AN AGENCY MODEL OF WELFARE AND DISABILITY ASSISTANCE

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ABSTRACT

Welfare programs including disability benefits have been considered as an efficient way of delivering transfers to the needy. This paper addresses the importance of administrative cost of welfare systems by focusing on an agency problem arising between the government and social workers, whose job is to use tagging to determine eligibility for welfare benefits. Tagging is imperfect in that it involves type I and II errors, and its accuracy depends on the effort of the social workers, which is private information. To induce positive effort, the government needs to monitor the social workers using a costly auditing procedure. Using the framework of optimal non-linear income taxation, we characterize second-best redistribution policies when the government can operate a welfare program alongside a negative income tax. The welfare program contains a disability benefit and a general welfare component. Welfare applicants that are tagged receive the disability benefit, while those who are untagged are accepted for general welfare benefits. It is emphasized that whether or not general welfare recipients, who may be the able or disabled, should be allowed to work is substantially affected by the nature of the administration cost of welfare programs and by indivisibilities that may exist in labor supply. Our ultimate objective is to contribute to the debate concerning transferring income to the poor using welfare programs versus negative income tax systems. This involves trading off the costs of administering welfare relative to the benefits of tagging.

KEY WORDS: tagging, welfare programs, disability benefit, social worker

JEL CLASSIFICATION: H2, I3

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I. INTRODUCTION

Financial assistance to the needy is typically delivered in two — via a negative income tax or via a welfare system. Many countries combine elements of the two systems as part of their overall programs of assisting the poor. While much of the literature on optimal redistribution focuses on negative income taxation (Mirrlees, 1971; Stiglitz, 1982), less attention has been devoted to optimal welfare programs. The optimal design of negative income tax and welfare systems are likely to differ because of the different institutions that deliver them. Negative income taxes are administered by the income tax authorities who rely on self-reporting by taxpayers (or transfer recipients) and on monitoring by random audits using criminal sanctions as a penalty. On the other hand, welfare systems are administered by agencies employing social workers who are responsible for determining which applicant are eligible for which types of benefits. Monitoring typically takes place administratively rather than through the legal system.

The purpose of this paper is to construct a simple model of the design of a welfare program that combines the standard optimal non-linear income tax framework with the administrative features of welfare systems. One such feature is that the eligibility of applicants is typically assessed on the basis of personal characteristics rather than being solely dependent on reported income as in negative income systems. The relevant characteristic will be taken to be disability status, but others can be imagined, such as state of health, employability, or asset wealth. The process of determining individual eligibility has been called ‘tagging’ by Akerlof (1978), a process which is imperfect. Two types of tagging errors can occur (Parsons, 1996): some of the needy may be regarded as not being eligible, while some of persons pretending to be eligible may be tagged. Despite this imperfection, the use of tagging does improve the information available to the government and thereby improves the ability to target transfers to those who need them most. To use the terminology of the literature on optimal redistribution, tagging relaxes the self-selection constraint which restricts the extent of redistribution to the disabled and thereby enhances the efficiency of redistribution.

In much of the previous work on tagging (Akerlof, 1978; Diamond and Sheshinski, 1995; Parsons, 1996), the accuracy of the tag, and therefore the probability of errors, has been taken as given. But the process of tagging is implemented by welfare agencies employing social workers, and we would expect that the accuracy of tagging depends on the

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1 Assistance to the poor may also include various forms of in-kind transfers. See, for example, the summary in van de Walle and Nead (1995). We concentrate on cash transfers in this paper.
amount of resources, including effort, devoted to it. The social workers obviously play an important role, since they are the ones who observe and apply the tag. But the motivation of social workers may be different from the government, and their actions are private information. This gives rise to a standard agency problem that contributes to the cost of administering welfare programs: the government needs to monitor the social workers through costly auditing to manipulate their incentives. The costs of administration must be set against the benefits of tagging to determine whether welfare systems are cost-effective means of delivering transfers to the needy.

This paper addresses the implications of there being agency costs of administering a welfare program, and analyzes how the structure of the welfare program is affected by its being alongside a redistributive income tax system. The welfare program we consider is one which contains two components, a disability benefit and a general welfare program. Tagging by social workers determines eligibility for the former. Applicants regarded as not being eligible for the disability benefit are allowed to participate in the general welfare program. Those who do not apply remain in the income tax system. The disability component is intended to target the disabled, and the general welfare program those who are not well-off for other reasons, in our case because they have low earnings potential.

The scheme described here is similar to that in many countries, including Canada, the one we are most familiar with. In the Canadian system, there is a welfare system administered by the provinces which exists alongside the progressive income tax system of the federal government. Although the latter has a system of refundable credits associated with it, for the most part those who rely on transfers obtain them from the welfare system. Eligibility for welfare is virtually automatic for those in need. Within the welfare system, there are different benefit levels according to whether recipients are deemed to be disabled or not. Eligibility for disability is determined administratively and involves some medical certification. In all provinces, the disabled receive more than those who are employable. For example, in 1995, a disabled person in Ontario received $11,759 compared with $8,126 for a single employable person. Welfare recipients are also allowed to earn minimal amounts of income without losing their welfare assistance. There is evidence that there are significant tagging errors in provincial welfare programs. An 1990–94 audit of Quebec’s welfare program revealed that mistakes in determining need had occurred in about 20 percent of the cases audited, and that about four-fifths of those errors involved excessive generosity (type II errors below). Similar evidence has been found in other countries. Duclos (1995) studies the take-up of Supplementary Benefits for the non-working poor administered by the Department of Social Security in the United Kingdom. Using estimates obtained from a maximum likelihood procedure, he finds that in 1985 (conditional on all units applying) 18.1% of individuals who were entitled to the benefit were rejected by the agency (type I errors), while 18.8% of individuals not entitled were accepted (type II errors). Our model is consistent with these characteristics of actual welfare programs.

Our approach shares an important feature with Parsons (1996), who analyzes the consequences of imperfect tagging for redistributive tax-transfer policy. One of our main
concerns is whether or not welfare recipients should be allowed to work.\(^2\) In Parsons (1996), the desirability of separating the households by allowing welfare recipients to work rests solely on the ability or/and taste for labor of the latter, and arises because of an assumed indivisibility of labor supply. In our model, including the costs of administering the tagging process by social workers allows us to identify other sources of benefit and cost from allowing the welfare recipients to work. These have to do with how the existence of working welfare recipients affects the information available to the government and the incentives facing social workers. Making the accuracy of tagging by social workers endogenous is critical to our analysis.

The issue of whether welfare recipients should be allowed, or induced, to work is relevant not only in welfare programs but in negative income tax systems as well. As is well-known from the optimal income tax literature, it may well be efficient to have lower ability persons not working, simply because their productivity is not enough to compensate for the loss in their utility from having them work (Mirrlees, 1971). Similar arguments apply in welfare programs as well. But there are also other considerations, as our analysis will make clear. Tagging is costly, and inducing welfare recipients to work may improve the efficiency of tagging processes used to sort different categories of welfare recipients. As well, it may serve as a device for assisting in the monitoring of the amount of effort that welfare agencies put into improving the accuracy of tagging. On the other hand, inducing low-ability persons to work in order to improve tagging may be excessively costly if there are indivisibilities of labor supply. The upshot is that it may be efficient to have welfare recipients work even though they should not in an optimal non-linear income tax system that uses no tagging, and vice versa.

**II. BASIC FEATURES OF THE MODEL**

The model we construct is highly stylized and contains a number of simplifications intended to focus the analysis on what we regard to be key elements. There are three ability-types of persons in the economy—high ability (2), low ability (1), and disabled (0).\(^3\) The distribution of ability-types is uniform: there are \(M\) of each of the three types. Ability is private information to each person; the government and the social agency know the distribution. Let \(C\) be consumption and \(Y\) income. The able persons have utility functions

\(^2\) In Parsons (1996), the only two categories of persons are the tagged and the untagged, where tagging is an imperfect signal of disability. He refers to a dual negative income tax system as one in which the tagged persons are allowed to work. Untagged persons always have the choice between working or not. He shows that it may or may not be preferable to allow the tagged to work. In our model, the tagged persons do not work. The issue is whether or not the untagged persons who enter the general welfare system should be allowed to work. The exact relation between our model and that of Parsons will become apparent as we proceed.

\(^3\) Adding more high ability-types adds nothing of substance to the problem we are addressing. They would simply become taxpayers under the non-linear progressive tax system, like high-ability persons. The properties of that system are well-known.
$V^i(C, Y)$ ($i = 1, 2$), which are increasing in $C$ and decreasing in $Y$, with marginal rates of substitution $-V^i_Y/V^i_C$ increasing in $Y$. Leisure and consumption are normal so that for any bundle $(C, Y)$, $-V^2_Y/V^2_C < -V^1_Y/V^1_C$ (the single-crossing property). We need not specify precisely the nature of the difference in $V^i(\cdot)$ between ability-types. One interpretation, as in the optimal income tax literature, is that it is due solely to differences in wage rates. But we can also allow for other interpretations, such as differences in taste for leisure. The disabled persons are unable to work. Their utility function is $V^0(C)$, an increasing and strictly concave function. It is assumed that for a common level of consumption, the level of utility of a disabled person is no more than that of either type of able person: $V^0(C) \leq V^i(C, 0)$, $i = 1, 2$. But the marginal utility of consumption is larger for the disabled: $V^0_C(C) \geq V^i_C(C, 0)$, $i = 1, 2$.

All output is produced by labor and is used for consumption. Though labor supply is variable, we assume that there is some discreteness involved in choosing to work. A person who works must earn at least a minimal amount of income $y$, so that $Y \geq y$. Such discreteness, which may reflect a minimum level of hours worked or/and some time costs relating to traveling, is consistent with casual empirical evidence. Whether or not this constraint is binding will turn out to have important implications for the design of the general welfare program, i.e., whether it is efficient to induce welfare recipients to work. If $Y \geq y$ is not binding, our analysis parallels the standard framework of optimal income tax models. If it is binding, our analysis captures some features of the existing tagging models of Akerlof (1978), Diamond and Sheshinski (1995), and Parsons (1996), all of whom abstract from the labor supply decision by assuming that the amount of labor supplied when working is fixed.

The objective of the government is to redistribute income from higher to lower ability-types. Our purpose will be to discover the form of policies that achieve a given amount of redistribution in the most efficient way, given the instruments and the information available to the government. The government operates a non-linear negative income tax of the standard form and can supplement it with a welfare program. The purpose of the welfare program is to take advantage of the information obtained by tagging. Whether or not such a program is desirable depends upon the cost of administering the welfare program relative to the benefits of tagging. For the purposes of presentation, we assume a welfare program is in place and analyze the costs and benefits of administering it efficiently. (In the absence of a welfare program, a non-linear income tax system would be used by itself, and the properties of it are well-known.) Because the purpose of the welfare program is to make the benefit to the disabled more generous than they would be under the non-linear negative income tax system, both the low-ability and the disabled will be included in the welfare program, leaving the high-ability persons to be dealt with by the income tax. Each person who applies for welfare is automatically considered for disability benefits;

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4 It is obvious that nobody would accept a job involving very few hours worked. Holding a job entails monetary costs (eating, traveling costs), time costs (time to travel), and utility costs (one would prefer being with his children) that all have some discreteness (Killingsworth, 1983).
those who choose not to apply stay in the income tax system. Eligibility for disability benefits is examined by the social worker using a tagging technology. Those who are not tagged are admitted to the general welfare program. The assumption that all applicants are considered for the disability benefit does not affect the generality of our analysis.

We follow the standard practice (Stiglitz, 1982) of characterizing the non-linear negative income tax by the consumption-income bundles offered by the government for each ability-type. The parameters of this system induce truthful revelation of preferences. This implies both that it is administratively possible to eliminate tax evasion by some combination of penalties and auditing, and that it is not efficient to allow some tax evasion.\(^5\) In the negative income tax system, the bundle intended for type \(i\) is denoted \((C_i, Y_i)\). Since the disabled persons cannot work, their bundle will be \((C_0, 0)\).

In the welfare program, the disability benefit for those who are tagged consists of the bundle \((g, 0)\). General welfare recipients, those who are untagged, may or may not be allowed to work. We refer to the case where they are allowed to work as Regime \(W\) and the one in which they cannot as Regime \(N\). Given that disabled persons cannot work, two alternative bundles, \((c_1, y_1)\) and \((c_0, 0)\), must be offered under general welfare Regime \(W\), where lower-case letters are used to distinguish welfare from negative income taxation. The bundle \((c_1, y_1)\) is intended for the low-ability persons. In Regime \(N\), the only bundle offered to the non-tagged welfare recipients is \((c_0, 0)\). It will turn out to be the case that \(g > c_0\) in our analysis, given our assumption that the government wishes to redistribute from the better-off to the less well-off. To simplify matters, we assume that the bundles \((c_1, y_1)\) and \((c_0, 0)\) offered in the welfare scheme are the same as the bundles \((C_1, Y_1)\) and \((C_0, 0)\) in the income tax system. This ensures that, given \(g > c_0\), both the low-ability and disabled persons apply for welfare. Thus, the bundles \((C_1, Y_1)\) and \((C_0, 0)\) are not really relevant for our analysis. To induce each of the ability-types to choose the bundle intended for them in the relevant program, appropriate self-selection constraints must be satisfied. These self-selection constraints are:

\[
V^2(C_2, Y_2) \geq V^2(c_1, y_1); \quad V^1(c_1, y_1) \geq V^1(c_0, 0) \tag{1}
\]

where the latter one only applies in Regime \(W\).

The tagging technology takes the following form. Following Akerlof (1978) and Parsons (1996), let \(\beta_i\) be the probability that a person of ability-type \(i\) is tagged. We assume that \(\beta_2 = 0\), so high-ability persons cannot be tagged (and thus do not apply for welfare). \(\beta_2 = 0\) along with equation (1) and the single crossing property ensures that the high-ability persons do not apply for welfare. For the disabled and low-ability persons, \(\beta_1\) and \(\beta_1\) are endogenously determined and fall in the range \(0 < \beta_1 \leq 1/2 \leq \beta_0 < 0\). Since \(\beta_0\) is the probability that the disabled will be (correctly) tagged, \((1 - \beta_0)\) is the probability

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\(^5\) Stiglitz (1987) has argued that allowing tax evasion may be efficient for one of two reasons. It may relax incentive constraints if lower ability persons are less risk-averse than higher ability types that are mimicking them, which seems unlikely. Or, it may be a form of randomization of the tax system to exploit the fact that the aggregate revenue function of the government may be convex in tax rates.
that they will be incorrectly untagged: these are the type I errors. Similarly, $\beta_1$ is the probability that a low-ability person will incorrectly receive a tag, or that a type II error will occur. The accuracy of tagging, that is, the magnitudes of $\beta_0$ and $1-\beta_1$, are increasing in the effort of the relevant social worker. To simplify matters, we assume that $\beta_0 + \beta_1 = 1$: increases in $\beta_0$ (reductions in type I errors) are accompanied by equal decreases in $\beta_1$ (reductions in type II errors). This allows us to write $\beta \equiv \beta_0 = 1 - \beta_1$, which simplifies both the algebra and the notation used in our analysis. Let $e$ be the effort of a social worker. Then $\beta(e)$ relates effort to the probability of the disabled being tagged and the able not being tagged, where $\beta(e)$ is an increasing function. We assume that $\beta(0) = 1/2$; that is, if no effort is taken, nothing is learned from tagging. (Recall that there are the same number of disabled as low-ability persons, so $\beta_0 = 1/2 = \beta_1$ is equivalent to tagging disabled and low-ability persons randomly.)

The government hires social workers, whose sole responsibility is to tag welfare applicants. The caseload of each social worker is drawn randomly from the pool of welfare applicants, which includes all $2M$ disabled and low-ability persons. Assuming that the number of social workers is large, the probabilities of a given welfare applicant being low-ability or disabled are $1/2$ each. The social worker receives a salary from the government of $S$ per client, but must pay a penalty of $k$ if either a type I or type II error in tagging is detected. The penalty $k$ is independent of type of error and of the total number of errors detected. The expected income received by the social worker for an applicant will then be either $S - k$ or $S$ depending on whether or not a tagging error has been detected.

The detection of tagging errors is of crucial importance in our analysis. There are two alternative ways that may be done. The first way is to take advantage of the fact that optimal redistribution schemes are like revelation mechanisms that induce persons to reveal their ability-types. If we allow welfare applicants who do not succeed at being tagged to work, the welfare system can be designed such that non-tagged low-ability persons choose to work and are therefore separated from the non-tagged disabled, who simply cannot work. The government knows which social worker is responsible for each welfare recipient in the welfare program because both tagged and untagged persons remain part of the caseload of the social worker who examined their eligibility. This is a key difference between the income tax system and the welfare program. It provides information that the government can use for the purpose of monitoring the social workers to detect errors in tagging, and penalizing them for such errors. Given that, in Regime W all type I errors can be detected costlessly. This is a major benefit from allowing non-tagged low ability-types to work.

Second, the government might randomly audit those welfare recipients who do not reveal themselves. This will include all $2M$ welfare recipients in Regime N and all $M$ tagged persons in Regime W. Let $q$ be the proportion of the relevant body of welfare recipients who are audited. If audited, the ability-type of a welfare recipient is revealed. Of course, auditing is costly to the government as discussed later. Given the audit probability $q$, if a social worker has committed an error, expected income will be given by $S - q k$. Given that auditing is costly, it is a standard result in the economics of crime that the cost of any given level of compliance can be reduced by increasing the penalty while reducing audit probabilities. To limit the extent of penalty, we impose a maximum value for the penalty
by assuming that \( k \leq S \). The argument is that making errors is not a criminal offense, so the most we can expect is that the salary of the social worker is confiscated. Given that auditing is costly, it is obvious that the constraint \( k \leq S \) will always be binding. Therefore, from now on we assume \( S = k \) and suppress \( S \) from our analysis.\(^6\)

The payoffs available to the social worker per welfare applicant in the various possible outcomes of Regimes \( W \) and \( N \) are shown in the appropriate columns in Figure 1. This figure is constructed for a given value of \( \beta \), that is, for a given effort level by the social worker. Since effort is reflected in \( \beta \) through the tagging function \( \beta(e) \), we can suppress effort and suppose that \( \beta \) is the object of choice by the social worker. We assume social workers are risk-neutral so that expected utility is expected income less the cost (disutility) of effort. Let \( \phi(\beta) \) be the cost per applicant of achieving a given level of \( \beta \), where \( \phi'(\beta) > 0 \), \( \phi''(\beta) > 0 \), and \( \phi(1/2) = \phi'(1/2) = 0 \). The latter implies that if the social worker exerts no effort, so the tag yields no information, there is no cost. From Figure 1, the expected utility of the social worker in the two Regimes is:

\[
EU^W(\beta, q, k) = \beta k + (1 - \beta)(1 - q)\frac{k}{2} - \phi(\beta) \quad (2)
\]
\[
EU^N(\beta, q, k) = \beta k + (1 - \beta)(1 - q)k - \phi(\beta) \quad (3)
\]

where the superscripts \( W \) and \( N \) denote the Regime. In equations (2) and (3) for the expected utility per applicant of the social worker, we are assuming that the social worker chooses to use the tagging technology, captured by \( \beta \), to assign disability status to applicants. In the following section, we need to consider the case where the social worker chooses not to use \( \beta \) to accept or reject applicants.

We need to be sure that the social worker will agree to accept employment. One way to do so that would be to impose a participation constraint that requires expected utility to be some minimal amount, say, zero. Instead, it turns out that, given the constraint on the penalty function \( k \leq S \), the expected utility of the social worker is always positive. Of course, we could impose a stricter participation constraint, but that would only complicate the analysis.

### III. BEHAVIOR OF THE SOCIAL WORKER

The payoffs of the social worker \( EU^W \) and \( EU^N \) given by (2) or (3) depend upon the parameters set by the government \( (k, q) \) and by the effort exerted by the social worker in each Regime. In this section we consider the behavior of the social worker. The following section takes up government policy.

**Regime W: Welfare Recipients Allowed to Work**

Social workers must decide whether to exert any effort at all and, if so, how much to exert. Assume that social workers choose some positive level of effort. They will choose \( \beta \) to

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\(^6\) Other less stringent constraints on \( k \) would also be possible. For example, we could set \( k \leq S - \Delta \) for some arbitrary value of \( \Delta \) without changing the essence of the argument.
maximize expected utility given by (2). The first-order condition is:

\[ k - (1 - q) \frac{k}{2} - \phi'(\beta) = 0. \tag{4} \]

The second-order condition requires \( \phi''(\beta) > 0 \), which we have already assumed. Equation (4), which determines the effort chosen by the social worker, can also be solved for the value of \( k \) that corresponds with given levels of \( \beta \) and \( q \) when the social worker is choosing the optimal level of effort. Solving (4) for \( k \) and substituting into (2), we obtain the level of \( EU^W \) attained by the social worker when optimal effort is exerted:

\[ EU^W(\beta, q) = \left( 2\beta + (1 - \beta)(1 - q) \right) \frac{\phi'(\beta)}{1 + q} - \phi(\beta). \]

Satisfaction of the second-order condition is not sufficient to ensure that the social worker will supply positive effort. Instead, the social worker may exert no effort and accept all applicants. Using the payoffs from Figure 1, expected utility will be:

\[ EU^A(q, k) = \left( 1 - \frac{q}{2} \right) k. \]

Intuitively, by accepting all applicants, no type I errors will be committed. A penalty will be imposed only if the social worker commits a type II error and is audited with probability \( q \).\(^7\) To induce the social worker to exert effort, an effort incentive constraint must be satisfied: \( EU^W(\beta, q) \geq EU^A(q, k) \), or, using (4) in \( EU^A(q, k) \),

\[ (2\beta + (1 - \beta)(1 - q)) \frac{\phi'(\beta)}{1 + q} - \phi(\beta) \geq (2 - q) \frac{\phi'(\beta)}{1 + q}. \tag{5} \]

Whether or not the effort incentive condition (5) is binding turns out to be important in choosing between the two Regimes and among other policy options.

Define by \( q^W(\beta) \) the audit probability that makes (5) binding. When (5) is an equality, we obtain:

\[ q^W(\beta) = \frac{\phi(\beta) + (1 - \beta)\phi'(\beta)}{\beta\phi'(\beta) - \phi(\beta)}. \tag{6} \]

It is easy to verify using \( \phi''(\beta) > 0 \) that \( q^W(\beta) \) is decreasing in \( \beta \). Also, \( \lim_{\beta \to 1/2} q^W(\beta) = 1 \). Using (6), (5) can be restated as \( q \geq q^W(\beta) \). Thus for a given \( \beta \), the government is constrained to choosing only those values of \( q \) such that \( q \geq q^W(\beta) \) if it wants to operate a welfare program. (If the social workers do not exert effort, there are no benefits from tagging.)\(^8\)

\(^7\) An alternative strategy of the social worker may be to reject all applicants. If so, the expected utility per client becomes \( EU^W(\text{reject}) = k/2 \). Obviously, this is less than \( EU^A \) so this alternative strategy would not be used.
Given this behavior of the representative social worker, we can determine administrative costs of the welfare program in Regime $W$. With $M$ being large, administrative costs can be treated as deterministic. Administrative costs include the expected income of the social workers (net of penalties) plus the cost of auditing a proportion $q$ of the $M$ tagged persons. Let the total cost of auditing be $C(qM)$, an increasing, convex function with $C(0) = 0$. Given the behavior of social workers as reflected in $EU^W(\beta, q)$, administrative costs per welfare recipient in Regime $W$ can be written:

$$AW^W(\beta, q) = (2\beta + (1 - \beta)(1 - q)) \frac{\phi'(\beta)}{1 + q} + \frac{C(qM)}{2M}$$

(7)

where admissible combinations of $\beta$ and $q$ must satisfy $q \geq q^W$ as defined by (6) above.

The properties of the administrative cost function $AW^W(\beta, q)$ are of some interest. Differentiating (7) with respect to $\beta$, it is straightforward to show that:

$$\frac{\partial AW^W(\beta, q)}{\partial \beta} = \phi'(\beta) + (2\beta + (1 - \beta)(1 - q)) \frac{\phi''(\beta)}{1 + q} > \phi'(\beta) > 0.$$

Not surprisingly, inducing greater effort from the social worker requires an increase in administrative costs. The derivative $\phi'(\beta)$ represents marginal administration cost when effort level is observable (perfect information). If information on effort is perfect, the government does not have to audit the social workers and $S$ would be chosen so that the individual rationality constraint is satisfied, that is, $S = \phi(\beta)$, which is also the administration cost in such a circumstance. When information is imperfect and thus there is penalty imposed, higher $\beta$ implies lower type I and II errors, which in turn leads to lower fine revenue. That is why the marginal cost of $AW^W(\beta, q)$ exceed that under full information. Differentiating (7) with respect to $q$ and simplifying yields:

$$\frac{\partial AW^W(\beta, q)}{\partial q} = \frac{C'(qM)}{2} - \frac{2}{(1 + q)^2} \phi'(\beta).$$

(8)

This satisfies $\partial^2 AW^W(\beta, q)/\partial q^2 > 0$, implying that administrative costs are strictly convex in $q$ as shown in Figure 2.

Now consider the government’s choice of $q$ to minimize administrative costs $AW^W(\beta, q)$ for any given value of $\beta$. If (5) is not binding, the government would select $q$ at the bottom of the U-shape of $AW^W(\beta, q)$. Denote this unconstrained level of $q$ as $q^{*W}(\beta)$. It satisfies $\partial AW^W(\beta, q)/\partial q = 0$ in (8) for $q^{*W}(\beta) > 0$. Since $\phi'(1/2) = 0$ and $C'(0)\geq 0$, it is easy to see that $q^{*W}(\beta) = 0$ may result for $\beta$ sufficiently close to $1/2$. The possibility of such a corner solution is not essential to our analysis, however. When $q^{*W}(\beta)$ does not exceed $q^W(\beta)$, the government is forced to choose the latter since the incentive constraint (5) would be violated otherwise. Therefore, if we denote cost-minimizing level of audit for a given $\beta$ in case $W$ as $q^W(\beta)$, we have $q^W(\beta) = \max\{q^{*W}(\beta), q^W(\beta)\}$. Figure 2 summarizes the two possibilities. In Panel A, the effort constraint is binding at the optimum, $q^{*W}(\beta) < q^W(\beta)$, and so, $q^W(\beta)$ must be chosen. On the other hand, in Panel B, the effort constraint is not binding so that the optimum $q^{*W}(\beta)$ can be chosen.
We have seen that $q^W(\beta)$ is decreasing in $\beta$ and that $\lim_{\beta \to 1/2} q^W = 1$. On the other hand, $q^*(\beta)$ is monotonically increasing in $\beta$. These curves are depicted in Figure 3. Let the level of $\beta$ that equates $q^W(\beta)$ and $\overline{q}^W(\beta)$ be $\bar{\beta}$: $q^W(\bar{\beta}) = \overline{q}^W(\bar{\beta})$. If $\bar{\beta} \in [1/2, \bar{\beta}]$, the effort constraint is binding and $\bar{q}(\beta)$ must be chosen, while if $\beta \in [\bar{\beta}, 1]$, the effort constraint is no longer binding and the optimum $q^*(\beta)$ can be chosen.\(^8\)

This characterizes the cost-minimizing values of $q^W(\beta)$. With some abuse of notation, denote by $A^W(\beta)$ the minimum value function for the government’s problem of choosing $q$ to minimize administrative costs subject to the effort incentive constraint. That is, $A^W(\beta) \equiv A^W(\beta, q^W(\beta))$. Note that when $q^W(\beta) = q^*(\beta)$, the envelope theorem implies $\frac{dA^W(\beta)}{d\beta} = \frac{\partial A^W(\beta, q^W(\beta))}{\partial \beta} > 0$ for $\beta > 1/2$; inducing higher effort increases administrative costs. But if the effort constraint is binding so $q^W(\beta) = \overline{q}^W(\beta)$,

$$\frac{dA^W(\beta)}{d\beta} = \frac{\partial A^W(\beta, q^W(\beta))}{\partial \beta} + \frac{\partial A^W(\beta, q^W(\beta))}{\partial q} \frac{dq^W}{d\beta}.$$  

Since the last term is negative, the sign of this expression is generally ambiguous. It is reasonable to suppose that administrative costs in Regime $W$ are as depicted in Figure 4. For example, if $\phi'(\beta)$ and $\phi''(\beta)$ are small enough at $\beta$ close to $1/2$, this will be the case. In this case, initially (in the left portion of the curve), the second term on the right-hand side of the previous expression dominates the first one so, administrative costs are decreasing. A point comes however where the first term becomes more important and so, the curve eventually becomes upward sloping. We will return to Figure 4 later.

**Regime N: Welfare Recipients not Allowed to Work**

Consider now the behavior of social workers in Regime $N$. Their expected utility is given by (3). For given $k$ and $q$, the social workers choose $\beta$ to maximize $EU^N(\beta, q, k)$. The first-order condition is simply:

$$qk - \phi'(\beta) = 0.$$  

The second-order condition is also satisfied here with $\phi'(\beta) > 0$. Solving (9) for $k$ and substituting into (3), we obtain the expected utility of the social worker when optimal effort is exerted:

$$EU^N(\beta, q) = \left( \beta + (1 - \beta)(1 - q) \right) \frac{\phi'(\beta)}{q} - \phi(\beta).$$

As in Regime $W$, a social worker may choose to provide no effort and to accept any applicant, reject any applicant, or accept applicants randomly. As before, $EU^A(q, k) = (1 - (q/2))k$. But, as opposed to Regime $W$, a social worker providing no effort ($\beta = 1/2$) but using the tagging technology also obtains the same expected utility: $EU^N(1/2, q, k) = (1 - (q/2))k$ (by (3)). This implies that the effort incentive constraint is never binding in

---

\(^8\) It could be the case that $\bar{\beta} = 1$. If so, $q^W(\beta) = \overline{q}^W(\beta)$ for all $\beta$. Though easy to analyze, this case is not particularly interesting. In what follows we assume that $\bar{\beta}$ is interior.
Regime $N$. This difference turns out to be an important distinction between Regimes $N$ and $W$.

Administrative costs per welfare client in Regime $N$ are again the expected income per client of the social worker plus the cost of auditing, where here a proportion $q$ all $2M$ tagged and untagged welfare recipients must be audited. Given the optimizing behavior of the social workers, administrative costs can be written:

$$A^N(\beta, q) = \left(\beta + (1 - \beta)(1 - q)\right)\frac{\phi'(\beta)}{q} + \frac{C(2qM)}{2M}.$$  \hfill (10)

Differentiating $A^N(q, \beta)$ with respect to $\beta$, again we can see $\partial A^N(q, \beta)/\partial \beta > \phi'(\beta)$. As in case $W$, administration costs are rising in $\beta$, and asymmetric information on effort makes the marginal administration cost exceed the marginal cost of effort to the social worker.

The government chooses audit effort to minimize $A^N(q, \beta)$ for any given level of $\beta$. Since the effort incentive constraint is always satisfied in this Regime, cost-minimizing level of $q$ is given as the solution to the following first-order condition:

$$C'(2qM) - \frac{\phi'(\beta)}{q^2} = 0.$$  \hfill (11)

Since $\partial^2 A^N/\partial q^2 > 0$, the solution to (11), $q^N(\beta)$, is a global minimum. Moreover, $q^N(\beta)$ is increasing in $\beta$ with $q^N(1/2) = 0$. Comparing (8) and (11), we can see that $q^N(\beta)$ can be greater or less than $q^W(\beta)$ for any value of $\beta > 1/2$. Figure 3 shows the case where $q^N(\beta) > q^W(\beta)$. This case applies, for instance, when the audit cost function $C(\cdot)$ is linear.

Denote the minimum value function to the government’s audit-choice problem in Regime $N$ by $A^N(\beta) \equiv A^N(\beta, q^N(\beta))$. We wish now to compare administrative costs of attaining a given level of effort in the two Regimes. The following Lemma, proven in the Appendix, is useful for comparing $A^W(\beta)$ and $A^N(\beta)$.

**Lemma 1:** For a given pair $(\beta, q)$, the administrative costs are lower when non-tagged welfare applicants can work than when they cannot: $A^W(\beta, q) < A^N(\beta, q)$.

Recalling that $\bar{\beta}$ is the value of $\beta$ such that the effort incentive constraint just becomes binding, the following proposition is proven in the Appendix:

**Proposition 1:** If $\beta \geq \bar{\beta}$, then $A^W(\beta) < A^N(\beta)$: administrative costs are lower when non-tagged welfare applicants can work than when they cannot.

In Figure 4, administrative costs in both Regimes are represented. Note that $A^N(\beta)$ is a strictly increasing curve and that given Proposition 1, it must cross $A^W(\beta)$ from below at some $\beta$ strictly smaller than $\bar{\beta}$. Proposition 1 leads us to conclude that when the government wants to implement a relatively higher level of $\beta$, it may find Regime $W$ in
which the welfare recipients are allowed to work preferable to Regime N. But we cannot claim that this would be always the case without examining the government optimization with respect to the parameters of the welfare program and the income tax system. The transition from one Regime to another involves a discrete change in resource allocation. We turn to this issue in the next section.

In the above discussion, we have already seen that \( \lim_{\beta \to 1/2} q^W = 1 \), while \( q^N(1/2) = 0 \). From (4) and \( \phi'(1/2) = 0 \), we have \( k = S = 0 \) in case \( W \) if \( \beta = 1/2 \). Solving (9) for \( k \) and substituting \( q^N(\beta) \) yields the penalty that is necessary to induce a given level of \( \beta: k^N(\beta) = \phi'(\beta)/q^N(\beta) = q^N(\beta)C'(2M q^N(\beta)) \), where we have used (11) to obtain the last equality. Obviously, \( k^N(1/2) = 0 \). This implies that \( A^W(\beta) \) approaches \( C(M)/2M \) as \( \beta \) approaches 1/2,\(^9\) while \( A^N(1/2) = 0 \). By the continuity of \( A^r(\beta) (r = W, N) \), we can conclude that when targeted \( \beta \) is relatively close to 1/2, Regime \( W \) is unambiguously more expensive than Regime \( N \) to administer, which may induce the government to choose the latter case.

**Proposition 2:** For \( \beta \) sufficiently close to 1/2, \( A^W(\beta) > A^N(\beta) \).

**IV. THE PROBLEM OF THE GOVERNMENT**

In the previous section, we found that the relative magnitude of administration costs in Regimes \( W \) and \( N \) depend on the effort put into tagging by the social workers. But the two Regimes also differ in their optimal welfare and income tax policies. To determine which of the two Regimes is superior, we need to optimize over these other government policy instruments. Unfortunately, exact comparisons become difficult since optimal allocations in Regimes \( W \) and \( N \) differ discretely. To obtain insight into the factors that favor one Regime relative to the other, we approach the problem in stages. First, we determine which welfare system requires the least net revenue to achieve a given combination of transfers to the disabled \( (g, c_0) \) and accuracy of tagging \( (\beta) \). Then, we consider the program that maximizes the weighted sum of utilities of each type of person. Finally, we examine an example in which the objective function attributes weight to the disabled only, and in which the able individuals have quasi-linear preferences.

**The Cost-Minimizing Welfare Scheme**

Suppose \( g, c_0 \) and \( \beta \) are all given, with \( g > c_0 \). Fixing \( \beta \) may be interpreted as having a target level for the proportion of the disabled that are entitled to the disability benefit. It also allows our approach to be compared with those in which \( \beta \) is exogenously given, such as Parsons (1996). The objective of finding the least-cost way of delivering given levels of support to the disabled is analogous to the criterion adopted by Besley and Coate (1992, 9 However, \( A^W(1/2) \neq C(M)/2M \) since there is no point in auditing (setting \( q = 1 \)) to get no information \( (\beta = 1/2) \). Note that the effort constraint (5) is satisfied for any \( q \) if \( \beta = 1/2 \). Hence, \( A^W(\beta) \) has a discontinuity at \( \beta = 1/2 \). In fact, \( A^W(1/2) = 0 \). This discontinuity is represented by the empty point at \( C(1) \) on the vertical axis in Figure 4.
In analyzing the use of workfare to screen workers of different ability types in welfare schemes.\textsuperscript{10}

In Regime $W$, total welfare program expenditures are given by:

$$T^W = E^W + 2MA^W(\beta)$$

$$= Mg + M(1 - \beta)c_0 + M\beta(c_1 - y_1) + M(C_2 - Y_2) + 2MA^W(\beta)$$

(12)

where $E^W$ is the amount of transfers net of taxes required to finance the given level of benefits to the tagged welfare recipients. Recall, using our notation $\beta_0 = \beta$ and $\beta_1 = 1 - \beta$, that $\beta_0 + \beta_1 = 1$, so that one half of welfare applicants are tagged, a proportion $\beta$ of which are disabled. Equation (12) reflects this. Similarly, in Regime $N$, total welfare program expenditures are:

$$T^N = E^N + 2MA^N(\beta) = Mg + Mc_0 + M(C_2 - Y_2) + 2MA^N(\beta).$$

(13)

Given $g$, $c_0$ and $\beta$, for each Regime the government will choose the values for its remaining instruments, $(c_1, y_1, C_2, Y_2)$, to minimize $T^W$ or $T^N$ subject to the self-selection constraints:

$$V^1(c_1, y_1) \geq V^1(c_0, 0)$$

(14a)

$$V^2(C_2, Y_2) \geq V^2(c_1, y_1).$$

(14b)

Constraint (14a) is relevant only in Regime $W$ since it ensures that low-ability workers will prefer to work when participating in the general welfare program. Constraint (14b) must be taken into account in both Regimes $W$ and $N$, although $(c_1, y_1)$ should be replaced by $(c_0, 0)$ in Regime $N$. The minimum income constraint also requires $y_1 \geq y$ in Regime $W$. (We assume that $Y_2 \geq y$ is not binding in the optimum, implying that type 2 persons have sufficiently high ability.)

It is worth highlighting an informational implication of the restriction $y_1 \geq y$. In absence of the agency problem, the government may prefer low-ability persons not to work because their ability is too low or because the self-selection constraint for the high-ability persons is tightened. But when agency problems are present, the government may want to induce the low-ability types to work to detect type I errors without auditing, even though it may be costly to do so: the disabled and the low ability individuals need to be separated in the general welfare. We return to this issue below. First we consider the government’s policy choices for each of the Regimes in turn.

\textsuperscript{10} In their model, which does not contain social workers, the criterion is to minimize the cost of delivering a given level of income to the poor. A slightly more general criterion which assumes a continuous distribution of abilities and maximizes an index of the extent to which households fall below some ‘poverty level’ of consumption is in Kanbur, Keen and Tuomala (1994).
The Lagrangian expression in Regime $W$ can be written as:
\[ L^W(c_1, C_2, y_1, Y_2, \lambda^y_1) = T^W - \mu^W_1(y_1 - y) - \lambda^y_1(V^1(c_1, y_1) - V^1(c_0, 0)) - \lambda^2(V^2(C_2, Y_2) - V^2(c_1, y_1)) \]
where $T^W$ is given by (12). The first-order conditions with respect to the government’s policy instruments are:
\[
\begin{align*}
    c_1 &: \quad \beta M - \lambda^W_1 V^1_c + \lambda^W_2 \hat{V}^2_c = 0 \\
    y_1 &: \quad -\beta M - \mu^W_1 - \lambda^W_1 V^1_y + \lambda^W_2 \hat{V}^2_y = 0 \\
    C_2 &: \quad M - \lambda^W_1 V^2_c = 0 \\
    Y_2 &: \quad -M - \lambda^W_2 V^2_y = 0
\end{align*}
\]
where $\hat{V}^2_c$, $\hat{V}^2_y$ represent the marginal utility of a high-ability mimic with respect to argument $c_1$, $y_1$. A similar convention will be used below for low-ability types when mimicking the disabled. From (15c) and (15d), we immediately obtain that standard result that the marginal tax rate at the top should be zero: $-V^2_c/V^2_c = 1$. It is also straightforward to show that low-ability individuals face a marginal tax rate that is between zero and 100 percent if $y_1 \geq y$ is not binding: $0 < -V^1_y/V^1_c < 1$. Otherwise, the sign of the marginal tax rate is ambiguous at $y_1 = y$. It may be negative, reflecting the need for a wage subsidy to induce low ability persons to choose $y_1 = y$.\(^{12}\)

Denote the minimum value function of the government’s problem in Regime $W$ by $T^W(\beta, g, c_0) = E^W(\beta, g, c_0) + 2MA^W(\beta)$. Applying the envelope theorem yields:\(^{13}\)
\[
\begin{align*}
    \frac{\partial E^W}{\partial \beta} &= ((c_1 - y_1) - c_0)M < 0 \quad \text{if } y_1 > y; \quad \frac{\partial E^W}{\partial g} = M; \quad \frac{\partial E^W}{\partial c_0} = (1 - \beta)M + \lambda^W_1 \hat{V}^1_c > M.
\end{align*}
\]

\(^{11}\) The marginal tax rate is not uniquely defined. Following Stiglitz (1982), an implicit marginal tax rate can be defined by $1 + V^1_y/V^1_c$.

\(^{12}\) Combining (15a) and (15b) yields:
\[
\lambda^W_1 V^1_c \left(1 + \frac{V^1_y}{V^1_c}\right) = \lambda^W_2 \hat{V}^2_c \left(1 + \frac{\hat{V}^2_y}{\hat{V}^2_c}\right) - \mu^W_1
\]

The first term is always positive due to the single-crossing property. If $\mu^W_1 > 0$ and thus $y_1 \geq y$ is binding, the second term may dominate the first one, which causes the marginal tax rate to be negative.

\(^{13}\) The first inequality obtains since $y_1 > y$ implies $MRS^1_{yc} < 1$ when evaluated at $(c_0, 0)$. An increase in $y_1$ beyond zero then allows the government to decrease its transfer to the low-ability individuals without violating their self-selection constraint. The third inequality follows from the fact that if leisure is not inferior, $V^i_y \leq 0$; we then have $\hat{V}^1_c \geq V^1_c$. By (15a), $\lambda^W_1 \hat{V}^1_c \geq \lambda^W_1 V^1_c > \beta M$, which leads to $\partial E^W/\partial c_0 > M$. 

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Thus, a one dollar increase in \( c_0 \) increases net costs by more than one dollar per general welfare recipient (because such an increase tightens the self-selection constraints for the two types of individuals). And, increasing \( \beta \) will reduce costs if \( y_1 \geq y \) is not binding. But it could increase costs if \( y_1 = y \); the net payment for the untagged able \( c_1 - y_1 \) might be higher than \( c_0 \) paid to the untagged disabled.\(^{14}\)

The Lagrangian expression for Regime \( N \) is:

\[
\mathcal{L}^N(C_2, Y_2, \lambda_2^N) = T^N - \lambda_2^N \left[V^2(C_2, Y_2) - V^2(c_0, 0)\right]
\]

where \( T^N \) is given by (13). The first-order conditions with respect to the policy instruments \( (C_2, Y_2) \) again yield (15c) and (15d), implying a zero marginal tax rate at the top. As above, let \( T^N(\beta, g, c_0) \equiv E^N(\beta, g, c_0) + 2MA^N(\beta) \) be the minimum value function of the government’s problem. From the envelope theorem:

\[
\frac{\partial E^N}{\partial \beta} = 0; \quad \frac{\partial E^N}{\partial g} = M; \quad \frac{\partial E^N}{\partial c_0} = M + \lambda_2^N \hat{V}_c^2 > M. \quad (17)
\]

From the expressions for \( \mathcal{L}^W(\cdot) \) and \( \mathcal{L}^N(\cdot) \), it is obvious that for a given \( \beta \), the administration costs have no effect on the cost-minimizing structure of the income tax schedule and the welfare program. On the other hand, whether the restriction \( y_1 \geq y \) is binding in Regime \( W \) does have an impact. First, suppose \( y_1 > y \). This implies that bunching is inefficient and the low-ability individuals should be working in Regime \( W \). Then, regardless of the difference between \( A^W(\beta) \) and \( A^N(\beta) \), separating the households in the general welfare program always reduces the cost of transfers net of taxes; that is, \( E^W(\beta, g, c_0) < E^N(\beta, g, c_0) \).\(^{15}\) Next, consider the case where the optimum requires \( y_1 = y \). The comparison between \( E^W(\cdot) \) and \( E^N(\cdot) \) becomes ambiguous: \( y_1 = y \) may be excessive for the low-ability persons and the government would have preferred to set \( y_1 \) below \( y \), but cannot. Therefore, \( E^W(\beta, g, c_0) > E^N(\beta, g, c_0) \) could hold: separating welfare recipients is more costly than allowing bunching at the bottom. To summarize:

**Lemma 2:** Given \( \beta, g, \) and \( c_0 \), if the minimum income constraint \( y_1 \geq y \) is not binding in the optimum of Regime \( W \), then \( E^W(\beta, g, c_0) < E^N(\beta, g, c_0) \). Otherwise, it can be the case that \( E^W(\beta, g, c_0) > E^N(\beta, g, c_0) \).

\(^{14}\) To illustrate conditions under which the minimum income requirement is binding, suppose preferences are quasi-linear, \( V^i(c, y) = c - h_i(y) \). Then, applying the envelope theorem to the problem of Regime \( W \), we obtain:

\[
M^{-1} \frac{\partial \mathcal{L}^W}{\partial y_1} = \beta(h_1'(y_1) - 1) + (h_1'(y_1) - h_2'(y_1)),
\]

where the second-order conditions are satisfied if \( h_1'(y) > h_2'(y) \) and \( h_1''(y) > h_2''(y) \). If \( (1 + \beta)h_1'(y) \geq \beta + h_2'(y) \) for the given \( \beta \), then \( \partial \mathcal{L}^W / \partial y_1 \) is strictly positive for any \( y_1 > y \), which implies that in the optimum, \( y_1 = y \).

\(^{15}\) In other words, \( \partial \mathcal{L}^W / \partial y_1|_{y_1 = y} < 0 \).
From Proposition 1, we know that for $\beta \geq \bar{\beta}$, $A^W(\beta) < A^N(\beta)$. On the other hand, if $\beta$ is sufficiently low, we know from Proposition 2 that $A^N(\beta) < A^W(\beta)$. By making use of Lemma 2 along with these propositions, $T^W(\beta, g, c_0)$ can be evaluated relative to $T^N(\beta, g, c_0)$. The comparison depends on whether $y_1 \geq y$ is effective in the optimum of Regime W. If $y_1 > y$ in the optimum, $E^W(\cdot) < E^N(\cdot)$ by Lemma 2. For $\beta$ such that $\beta \geq \bar{\beta}$, it must be that $T^W(\beta, g, c_0) < T^N(\beta, g, c_0)$ so it is cost-minimizing to induce the low-ability persons to work in the general welfare system. For sufficiently small $\beta$, the comparison becomes ambiguous. If the difference in the administration costs outweighs the inefficiency caused by bunching at the bottom, Regime N will be less costly than Regime W.

Suppose now that $y_1 = y$. Then, from Lemma 2, $E^W(\cdot) > E^N(\cdot)$ may be the case. This possibility implies that even if $\beta \geq \bar{\beta}$ so that $A^W(\beta) < A^N(\beta)$, the relationship between $T^W(\cdot)$ and $T^N(\cdot)$ may be ambiguous. When $\beta$ is sufficiently low, so $A^W(\beta) > A^N(\beta)$, and if $E^W(\beta, g, c_0) > E^N(\beta, g, c_0)$, then it is clearly the case that $T^W(\cdot) > T^N(\cdot)$. The following proposition summarizes this discussion.

**Proposition 3:**

(i) Suppose $y_1 > y$ in the optimum of Regime W. Then, for $\beta \geq \bar{\beta}$, Regime W is less costly than Regime N. For $\beta$ small enough, it is possible that Regime N is less costly than Regime W.

(ii) When $y_1 = y$ causes $E^W(\cdot) > E^N(\cdot)$, even for $\beta \geq \bar{\beta}$, $T^W(\cdot) > T^N(\cdot)$ may be possible, while this is definitely the case for a sufficiently small $\beta$.

**The Pareto-Efficient Welfare Scheme**

We now turn to the more general problem of a government maximizing a welfare function of the form:

$$SW = \beta V^0(g) + (1 - \beta) V^0(c_0) + \delta_1((1 - \beta) V^1(g, 0) + \beta V^1(c_1, y_1)) + \delta_2 V^2(C_2, Y_2)$$

(18)

where $(c_1, y_1)$ reduces to $(c_0, 0)$ in Regime N. This can be interpreted as the objective function that would result in an allocation of resources along the economy’s second-best utility possibilities frontier (UPF). To narrow the range of alternatives, assume that $1 \geq \delta_1 \geq \delta_2$, implying that redistribution goes from the able to the disabled individuals.16 This, in turn, ensures that the incentive constraints used in the previous sections and subsections are still the relevant ones. To begin with, we hold $\beta$ fixed and characterize the optimal policy in each regime, given $\beta$. We subsequently return to the choice of $\beta$ in each Regime.

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16 This is analogous to assuming that the objective function $SW$ corresponds with a social welfare function that is a function of expected utilities and is increasing and quasi-concave. An alternative, more complicated, formulation would treat social welfare as a function of *ex post* utilities and attach separate welfare weights to the utility of each household in each state.
In Regime $W$, the government chooses $\{g, c_0, c_1, y_1, C_2, Y_2\}$ to maximize $SW$ given by (18) subject to

\[
\begin{align*}
g &\ge c_0 \\
y_1 &\ge \underline{y} \\
V_1(c_1, y_1) &\ge V_1(c_0, 0) \\
V_2(C_2, Y_2) &\ge V_1(c_1, y_1) \\
M(Y_2 - C_2) + M\beta(y_1 - c_1) - M(1 - \beta)c_0 - Mg - 2MA^W(\beta) &\ge 0
\end{align*}
\]

where the multiplier assigned to a constraint appears as its equation label. For a given $\beta$, the administration costs are already minimized and summarized in $A^W(\beta)$, depicted as the U-shaped curve in Figure 4. From the first-order conditions to this problem (shown in the Appendix), the optimal policy in Regime $W$ has the following properties. As in the cost-minimization problem, the marginal tax rate is zero at the top: $-V^2_y/V^2_c = 1$. If the constraint $y_1 \ge \underline{y}$ is not binding, the marginal tax rate is positive at $(c_1, y_1)$. If it is binding ($\mu^W_1 > 0$), the marginal tax rate for low-ability individuals may be negative so as to induce them to supply a positive amount of labor. The optimal values for the government’s instruments are denoted $g^W(\beta)$, $c^W_0(\beta)$, and so on. These yield the maximum value function $SW^W(\beta)$. The following Lemma relating $g^W(\beta)$ and $c^W_0(\beta)$ is proven in the Appendix:

**Lemma 3:** In the optimum of case $W$, $g^W(\beta) > c^W_0(\beta)$.

In Regime $N$, the government chooses $\{g, c_0, C_2, Y_2\}$ to maximize

\[
\beta V^0(g) + (1 - \beta)V^0(c_0) + \delta_1 \left( (1 - \beta)V^1(g, 0) + \beta V^1(c_0, 0) \right) + \delta_2 V^2(C_2, Y_2)
\]

subject to

\[
\begin{align*}
g &\ge c_0 \\
V^2(C_2, Y_2) &\ge V^1(c_1, y_1) \\
M(Y_2 - C_2) - Mc_0 - Mg - 2MA^N(\beta) &\ge 0.
\end{align*}
\]

Denote the solutions to this problem as $g^N(\beta)$, $c^N_0(\beta)$, etc. The maximum value function for this problem is $SW^N(\beta)$. Using a proof similar to that of Lemma 3, we can derive the following:

**Lemma 4:** $g^N(\beta) > c^N_0(\beta)$ for any value of $\beta$.

In the final stage of each Regime, the government optimizes $SW^W(\beta)$ or $SW^N(\beta)$ with respect to $\beta$. For Regime $W$, the envelope theorem allows us to obtain the first-order
condition for the optimal $\beta$, denoted by $\beta^W$:

$$
\left( V^0 \left( g^W(\beta) \right) - V^0 \left( c_0^W(\beta) \right) \right) - \delta_1 \left( V^1 \left( g^W(\beta), 0 \right) - V^1 \left( c_1^W(\beta), y_1^W(\beta) \right) \right) + \lambda^W_3 \left( c_0^W(\beta) - \left( c_1^W(\beta) - y_1^W(\beta) \right) \right) = \lambda^W_3 \frac{2M}{\partial \beta} \frac{\partial A^W(\beta)}{\partial \beta}. \tag{19}
$$

The left-hand side is the marginal benefit from increasing the accuracy if tagging, while the right-hand side represents the marginal administrative cost from so doing. The first term of the left-hand side is positive by Lemma 3. The second is negative since $\lambda^W_3 > 0$ in the optimum, which implies $V^1(c_1^W, y_1^W) = V^1(c_0^W, 0) < V^1(g^W, 0)$. The assumption $V^0_c(c) \geq V^1_c(c, 0)$ along with $\delta_1 \leq 1$ ensures, however, that the sum of the first and second terms is positive.\footnote{To see this, note that the first two terms in (19) can be rewritten as:}

$$
\left( V^0 \left( g^W(\beta) \right) - V^0 \left( c_0^W(\beta) \right) \right) - \delta_1 \left( V^1 \left( g^W(\beta), 0 \right) - V^1 \left( c_1^W(\beta), 0 \right) \right) = \int_{c_0^W(\beta)}^{g^W(\beta)} \left( V^0_c(c) - \delta_1 V^1_c(c, 0) \right) dc.
$$

Obviously, the right-hand side is positive, and so is the left-hand side.

\footnote{It is technically possible that the curve $A^W(\beta)$ be declining throughout its range. In that case, $\beta^W = 1$ as long as $y_1 \geq y$ is not sufficiently binding. We do not discuss that case further, though it can also give rise to either Regime being preferred.}

The left-hand side of (19) is unambiguously positive. That implies that $\partial A^W(\beta)/\partial \beta > 0$ at $\beta = \beta^W$ if $y_1^W(\beta^W) > y$. This is depicted in Figure 4, where $\beta^W$ is assumed to be in the rising portion of the $A^W(\beta)$ curve.\footnote{It is technically possible that the curve $A^W(\beta)$ be declining throughout its range. In that case, $\beta^W = 1$ as long as $y_1 \geq y$ is not sufficiently binding. We do not discuss that case further, though it can also give rise to either Regime being preferred.} If the minimum income constraint $y_1 \geq y$ is binding, it is conceivable that the left-hand side of (19) is negative, in which case $\beta^W$ is on the falling portion of the $A^W(\beta)$ curve. The solution to (19) yields a value of social welfare $SW^W(\beta^W)$. Analogously, the necessary condition for the optimal $\beta$ in Regime $N$ is given by:

$$
\left( V^0 \left( g^N(\beta) \right) - V^0 \left( c_0^N(\beta) \right) \right) - \delta_1 \left( V^1 \left( g^N(\beta), 0 \right) - V^1 \left( c_0^N(\beta), 0 \right) \right) = \lambda^N_3 \frac{2M}{\partial \beta} \frac{\partial A^N(\beta)}{\partial \beta}. \tag{20}
$$
Since individuals are not separated in the general welfare program, a marginal change in $\beta$ does not affect the payments in that program. Given that $g^N(\beta) > g^N_0(\beta)$ by Lemma 4, the first term of the left-hand side is positive and the second term is negative. The right-hand side is clearly positive since administration costs are unambiguously increasing in $\beta$ in this Regime (see Figure 4). Let $\beta^N$ be the solution obtained by solving (20). At this optimum, social welfare is given by $SW^N(\beta^N)$.

It is difficult to compare $SW^W(\beta^W)$ and $SW^N(\beta^N)$ in general. But it is possible to stipulate circumstances under which Regime $W$ will be preferred over Regime $N$, and vice versa. For example, suppose that $\beta^N$ is greater than $\tilde{\beta}$. For this range of $\beta$, we have seen that $A^W(\beta) < A^N(\beta)$. In addition, if $y_1 \geq \tilde{y}$ would not be binding in Regime $W$ when $\beta = \beta^N$, then efficiency can be improved by separating the individuals in the general welfare program. That is, starting in the optimum of Regime $N$, social welfare can be improved upon by moving to Regime $W$. Therefore, $SW^W(\beta^N) > SW^N(\beta^N)$. And, since $SW^W(\beta^W) > SW^W(\beta^N)$, we can conclude that $SW^W(\beta^W) > SW^N(\beta^N)$ and that Regime $W$ is superior to $N$.

On the other hand, there are two possible circumstances in which Regime $N$ might be preferred to Regime $W$. Suppose first that the minimum income constraint $y_1 \geq \tilde{y}$ is not binding, but that $\beta^N$ is low enough such that $A^N(\beta^N) < A^W(\beta^N)$. Then, it is may be that $A^N(\beta^N) < A^W(\beta^W)$, so administrative costs are lower in Regime $N$ than in Regime $W$. This case is depicted in Figure 4. If the saving in administrative costs by adopting Regime $N$ exceeds the benefits from the better information ($\beta^W > \beta^N$) and from separating the low-ability persons from the disabled in the general welfare program in Regime $W$, the former will be preferred. Alternatively, suppose that $y_1 \geq \tilde{y}$ is binding in the optimum of Regime $W$. This makes it more costly to separate low-ability types from the disabled, and may make Regime $N$ better than Regime $W$ even if $A^N(\beta^N) > A^W(\beta^W)$. So it is possible that Regime $N$ is superior to Regime $W$ ($SW^N(\beta^N) > SW^W(\beta^W)$). Generally, there will be some $\tilde{y}$ large enough to make Regime $N$ superior to Regime $W$. To summarize:

**Proposition 4:**

(i) If $\beta^N \geq \tilde{\beta}$ and $y_1^W(\beta^N) > y$, then Regime $W$ is superior to Regime $N$ ($SW^W(\beta^W) > SW^N(\beta^N)$).

(ii) If $\beta^N$ is sufficiently low or if $y_1^W(\beta^W) \geq \tilde{y}$ is sufficiently binding, it is possible that Regime $N$ is superior to Regime $W$ ($SW^N(\beta^N) > SW^W(\beta^W)$).

It is obviously difficult to be more precise about when Regime $W$ or Regime $N$ would be preferred. That depends upon the relevant parameters of the economy. To illustrate the possibility that either regime might be preferable we next provide a simple illustrative example.

**An Illustrative Example**

A simple example illustrates the properties of an optimal welfare program alongside an optimal income taxation and provides a more concrete welfare comparison between the
two Regimes. Suppose that all able persons have quasi-linear preferences of the form
\[ V^i(c, y) = c - h_i(y) \] (i = 1, 2) where \( h_i(y) \) is the disutility of supplying income and has the following properties \( h'_1(y) > h'_2(y), \ h''_1(y) > h''_2(y) \) and \( h_1(0) = h_2(0) = 0 \). This would be consistent with both persons having the same convex disutility-of-labor function \( h(L) \), such that \( h_i(y) = h(y/w_i) \). The utility function of the disabled is assumed to be logarithmic: \( V^d(c) = \log c \). Suppose also that the government cares only about the disabled, so the objective function is the expected utility of disabled persons: \( \beta \log g + (1 - \beta) \log c_0 \). After some manipulation, we can obtain the maximized expected utilities of the disabled, given \( \beta \), for each Regime by:

\[
SW^W(\beta) = \beta \log \beta + (1 - \beta) \log \left( \frac{1 - \beta}{2} \right) + \log \left( \frac{\nabla^2 + H(\beta, y) - 2A^W(\beta)}{2} \right) \quad (21a)
\]

\[
SW^N(\beta) = \beta \log \beta + (1 - \beta) \log \left( \frac{1 - \beta}{2} \right) + \log \left( \frac{\nabla^2 - 2A^N(\beta)}{2} \right) . \quad (21b)
\]

where \( \nabla^2 \equiv \max\{y - h_2(y)\} \) and \( H(\beta, y) \equiv \max_{y_1 \geq y}\{\beta y_1 - (1 + \beta)h_1(y_1) + h_2(y_1)\} \). The term \( H(\beta, y) \) can be interpreted as the net reduction in cost from separating the low-ability from the disabled in the welfare system.

We can confirm with numerical examples that either regime \( N \) or Regime \( W \) may be preferred under reasonable circumstances. Let the cost of auditing a proportion \( q \) of \( R \) welfare recipients be given by \( C(qR) = c(qR)^2 \). For simplicity, we normalize the number of individuals of each type to unity (i.e., \( M = 1 \)). Also, suppose that to obtain accuracy \( \beta \), social workers have to exert effort costing \( \rho(\beta - 1/2)^3 \). Finally, suppose that to obtain income \( y \), able individuals have to incur a cost \( h_i(y) = (\sigma_i/2)y^2 \), with \( \sigma_1 > \sigma_2 \) reflecting the fact that low-ability individuals are less efficient at generating income than high-ability individuals. Thus for the able persons, \( V^i(c, y) = c - (\sigma_i/2)y^2 \), while for the disabled \( V^d(c) = \log c \). By specifying parameter values for \( \alpha, \rho, \gamma, \sigma_1, \sigma_2, \) and \( y \), we use equations (21a) and (21b) to calculate the values of \( \beta^W \) and \( \beta^N \) that maximize \( SW^W \) and \( SW^N \) in the two Regimes. Consider the following three cases.

i) \( SW^W > SW^N \)

Suppose that the parameters take the following values: \( \alpha = 3, \rho = 5, \gamma = 5, \sigma_1 = 1, \) and \( \sigma_2 = 0.1 \). Initially let the minimal income requirement \( y \) be very low (say \( = 0.1 \)) so it is not binding. Then, social welfare in the optimum of Regime \( W \), reached at \( \beta^W = .837 \) is well above social welfare in the optimum of Regime \( N \), reached at \( \beta^N = .644 \) (\( SW^W = .840; \ SW^N = .644 \)). Thus, Regime \( W \) is superior to Regime \( N \) for those parameters. The value for \( \beta \) turns out to be \( .885 \), implying that the effort incentive constraint is binding in the optimum of Regime \( W \).

ii) \( SW^N > SW^W; \ y_1 = y \)

Now suppose that the minimal income requirement \( y \) is increased to \( y = 2 \), but all other parameter values are kept the same. The minimal income requirement then becomes a
binding constraint for the government. In this case, social welfare in the optimum of Regime $N$, which is unchanged, is above social welfare in the optimum of Regime $W$ ($SW^W = .199; SW^N = .644$). We also obtain $\beta^W = .799 < \beta$, so the effort incentive constraint is binding.

iii) $SW^N > SW^W; y_1 > y$

As mentioned above, social welfare could be larger in Regime $N$ even when $y_1 \geq y$ is not a binding constraint. If so, the superiority of Regime $N$ over Regime $W$ comes from the tightness of the effort incentive constraint when $\beta$ is sufficiently low. Consider the following parameter values: $\alpha = 5, \rho = 20, \gamma = 5, \sigma_1 = 1$, and $\sigma_2 = 0.1$. Suppose again that the minimal income requirement is not constraining ($y$ is chosen to be small). Then, $SW^N = .599$ compared with $SW^W = .480$. In the optimum of the two regimes, $\beta^N = .564$, while $\beta^W = .738 < \beta = .836$. It is interesting to note that the level of social welfare (here, expected utility of the disabled) that would be obtained by dispensing with welfare and relying solely on a non-linear negative income tax system is lower than $SW^N$ in this case. (Social welfare is .562, which is above $SW^W$ but below $SW^N$.) This is so despite the fact that under the negative income tax system, there would be no bunching at the bottom, unlike with Regime $N$.

Given these various benefits and costs, whether or not Regime $W$ is preferred to Regime $N$ is obviously an empirical issue. Reasonable circumstances can be imagined in which either Regime is the preferred one. Moreover, whether or not either Regime is preferred to a simple negative income tax system that does not take advantage of tagging is also an empirical question. It depends upon whether the costs of administering the tagging system, $A^W$ or $A^N$, outweigh the advantages of tagging in terms of better targeting of transfers to the needy.\textsuperscript{19} One interesting point to note is the following. It is well-known that in a negative income tax scheme, it may be efficient not to have the low-ability people working.\textsuperscript{20} But is may be the case that Regime $W$ dominates either the negative income tax or Regime $N$. That is, it may be optimal to induce low-ability persons to work in order to implement tagging in the most efficient way. And this may be so even if the constraint $y_1^W(\beta) \geq y$ is binding. The reason is that forcing the low-ability persons to work allows the government to detect type I errors and therefore reduces the cost of monitoring the

\textsuperscript{19} Note that if $Y_1 = 0$ in the optimum of a single negative income tax system (so there is bunching at the bottom), the introduction of a welfare program is unambiguously welfare enhancing. Let $(C_0^I, Y_1^I) = (0)$ and $(C_1^I, Y_2^I)$ be the solution to the government optimization when only the negative income tax is operated. Keep the bundles unchanged and introduce a Regime $N$ welfare program with $\beta = 1/2$ ($g = C_0^I$). Since $\beta_2 = 0$, the high-ability person will not apply for the disabled benefit while the disabled and the low-ability person may apply. Thus the initial allocation is still incentive compatible. Note that $A^N(1/2) = 0$, so the initial allocation is feasible after the welfare program is introduced. For $\beta = 1/2$, by Lemma 6, we know $g > y_0$ in the optimum, which implies $SW^N(1/2) > SW$ (negative income tax). We have $\text{Max}(SW^W(\beta^W), SW^N(\beta^N)) \geq SW^N(1/2)$ and thus we can get the desired result.

\textsuperscript{20} This was apparent in Mirrlees (1971) original analysis.
social workers. By the same token, the converse is true. Regime N may be preferred even if under the negative income tax system it would be efficient for the low-ability persons to work. Thus, the various benefits and costs of employing a tagging technology allow virtually anything to happen.

V. EXTENSIONS

Our analysis has been conducted in the simplest of settings. It is worth contemplating how the structure of welfare programs and the robustness of our results might be affected by introducing some natural complications and extensions to the model. In this section we briefly consider the implications of the following factors: altruistic preferences for the disabled among the social workers, workfare and allowing tagged persons to work. The discussion will be primarily intuitive and will rely on lessons that have been learned from the more formal modeling of the previous sections.

Altruistic Social Worker Preferences

It is reasonable to suppose that social workers may attach particular weight to the welfare of the disabled; this may have motivated the choice of social work as a profession in the first place. For simplicity, suppose such altruism does not extend to the low-ability persons. The existence of altruism can affect both the tightness of the effort incentive constraint and the choice of effort by the social worker. Consider the former first. The social worker must decide whether or not to exert effort before knowing anything about an applicant. Ex ante, the probability of a given applicant being disabled is one-half. If an applicant is accepted without tagging, the expected altruistic benefit to the social worker is simply \( V^A(g)/2 \). This must be added to \( EU^A \) defined earlier to determine the expected utility of accepting all applicants and exerting no effort. Alternatively, suppose that the social worker participates in the tagging technology. Again, ex ante the probability that a given client will be disabled is one-half, and therefore the probability that a tagged applicant is actually disabled is \( \beta/2 \). Similarly, the probability that an untagged applicant is disabled is \( (1 - \beta)/2 \). Thus, the expected altruistic benefit to the social worker per applicant is \( \beta V^A(g)/2 + (1 - \beta)V^A(c_0)/2 \). This must be added to \( EU^W \) (given by (2)) to determine the expected utility of engaging in the tagging technology. Since \( g > c_0 \), the altruistic part is larger when the social worker accepts all applicants than when he uses the tagging technology. This implies that altruism will cause the social worker to have more incentive to accept them than when he is selfish, which in turn implies that the effort incentive constraint is tightened in Regime W (and it can even make it effective in Regime N).\(^{21}\)

\(^{21}\) The presence of altruism makes the lowest levels of \( \beta \) non-implementable in both Regimes. First consider Regime W and recall that \( q^W(1/2) = 1 \) when the social worker is completely selfish: \( q = 1 \) equates the non-altruistic component of expected utility from tagging with that from accepting all applicants. In the present context, however, the gain from the altruistic component in \( EU^A \) is still higher than the one in \( EU^W \) for \( \beta = 1/2 \), which leads to \( EU^A > EU^W(-) \) even with \( q = 1 \). The same is true for Regime N: the non-altruistic component is
Besides affecting the effort constraint, altruism for the disabled can also influence the social worker's choice of effort in tagging. It is easy to see that an increase in $\beta$ raises the value of the altruistic component by $(V^0(g) - V^0(c_0))/2$, and this must be added to $\partial EU^j/\partial \beta (j = W, N)$. This implies that $q$ and $k = S$ need not be set so high relative to the one in the basic model to implement a given $\beta$, which in turn implies lower administration costs in both Regimes for $\beta$ such that the effort constraint is not effective. In addition, since $\partial EU^j/\partial \beta (j = W, N)$ is increasing in $g$ and decreasing in $c_0$, higher (lower) $g$ or lower (higher) $c_0$ brings more (less) effort by the social worker.

Overall effect of the altruistic motive on administration costs (after the government solves the cost minimization for given $\beta$, income tax and social policy as described in section III) is not straightforward in either Regime. To summarize, an increase in $g$ and/or a decrease in $c_0$ tightens the effort constraint, while encouraging effort given that the social workers use tagging. Therefore, administration costs can be influenced by $g$ and $c_0$ as well as by $\beta$.

**Workfare**

As noted by Besley and Coate (1992), requiring welfare recipients to work may serve as a screening device to separate the needy from other potential beneficiaries. In our context, workfare has somewhat different implications. Suppose that all untagged applicants are required to work a certain amount of hours, say $y_1 \geq y$, to qualify for the welfare transfer of $c_1 - y_1$. The untagged disabled cannot work so cannot receive welfare; they are left to the negative income tax system where they receive the bundle $(C_0, 0)$. When the self-selection constraint $V^1(c_1, y_1) \geq V^1(C_0, 0)$ is satisfied, the untagged households are separated as in Regime $W$. But there is critical difference between Regime $W$ and workfare. In the former, by observing income $y_0 = 0$, the government can detect type I errors by each social worker since it can identify the social worker who is responsible for the error. In the present context, however, by observing $Y_0 = 0$ in the income tax system, the government cannot trace the source of the type I error without auditing because the tie between the social worker and the untagged disabled person is broken. Therefore, the informational structure is the same as in Regime $N$, and the minimized administration cost for a given $\beta$ is $A^N(\beta)$ under workfare. Except for the fact that $c_0$ is replaced by $C_0$, we obtain the same set of participation and self-selection constraints as in Regime $W$.

When the minimum income constraint $y_1 \geq y$ is not binding for a given $\beta$ so it is efficient to separate the low-ability from the disabled, workfare is advantageous relative to Regime $N$: it can achieve the benefits of separating while keeping the administration cost unchanged from Regime $N$. In our example in the previous section, for a given $\beta$, the same for tagging as for accepting both for $\beta = 1/2$, while the altruistic gain is higher for the latter than for the former.

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22 We assume the labor done by the participants in the workfare is productive, in contrast with Besley and Coate (1992) who assume it has no productive value.
maximized social welfare under workfare is given by:

\[
SW^W(\beta) = \left(\beta \log \beta + (1 - \beta) \log \left(\frac{1 - \beta}{2}\right)\right) + \log \left(W^2 + H(\beta, y) - 2A^N(\beta)\right)
\]

The comparison between \(SW^W(\beta)\) and \(SW^N(\beta)\) is solely dependent on the sign of \(H(\beta, y)\). If \(H(\beta, y) > 0\) (that is, the net gain from inducing the low-ability persons to work is positive), workfare is preferable to Regime \(N\), and vice versa. On the other hand, the relation between \(SW^W(\beta)\) and \(SW^W(\beta)\) is determined by the size of \(A^N(\beta)\) relative to \(A^W(\beta)\). It is immediately apparent that \(SW^W(\beta) > SW^W(\beta)\) if and only if \(A^N(\beta) < A^W(\beta)\).

Once \(\beta\) is optimized for each Regime, welfare comparisons are limited and generally ambiguous as in the basic model. Let \(\beta^{WF}\) be the optimal level of \(\beta\) under workfare. We can say that \(SW^W(\beta^{WF}) < SW^W(\beta^{WF})\) holds if \(A^W(\beta^{WF}) > A^N(\beta^{WF})\). And, \(SW^W(\beta^{WF}) > SW^W(\beta^{WF})\) if \(A^W(\beta^{WF}) < A^N(\beta^{WF})\). Thus, as in Besley and Coate (1992), workfare can be a useful policy instrument in the right circumstances. Of course, the case for workfare will be weakened if the required work is less unproductive than market work. And, it can be strengthened if the workfare requirement allows a smaller amount of work that the minimum required to earn \(y\) in the private sector.

**Tagged Persons Allowed to Work**

We have so far considered the case where the welfare recipients can work in the general welfare program (Regime \(W\)) but tagged persons must accept the same bundle \((g, 0)\) in the disability program. The presence of type II error may justify allowing welfare recipients to work in the disability benefit. Parsons (1996), who abstracts from administration costs and who assumes that persons who work have no control over the amount they work and earn, shows that it may be efficient to allow tagged persons to work if the cost of separating the able from the disabled persons is not too great. The latter depends upon the preferences and ability of the tagged able persons, and upon the amount of work effort required. In our model, we have the additional complications of administrative costs and variable earnings. Following the convention of Parsons, let us refer to the case where people are separated in all three programs (disability, general welfare, and income tax) as ‘triple negative income taxation’ and denote it by superscript ‘\(T\)’.

The benefit of offering two bundles \((g_0, 0)\) and \((g_1, y_1^T)\) (with \(y_1^T \geq y\)) in the disability benefit and inducing the low-ability persons to choose the latter bundle is that it enables the government to detect type II errors. In Regime \(W\), the government can already identify type I error, so now both types of errors can be detected without auditing. In this triple negative income tax system, the expected utility of each social worker when using the tagging technology is given by \(EU^T(\cdot) = k\beta - \phi(\beta)\). Alternative strategies, such as accepting all applicants, are not useful now since all errors can be detected. Therefore, the government is not confronted with the effort incentive constraint. Minimized administration costs for a given \(\beta\) is \(A^T(\beta) = \beta\phi'(\beta)\).

Despite the fact that both type I and type II errors can be traced automatically, administration costs \(A^T(\beta)\) differs from that under full information \(\phi(\beta)\). This is due to
the constraint \( S \geq k \). When the restriction of \( y_1 \), \( y_1 \overset{T}{\geq} y \) is either not binding in the optimum or not so significant, the triple negative income tax system is preferred to other welfare schemes. It should be emphasized that the advantage of this sort of income tax system is not only in improving the allocation by the separation of households, but also in saving the administration cost by improving information.
Errors
Regime W | Regime N
\[
\begin{array}{c|cc}
\text{Errors} & 0 & k & k \\
\text{Regime W} & \text{Regime N} \\
\hline
\text{type I} & 0 & (1-q)k \\
\text{type II} & (1-q)k & (1-q)k \\
\end{array}
\]

Figure 1
Figure 2
Figure 3
Figure 4

$A^W(\beta), A^N(\beta)$

$A^N(\beta)$

$A^W(\beta)$

$\epsilon(1)$

$A^W(\beta^W)$

$A^N(\beta^N)$

$\frac{1}{2}$

$\beta^N$  $\bar{\beta}$  $\beta^W$  $\beta$
APPENDIX

**Proof of Lemma 1:** Combining (7) and (10), it is possible to obtain:

\[ A^N(\beta, q) - A^W(\beta, q) > (\beta + (1 - \beta)(1 - q)) \frac{\phi'(\beta)}{q} - (2\beta + (1 - \beta)(1 - q)) \frac{\phi'(\beta)}{1 + q} \]

\[ = \frac{\phi'(\beta)}{q(1 + q)} ((1 + q) \cdot (\beta + (1 - \beta)(1 - q)) - q \cdot (2\beta + (1 - \beta)(1 - q))) \]

\[ = \frac{\phi'(\beta)}{q(1 + q)} (\beta - (1 - \beta)(1 - q)) = \frac{\phi'(\beta)}{q(1 + q)} (1 - q) \geq 0 \]

Therefore, \( A^N(\beta, q) - A^W(\beta, q) > 0 \). QED

**Proof of Proposition 1:** For \( \beta \geq \bar{\beta} \), \( q^W(\beta) = q^*W(\beta) \). Using this,

\[ A^N(\beta) \equiv A^N(\beta, q^N(\beta)) > A^W(\beta, q^N(\beta)) \geq A^W(\beta, q^*W(\beta)) \equiv A^W(\beta) \]

The inequality comes from Lemma 1. QED

**Proof of Lemma 3:** The first-order conditions in Regime W are:

\[ g : \quad \beta V_c^0(g) + \delta_1 (1 - \beta) V_c^1 + \mu_0^W - \lambda_3^W M = 0 \quad (A - 1) \]

\[ c_0 : \quad (1 - \beta) V_c^0(c_0) - \mu_0^W - \lambda_1^W V_c^1 - \lambda_3^W (1 - \beta) M = 0 \quad (A - 2) \]

\[ c_1 : \quad \delta_1 \beta V_c^1 + \lambda_1^W V_c^1 - \lambda_2^W V_c^2 - \lambda_3^W M = 0 \quad (A - 3) \]

\[ y_1 : \quad \delta_1 \beta V_y^1 + \mu_1^W + \lambda_1^W V_y^1 - \lambda_2^W V_y^2 + \lambda_3^W M = 0 \quad (A - 4) \]

\[ C_2 : \quad \delta_2 V_c^2 + \lambda_2^W V_c^2 - \lambda_3^W M = 0 \quad (A - 5) \]

\[ Y_2 : \quad \delta_2 V_y^2 + \lambda_2^W V_y^2 + \lambda_3^W M = 0 \quad (A - 6) \]

where \( V_c^0 = V^1(g, 0) \) and \( V_c^1 \) is the marginal utility of a type i mimicker with respect to argument \( a \). From (A - 2) and (A - 3),

\[ (1 - \beta) V_c^0(c_0) + \delta_1 \beta V_c^1 - \mu_0^W - \lambda_1^W (\tilde{V}_c^1 - V_c^1) - \lambda_2^W V_c^2 - \lambda_3^W M = 0 \]

Subtracting (A - 1) from this equation gives:

\[ ((1 - \beta) V_c^0(c_0) + \delta_1 \beta V_c^1) - (\beta V_c^0(g) + \delta_1 (1 - \beta) V_c^1) = 2\mu_0^W + \lambda_1^W (\tilde{V}_c^1 - V_c^1) + \lambda_2^W \tilde{V}_c^2. \]

Since \( c_0 < c_1 \) (because of the incentive constraint \( V^1(c_1, y_1) \geq V^1(c_0, 0) \) and \( V_{cy}^1 \leq 0 \), the right-hand side is strictly positive: \( V_c^1(c_1, y_1) = V_c^1 \leq \tilde{V}_c^1 = V_c^1(c_0, 0) \). Suppose that \( g = c_0 \) in the optimum. Then the left-hand side is:

\[ (1 - 2\beta) V_c^0(c_0) + \delta_1 (\beta V_c^1(c_1, y_1) - (1 - \beta) V_c^1(c_0, 0)) \leq (1 - 2\beta)(V_c^0(c_0) - \delta_1 V_c^1(c_0, 0)) \]

30
where the inequality obtains because $V^1_c(c_1, y_1) \leq V^1_c(c_0, 0)$. Because $\delta_1 \leq 1$, $\beta \geq 1/2$, and $V^0_c(c) \geq V^1_c(c, 0)$, it is clear that $(1 - 2\beta)(V^0_c(c_0) - \delta_1 V^1_c(c_0, 0)) \leq 0$. This, in turn, implies that left-hand side becomes non-positive. This, however, contradicts the fact that the right-hand side is strictly positive. QED

**Proof of Lemma 4:** In Regime $N$, the first-order conditions for $g$, $C_2$ and $Y_2$ are the same as $(A - 1)$, $(A - 5)$ and $(A - 6)$. The first-order condition with respect to $c_0$ is:

$$c_0 : \quad (1 - \beta)V^0_c(c_0) + \delta_1 \beta V^1_c - \mu^N_0 - \lambda^N_2 \tilde{V}^2_c - \lambda^N_3 M = 0. \quad (A - 2')$$

The proof is analogous to the one of Lemma 3. QED
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