

ECON 421
Financial Crises

Fall 2015

A Basic Investment Problem

Population is fixed at $N = 2$ with equal number of investors and entrepreneurs.

Investors:

- ▶ have endowments y when young
- ▶ either rent endowment out to entrepreneurs ...
- ▶ ... or store with gross return δ

Entrepreneurs:

- ▶ own technology
- ▶ return $f(k) = Ak^\alpha$ where $\alpha \in (0, 1)$ when old
- ▶ rent k from investors when young at interest rate R

Consumption takes place only when old.

Capital fully depreciates after one period.

Competitive Capital Markets

Entrepreneurs maximize profits taking interest rate R as given.

$$\max_k f(k) - Rk$$

FOC:

$$f'(k) = R$$

Investors take R as given and supply all funds as long as $R > \delta$.

Assume $f'(y) > \delta$.

In equilibrium, we have

- ▶ $k^* = y$ and $R = f'(y)$
- ▶ investors consume returns: $f'(y)y = \alpha Ay^\alpha$
- ▶ entrepreneurs consume profits: $(1 - \alpha)Ay^\alpha$

ENV I – A Simple Default Problem

Entrepreneurs survive with probability $\rho \in (0, 1)$.

Assume that investors have a well-diversified portfolio of projects so that the average return is ρR per unit of investment.

Entrepreneurs still maximize their expected return

$$\max_k \rho(f(k) - Rk)$$

so that $f'(k) = R$.

But investors only invest in the portfolio as long as

$$\rho R \geq \delta.$$

Result:

For $\delta > \rho f'(y)$, in equilibrium funding to entrepreneur is given by

$\rho f'(k) = \delta$ or $k = \left(\alpha \frac{\rho A}{\delta}\right)^{\frac{1}{1-\alpha}}$ and thus increasing in ρ .

Shocks in ENV I

There is a **lack of supply of funds**.

Suppose ρA falls.

If $\delta > \rho f'(y)$, the fall in output by entrepreneurs is **amplified**.

Why?

- ▶ capital input is reduced due to default risk

This is efficient.

- ▶ return on storage $>$ expected return on investment

Policy?

- ▶ investment can be increased by **reducing** δ , but this is **inefficient**

To see the last point consider a social planner's problem

$$\max_{k \in [0, y]} (y - k)\delta + \rho f(k)$$

The first-order condition again yields

$$\delta = \rho f'(k)$$

Changing δ changes output according to

$$y - k + \left(-\frac{\partial k}{\partial \delta}\right) \delta + \rho f'(k) \frac{\partial k}{\partial \delta} > 0$$

by the envelope theorem (see FOC condition above).

ENV II – Default Incentives

Suppose now that entrepreneurs can decide not to repay.

In that event, they forgive a fraction $\rho \in (0, 1)$ of output.

Interpretation:

- ▶ entrepreneurs can appropriate all gross returns ...
- ▶ ... unless they pledge collateral
- ▶ ρ is the fraction of value that can be pledged

How much funding would an entrepreneur at most receive?

$$\begin{aligned} & \max_k f(k) - Rk \\ & \text{subject to} \\ & Rk \leq \rho f(k) \end{aligned}$$

The constraint is called a borrowing (or collateral) constraint which restricts the demand of capital.

The Impact of Borrowing Constraints

The first-order conditions is given by

$$f'(k) = R(1 + \lambda)$$

where λ is the Lagrange multiplier on the constraint.

The demand for funds is restricted whenever the borrowing constraint binds ($\lambda > 0$).

Hence, there will be excess supply at any $R > \delta$. In equilibrium, we need to have then $R = \delta$.

Result:

- 1) $\lambda = 0$ implies $R = f'(y) > \delta$.
- 2) $\lambda > 0$ implies $f'(k) > R = \delta$.

The critical value for ρ such that the constraint is binding is given by

$$Ry = f'(y)y = \alpha Ay^\alpha = \rho Ay^\alpha$$

or

$$\alpha = \rho.$$

Hence, when $\rho < \alpha$, the constraint is binding and investment is given by

$$Rk = \delta k = \rho Ak^\alpha$$

or

$$k = \left(\frac{\rho A}{\delta} \right)^{\frac{1}{1-\alpha}}.$$

Hence, tighter financial market conditions (low ρ) reduce output.

The marginal return on capital $f'(k)$ exceeds the interest rate $R = \delta$.

Shocks in ENV II

There is a lack of **demand of funds**.

Suppose now ρA falls.

- ▶ A always matters
- ▶ ρ only when borrowing constraint binds

When $\rho < \alpha$, the fall in output by entrepreneurs is again **amplified**.

Why?

- ▶ capital is reduced as borrowing constraint becomes more binding

Policy?

- ▶ investment can be increased by **reducing** δ can be **efficient**
- ▶ why? $f'(k) > R = \delta$
- ▶ but income is being redistributed from investors to entrepreneurs