1)$

There is a storage technology with gross return $r = n = 1$.

- no dynamic inefficiency

Optimal Allocation

An efficient allocation maximizes the utility of a representative generation.

We have

$$\max_{c_1,c_2,s} \log c_1 + \beta \log c_2$$

subject to

$$AC_1^{\omega}N + rs = c_1 + c_2 + s$$

$$C_1 = c_1$$

FOC:

$$\frac{c_2}{\beta c_1} = 1 - \omega A c_1^{\omega-1}$$

The key insight is that the MRS satisfies

$$\frac{u'(c_1)}{\beta u'(c_2)} < 1$$

so that the consumption profile is tilted towards current consumption.

The optimal allocation sets

$$\begin{aligned} c_1 &= \left(A\frac{1+\beta\omega}{1+\beta}\right)^{\frac{1}{1-\omega}} \\ c_2 &= \beta\left(\frac{1-\omega}{1+\beta\omega}\right)c_1 \end{aligned}$$

Note that savings are irrelevant due to $r = n = 1$.

Competitive Equilibrium

The young generation solves the problem

$$\max_{c_1, c_2, s} \log c_1 + \beta \log c_2$$

subject to

$$c_1 + s = w$$

$$c_2 = rs$$

where w is the wage rate.

The first-order condition is given by

$$\frac{c_2}{\beta c_1} = 1$$

Individual demand is given by

$$c_1 = \frac{1}{1+\beta} w$$

$$c_2 = \frac{\beta}{1+\beta} w$$

Firms make zero profits when

$$w = AC_1^{\omega}.$$

We use ndividual demand to determines aggregate demand

$$C_1 = c_1 N$$

which yields total income from work as

$$w = Ac_1^{\omega}.$$

Hence

$$c_1 = \frac{1}{1+\beta} A c_1^{\omega}$$

or

$$c_1 = \left(A \frac{1}{1+\beta} \right)^{\frac{1}{1-\omega}}$$

and

$$c_2 = \beta c_1$$

Comparison

Current aggregate demand is always inefficiently low in equilibrium.

$$c_1^* = \left(A\frac{1+\beta\omega}{1+\beta}\right)^{\frac{1}{1-\omega}} > c_1 = \left(A\frac{1}{1+\beta}\right)^{\frac{1}{1-\omega}}$$

For $\omega \to 1$, the demand externality gets stronger so that the inefficiency becomes larger.

For $\beta \to 0$, the inefficiency disappears.

"Savings" can be too low or too high depending on the income effect that arises from higher output due to the demand externality.

When $\omega \to 0$ ($\omega \to 1$), we have that $c_2^* < c_2$ ($c_2^* > c_2$).

Gov't Policy – Taxes & "Interest Rates"

Gov't policy:

- (Pigouvian) tax on savings: τs
- lump-sum transfer to young generation: T

Gov't budget constraint:

$$T = \tau s$$

Household's problem:

$$\max_{c_1, c_2, s} \log c_1 + \beta \log c_2$$

subject to

$$c_1 + s = w + T$$

$$c_2 = r(1 - \tau)s$$

The first-order condition is then given by

$$\frac{c_2}{\beta c_1} = (1 - \tau)$$

and from the life-time budget constraint

$$c_1 = \frac{1}{1+\beta}(w + T).$$

The gov't budget constraint implies that in equilibrium

$$c_1 = \frac{1}{1+\beta} w \sum_{n=0}^{\infty} \left(\frac{\tau\beta}{1+\beta} \right)^n$$

and using c_1^*, we obtain

$$\tau = \omega \frac{1+\beta}{1+\beta\omega} > 0.$$

Result:

The tax on savings lowers the rate of return on savings (or interest rate) to $(1 - \tau)$. Furthermore, as the propensity to save or externality increases, so does the tax ($\partial\tau/\partial\beta > 0$ and $\partial\tau/\partial\omega > 0$).

Conclusion

1) Demand externalities give a reason for gov't policy that supports aggregate demand.

2) A shift to more savings ...

(even if preferred by households because of higher β)

... calls for an adjustment in interest rates/taxes.

3) The reason is that individual households do not take into account the negative externality that more savings have on income.