ECON 421

The OG Model

Queen's University

Winter 2015

Goal of this Course

- What are the effects of public policy on the aggregate economy?
- Consider several areas: fiscal policy, monetary policy, social security, environmental policy, etc.
- Why is there a need for public policy in the first place?
- What are the limits of these policies?

Issue: need a model where policy plays a role.

For the most part we will rely on the **Overlapping Generations Model**.

The OG Model

Time $t = 0, 1, 2, \ldots$

N_t people are born of generation t.

Alive for two periods, t and $t + 1$.

	$t = 0$	$t = 1$	$t = 2$	$t = 3$
Gen. $-$	X			
Gen. 0	X	X		
Gen. 1		X	X	
Gen. 2			X	X
Gen. 3				X

Endowments:

- $y_t(t)$ when young
- $y_t(t+1)$ when old

Consumption:

- $c_t(t)$ when young
- $c_t(t+1)$ when old

Preferences:

- $u(c_t(t), c_t(t+1))$

Initial generation:

- N_{-1} people with $y_{-1}(0)$ endowment
- $u(c_{-1}(0))$ preferences

Assumptions:

- $N_t = n^t N_0$
- either $u(c_t(t)) + \beta u(c_t(t+1))$, where $\beta \in (0,1]$ ...
- ... or $\lambda u(c_t(t)) + (1-\lambda) u(c_t(t+1))$, where $\lambda \in [0,1]$
- $u' > 0$ and $u'' < 0$

There is a **private incentive** to trade over time.

But one needs a technology to do so.

There is a **social incentive** to trade over time.

But one needs coordination to do so.

Pareto Optimal Allocations

Definition: An allocation is Pareto-optimal if (i) it is feasible and (ii) there is no other allocation that makes everybody at least well off and someone better off (pareto-dominates).

Allocation: $c = (c_1(0), (c_0(0), c_0(1)), (c_1(1), c_1(2)), \dots)$

Stationary Allocation:

- independent of time t
- same across all generations
- $c_t(t) = c_1$ and $c_t(t+1) = c_2$ for all t

Feasible Allocation:

$$N_t c_t(t) + N_{t-1} c_{t-1}(t) \leq N_t y_t(t) + N_{t-1} y_{t-1}(t) \text{ for all } t$$

With stationarity:

$$c_1 + \frac{c_2}{n} = y_1 + \frac{y_2}{n}$$

Allocation $\tilde{c}$ dominates allocation c if and only if

$$\begin{aligned} u(\tilde{c}_t(t)) + \beta u(\tilde{c}_t(t+1)) &\geq u(c_t(t)) + \beta u(c_t(t+1)) \text{ for all } t \\ u(\tilde{c}_t(t)) + \beta u(\tilde{c}_t(t+1)) &> u(c_t(t)) + \beta u(c_t(t+1)) \text{ for some } t \end{aligned}$$

Important! The initial old generation matters too

$$\tilde{c}_{-1}(0) \geq c_{-1}(0)$$

The Set of Feasible Allocation

$$
\begin{aligned}
N_t c_t(t) + N_{t-1} c_{t-1}(t) &\leq N_t y_t(t) + N_{t-1} y_{t-1}(t) \equiv Y_t \\
N_{t+1} c_{t+1}(t+1) + N_t c_t(t+1) &\leq N_{t+1} y_{t+1}(t+1) + N_t y_t(t+1) \equiv Y_{t+1}
\end{aligned}
$$

With stationary endowments we have

$$Y_{t+1} = nY_t$$

The set of per-capita (gen. t) feasible allocations is then constant across time

$$c_t(t) + \frac{c_{t-1}(t)}{n} = \frac{Y_t}{N_t} \equiv y$$

With stationary allocations, the frontier of the set of feasible allocation is given by

$$c_2 = n\left(y - c_1\right)$$

Indifference Curves

Initial old generation only likes period 2 consumption.

Other generations, fix utility level $u(c_t(t), c_t(t+1)) = \bar{u}$.

Slope of indifference curves are given by

$$u_1' dc_t(t) + u_2' dc_t(t+1) = 0$$
$$\frac{dc_t(t+1)}{dc_t(t)} = -\frac{u_1'}{u_2'}$$

For time-separable utility function and stationarity this yields

$$\frac{dc_2}{dc_1} = -\frac{u'(c_1)}{\beta u'(c_2)}$$

Conditions for Pareto-Optimality

(i) Allocation needs to be on the boundary of the feasible set.

(ii) MRS = MRT

Here:

$$
\begin{aligned}
c_t(t+1) &= n\left(y_t - c_t(t)\right) \\
-\frac{u'(c_t(t))}{\beta u'(c_t(t+1))} &\leq -n
\end{aligned}
$$

ADD GRAPH

How can we compute the stationary PO allocation?

$$\max_{c_1, c_2} u(c_1) + \beta u(c_2)$$

subject to

$$c_1 + \frac{c_2}{n} = y$$

Lagrangian/FOC:

$$\begin{aligned} u'(c_1) - \lambda &= 0 \\ \beta u'(c_2) - \frac{\lambda}{n} &= 0 \\ c_1 + \frac{c_2}{n} = y & \end{aligned}$$

Hence:

$$\begin{aligned} \frac{u'(c_1)}{\beta u'(c_2)} &= n \\ c_1 &= n(y - c_2) \end{aligned}$$