Queen's Economics Department Working Paper No. 34

TRADE WITH A PRODUCED TRANSPORTATION GOOD

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3-1971

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March 1971

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I. Introduction

The analysis of transportation costs in interregional or international trade has been focussed principally on the effects of transport costs on relative commodity prices between traders. Viner considered this problem in his Studies [1937; pp. 468ff]; Samuelson examined the effects of transport costs on the terms of trade in the context of "the transfer problem" [1954]. Mundell [1957] reworked Samuelson's analysis in terms of geometry.[1] More recently, however, Hadley and Kemp [1966] have proved the existence of a general equilibrium in a m country, n commodity model where transportation is a produced good. Pearce [1970; Chapts. 11 and 16] has analysed cases when transport costs preclude trade and presented notes on the effects of changes in transportation costs on factor prices in a general trade model.[2]

This paper is in one sense the analysis of a particular case of the Hadley and Kemp model. We shall focus on two region or country trade with two orthodox commodities plus a produced transport commodity. We shall be concerned with three topics: the welfare effects of trade with endo-

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*I am much indebted to Keith Acheson and Dan Usher for criticisms and comments but neither person should be implicated with any remaining errors.

genous transport costs, the balance of payments equilibrium, and factor pricing problems. The two principal results of the analysis will be that countries can become collectively worse off in trade than in autarky given endogenously produced transport goods and second that international payments will in general never be in balance in a world with endogenous transport goods. The first result can be explained as follows: there are gains from trade when factor endowments differ but there are costs of trade when transport goods must be provided. The process of decentralized decision-making in the face of observed relative international commodity prices can in general induce people in different countries to trade and in a wide class of cases the respective societies will consume more in transportation goods in the form of factors foregone than can be gained from trade. This can then be considered as a basic immiserization result.

We will observe that balance of payments disequilibrium is a general result with trade with endogenous transport costs and finally that reductions in transport costs may not result in factor prices moving closer together.

2. The Model and Equilibrium

Our model will be comprised of two countries, each producing three commodities, the commodity numbered

1 in each country will be a transportation good. This transportation good will be entirely consumed or evaporated (the term is Samuelson's) in the process of transporting either of the other two commodities between countries. The transportation good will be tradable and it will be assumed that no transport costs are incurred in the transit. This latter assumption is not essential. Only commodities numbered 2 and 3 enter the utility functions of the societies in the respective countries. The model is similar to that developed by Samuelson [1954] and Mundell [1957] except that transportation is here introduced as a third produced good whereas Samuelson introduced it as a part of the two traded commodities which evaporated in transit. The Samuelson model will be illustrated as a special case of the model in this paper in Section 4.

Let the production functions for commodities 1,2 and 3 in the home and foreign country be

$$(2.1) \quad y_i = f_i(l_i, k_i, n_i); \; y_i^* = f_i^*(l_i^*, k_i^*, n_i^*) \qquad (i=1,2,3)$$

respectively, where y_i denotes the home output of commodity i, l_i the labor input, k_i the capital input, and n_i the resource input into the home production of commodity i. Starred quantities denote the corresponding entities in the foreign country f_i and f_i^* are assumed to be concave, positively homogeneous of the first degree and differentiable.

The factors of production are assumed to be subject to the restrictions

$$(2.2)\quad l_1+l_2+l_3 \leqq l \quad ; \quad l_1^*+l_2^*+l_3^* \leqq l^*$$

$$k_1+k_2+k_3 \leqq k \quad ; \quad k_1^*+k_2^*+k_3^* \leqq k^*$$

$$n_1+n_2+n_3 \leqq n \quad ; \quad n_1^*+n_2^*+n_3^* \leqq n^*$$

where l, l^* are the fixed endowments of labour in the two countries, k and l^* the endowments of capital, and n and n^* the endowments of resources.

Factor-product coefficients will be denoted by

$$(2.3)\quad \begin{array}{lll} a_i = \dfrac{l_i}{y_i} \; ; & b_i = \dfrac{k_i}{y_i} \; ; & c_i = \dfrac{n_i}{y_i} \\ a_i^* = \dfrac{l_i^*}{y_i^*} \; ; & b_i^* = \dfrac{k_i^*}{y_i^*} \; ; & c_i^* = \dfrac{n_i^*}{y_i^*} \end{array} \qquad (i=1,2,3)$$

when y_i, $y_i^* > 0$. From the assumed homogeneity,

$$(2.4)\quad f_i(a_i,b_i,c_i) = 1; \quad f_i^*(a_i^*,b_i^*,c_i^*) = 1 \qquad (I=1,2,3).$$

Let w,r,s and w^*,r^*,s^* denote the wage rate, rental of capital, and rental of resources in the respective countries. The minimum unit cost functions are defined for non-negative w,r,s and w^*,r^*,s^*, by

$$(2.5)\quad g_i(w,r,s) = \min \quad wa_i+rb_i+sc_i : f_i(a_i,b_i,c_i) = 1$$

$$g_i(w^*,r^*,s^*) = \min \quad w^*a_i^*+r^*b_i^*+s^*c_i^* :$$

$$f_i^*(a_i^*,b_i^*,c_i^*) = 1$$

These are also homogeneous of degree one, and concave (cf. Uzawa [1964]). The factor-product ratios entering the minimum unit cost functions (2.5) depend on the factor prices: when the dependence is unique, the functional relationship will be denoted by

$$(2.6)\quad \begin{matrix} a_i(w,r,s), b_i(w,r,s), c_i(w,r,s) \\ a_i^*(w^*,r^*,s^*), b_i^*(w^*,r^*,s^*), c_i^*(w^*,r^*,s^*) \end{matrix} \qquad (i=1,2,3)$$

where the functions are defined for positive w,r,s, w^*,r^*,s^*.

Let p_2 and p_3 denote the price of the second and third commodities relative to the first in the home country. p_2^* and p_3^* will be the corresponding prices in the foreign country. Admissable prices for commodities 2 and 3 must satisfy the conditi on that the difference between p_i and p_i^* must equal the cost of transporting a unit of i between the countries. Let t_i be the physical amount of the transportation good required to make a round trip in order to deliver a unit of commodity i on one way,

$(i=2,3)$. Since the price of the transportation good is unity, then admissable prices are defined by

$$2.7) \quad p_2 = p_2^* + t_2 > 0$$

$$p_3 = p_3^* - t_3 > 0$$

when commodity 2 is exported by the foreign country and commodity 3 is exported by the home country.

It will be recalled[3] that a point

$$(2.8) \quad (Y_1, Y_2, Y_3) = (y_1 + y_1^*, y_2 + y_2^*, y_3 + y_3^*)$$

of efficient world production is one for which there exists a vector $p = (1, p_2, p_3, 1, p_2^*, p_3^*) > 0$ such that

$$(2.9) \quad V = y_1 + p_2 y_2 + p_3 y_3 + y_1^* + p_2^* y_2^* + p_3^* y_3^*$$

is a maximum subject to (2.1) and (2.2). An efficient point with diversification (the term is due to Chipman) is defined as a point of efficient world production such that

$$(2.10) \quad y_i > 0, \; y_i^* > 0 \qquad (i=1,2,3)$$

that is, each country produces a positive amount of each commodity. We shall deal with the case in which such an efficient point exists.

A necessary and sufficient condition that there be some factor endowments l, l^*, k, k^*, n, n^* for which an efficient point with diversification exists, is that the system of equations

$$2.11) \quad g_1(w,r,s) = 1 \quad ; \quad g_1^*(w^*,r^*,s^*) = 1 \quad ;$$
$$g_2(w,r,s) = p_2 \quad ; \quad g_2^*(w^*,r^*,s^*) = p_2^* \ ;$$
$$g_3(w,r,s) = p_3 \quad ; \quad g_3(w^*,r^*,s^*) = p_3^* \ ;$$

have a solution $w \geqq 0$, $w^* \geqq 0$, $r \geqq 0$, $s \geqq 0$ $s^* \geqq 0$.

From the assumptions of differentiability and strict concavity of the production functions, it follows (cf. Shephard [1953; p11]) that the minimum unit cost functions are differentiable for $w > 0$, $w^* > 0$, $r > 0$, $r^* > 0$, $s > 0$, $s^* > 0$, with partial derivatives equal to

$$\frac{\partial g_i(w,r,s)}{\partial w} = a_i(w,r,s); \quad \frac{\partial g_i^*(w^*,r^*,s^*)}{\partial w^*} = a_i^*(w^*,r^*,s^*);$$

$$\frac{\partial g_i(w,r,s)}{\partial r} = b_i(w,r,s); \quad \frac{\partial g_i^*(w^*,r^*,s^*)}{\partial r^*} = b_i^*(w^*,r^*,s^*);$$

$$\frac{\partial g_i(w,r,s)}{\partial s} = c_i(w,r,s) \quad \frac{\partial g_i^*(w^*,r^*,s^*)}{\partial s^*} = c_i^*(w^*,r^*,s^*)$$

Suppose (2.11) has a positive solution $\bar{w} > 0$, $\bar{w}^* > 0$, $\bar{r} > 0$, $\bar{r}^* > 0$, $\bar{s} > 0$, $\bar{s}^* > 0$; then the cost-minimizing factor-product ratios are these particular factor prices will be denoted

$$(2.12) \quad \bar{a}_i = a_i(\bar{w},\bar{r},\bar{s}); \quad \bar{a}_i^* = a_i^*(\bar{w}^*,\bar{r}^*,\bar{s}^*)$$
$$\bar{b}_i = b_i(\bar{w},\bar{r},\bar{s}); \quad \bar{b}_i^* = b_i^*(\bar{w}^*,\bar{r}^*,\bar{s}^*)$$
$$\bar{c}_i = c_i(\bar{w},\bar{r},\bar{s}); \quad \bar{c}_i^* = c_i^*(\bar{w}^*,\bar{r}^*,\bar{s}^*)$$

For these $\bar{a}_i, \bar{b}_i, \bar{c}_i, \bar{a}_i^*, \bar{b}_i^*, \bar{c}_i^*$, the given solution $\bar{w}, \bar{w}^*$, $\bar{r}, \bar{r}^*, \bar{s}, \bar{s}^*$ of (2.11) must be a solution of the linear system.

$$(2.13) \quad \begin{bmatrix} \bar{a}_1 & \bar{b}_1 & \bar{c}_1 & 0 & 0 & 0 \\ \bar{a}_2 & \bar{b}_2 & \bar{c}_2 & 0 & 0 & 0 \\ \bar{a}_3 & \bar{b}_3 & \bar{c}_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{a}_1^* & \bar{b}_1^* & \bar{c}_1^* \\ 0 & 0 & 0 & \bar{a}_2^* & \bar{b}_2^* & \bar{c}_2^* \\ 0 & 0 & 0 & \bar{a}_3^* & \bar{b}_3^* & \bar{c}_3^* \end{bmatrix} \begin{bmatrix} w \\ r \\ s \\ w^* \\ r^* \\ s^* \end{bmatrix} = \begin{bmatrix} 1 \\ p_2 \\ p_3 \\ 1 \\ p_2 - t_2 \\ p_3 + t_3 \end{bmatrix}$$

Recall that a non-negative solution of 2.13 indicates that an efficient point with diversification exists or that (2.9) has a maximum subject to (2.1) and (2.2). We are assuming the required solution exists.[5]

We shall choose our aggregate utility functions to be

$$(2.14) \quad \begin{aligned} u &= x_2^{\theta_2} x_3^{\theta_3} \\ u^* &= x_2^{*\theta_2} x_3^{*\theta_3} \end{aligned} \qquad \left(\theta_j > 0, \ \sum_{j=2}^{3} \theta_j = 1\right)$$

where these functions[6] are defined over positive values of commodity i (x_i) consumed. Note that commodity 1, the transportation good yields no utility directly to consumers. One cannot "live on" transportation goods.

A <u>trade equilibrium</u> will be a pair of vectors

$(1, p_2, p_3, 1, p_2^*, p_3^*)$ which satisfies (2.7) and $\quad (y_1, y_2, y_3, y_1^*, y_2^*, y_3^*)$

which define an efficient point with diversification and satisfy

$$\frac{\frac{\partial u}{\partial x_2}}{\frac{\partial u}{\partial x_3}} = \frac{p_2}{p_3}$$

or

$$\theta_2 p_3 x_3 - \theta_3 p_2 x_2 = 0 \tag{2.15}$$

and

$$\frac{\frac{\partial u^*}{\partial x_2^*}}{\frac{\partial u^*}{\partial x_3^*}} = \frac{p_2^*}{p_3^*}$$

or

$$\theta_2 p_3^* x_3^* - \theta_3 p_3^* x_3^* = 0 \tag{2.16}$$

and

$$y_1 + y_1^* = x_1 + x_1^* \tag{2.17}$$

$$y_2 + y_2^* = x_2 + x_2^* \tag{2.18}$$

$$y_3 + y_3^* = x_3 + x_3^* \tag{2.19}$$

$$x_1 = t_3(y_3 - x_3) \tag{2.20}$$

$$x_1^* = t_2(y_2^* - x_2^*) \tag{2.21}$$

Conditions (2.15) and (2.16) are price equilibrium conditions in consumption of commodities 2 and 3 in the home and foreign country respectively. Condition (2.17) is a commodity balance relation for the transportation good: production must equal the amount required in trade. For example $x_1^* = t_2(y_2^*-x_2^*)$ is the physical amount of transportation good demanded to transport exports $(y_2^*-x_2^*)$. Conditions (2.18) and (2.19) are commodity balance relations for commodities 2 and 3: the total amount produced must equal the total amount consumed. Conditions (2.20) and (2.21) are transportation good use relations. For example the home country consumes or evaporates $x_1 = t_3(y_3-x_3)$ amount of transportation good in exporting amount (y_3-x_3) of commodity 3.[7]

The existence of a trade equilibrium is assured since the model satisfies the basic theorem of Debreu which in turn permits the application of the fixed point theorem of Kakutani.[8]

Observe that an equilibrium in the payments balance between countries does not in general obtain in a trade equilibrium. Condition (2.23) or the transportation good equilibrium condition in one sense displaces the payments equilibrium. The production decisions by transportation good producers and the demands by transporters cause condition (2.23) to obtain in equilibrium. In the familiar trade model with zero transport costs, the activity of trading equalizes product prices between countries and co-incidentally causes an equilibrium or balance to obtain in international payments.[9]

3. Welfare Aspects of a Trade Equilibrium

The equilibrium developed in Section 2 is illustrated in Figure 1 for the case of the home country.

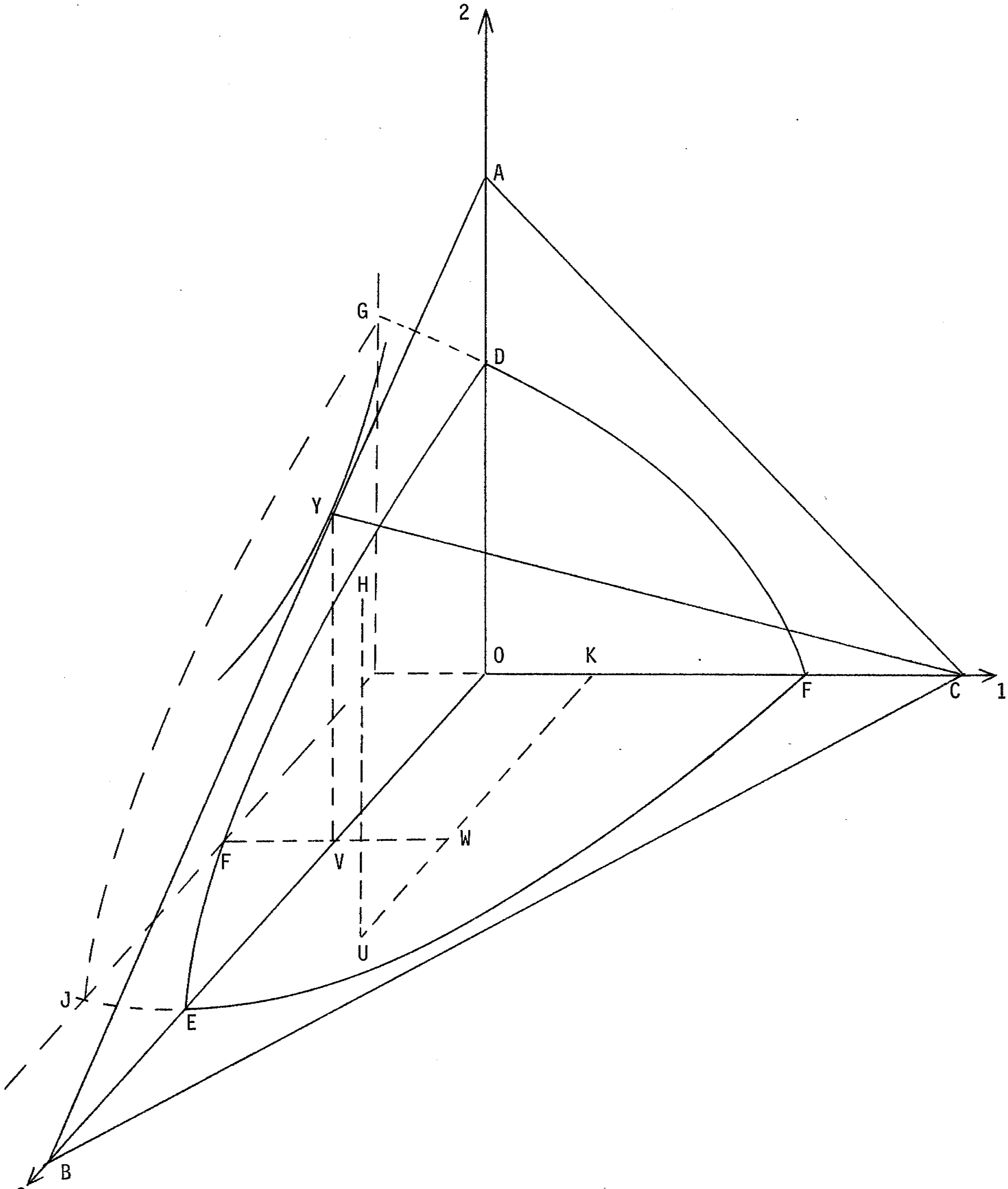

Figure 1.

In Figure 1, the axes are numbered in accordance with the commodities. GFJ is the production possibility surface in the home country. ABC is the price plane indicating the relative prices of commodities in the home country in equilibrium. This price plane will differ in attitude from that in the foreign country because of the existence of transport costs. ABC is tangent to the production possibility surface at point H. The home country will produce UK of commodity 3 in equilibrium. (The production of commodity 2 is not indicated.) The community indifference curve is defined only in the plane of commodities 2 and 3. The possible tangency points of the community indifference curve and the price plane trace out line YC. The consumption point in equilibrium for the home country is at Y. UW indicates the amount of commodity 3 exported.

The production possibility block is projected back along the transportation good axis to indicate that a portion of output (and factors) have been consumed or evaporated in the process of trade. In fact amount TV of the transportation good has been used up in exporting UW of commodity 3. The ratio TV/UW equals t_3 or the physical amount of transportation good required to transport a unit of commodity 3 between the countries.[10] Amount UW of transportation good is exported from the home country to the foreign country.

Let us briefly review the nature of equilibrium in the familiar two country, two commodity, two factor trade model. The basic theorems are as follows. (1) Profit maximization by producers causes the economy of each country to attain an efficient point in the respective country and commodity prices exist at the efficient points.[11] (2) Profit maximization by inter-country commodity transporters causes commodity prices to equalize between countries when there are no impediments to trade in the form of tariffs, taxes, or transport costs. We have a series of other theorems which do not have a behavioral foundation analogous to that in (1) and (2).

(3) Factor prices are equalized between countries in equilibrium given certain assumptions on technology and factor endowments. (4) The welfare in each country is maximized in equilibrium given certain assumptions on individual and group utility functions. (5) The international payments between countries are in balance in equilibrium. Propositions (3), (4) and (5) can usefully be considered as corollaries to propositions (1) and (2) combined.

With a two country, three commodity, three factor trade model where one commodity is a transportation good, we have seen that profit maximization by producers in the respective countries causes efficient points to obtain. Transportation good producers are included. Secondly, profit maximization by transporters of commodities between countries causes commodity prices on goods 2 and 3 to differ between countries in equilibrium by exactly the cost of transport when such goods are in fact traded.

Factor prices will never be equalized when transportation costs exist, Pearce [1970; p. 356]. We shall consider this issue in detail in Section 5. We noted that balance in international payments will not obtain. We have been assuming that an interior solution to our problem obtains or an equilibrium with diversification. For such an equilibrium to obtain, certain restrictions must be imposed on factor endowments, the technology of production and the technology of transportation. These restrictions are neither unusual nor severe. However no restriction is imposed on the relative magnitudes of commodities produced. In other words, if a trade equilibrium can exist with the given data concerning factor endowments and technology, such an equilibrium will obtain because of the motivation of profit maximization. There is no restriction on the amount of transportation good consumed in a trade equilibrium. In fact, factor endowments and technology permitting, a trade equilibrium can conceivably obtain when say ninety per cent of each country's factors are being consumed in transportation. There is nothing in the model to rule this case out and in fact profit maximization by producers and transporters will cause it to obtain. Clearly, a society may possibly attain a higher social indifference curve by producing in autarky or not participating in trade and thus avoiding producing or consuming any transportation goods. The factors foregone in the production of transportation goods can be used for the production of the commodities 2 and 3. These latter goods enter the utility functions of individuals and the society as a whole whereas transportation goods do not.

To summarize, if factor endowments and technology are such as to permit a trade equilibrium with diversification to obtain, profit maximization by producers and transporters will cause that equilibrium to obtain, and at that trade equilibrium, there is a determinable amount of transportation good produced in both countries. There is no economic mechanism which restricts the proportions in which resources are devoted to transportation goods relative to commodities which yield utility directly, namely commodities 2 and 3.[12]

In a trade quilibrium when there is a produced transportation good, both countries can be better off than in autarky, both can be worse off than in autarky, or one can be better off and the other worse off than in autarky. We can adapt Figure I to show the home country better off in trade than in autarky or worse off in trade than in autarky. We take Figure I and twist the diagram so that the transportation good axis is projecting from the page at a right angle to the page. Figure 2 results.

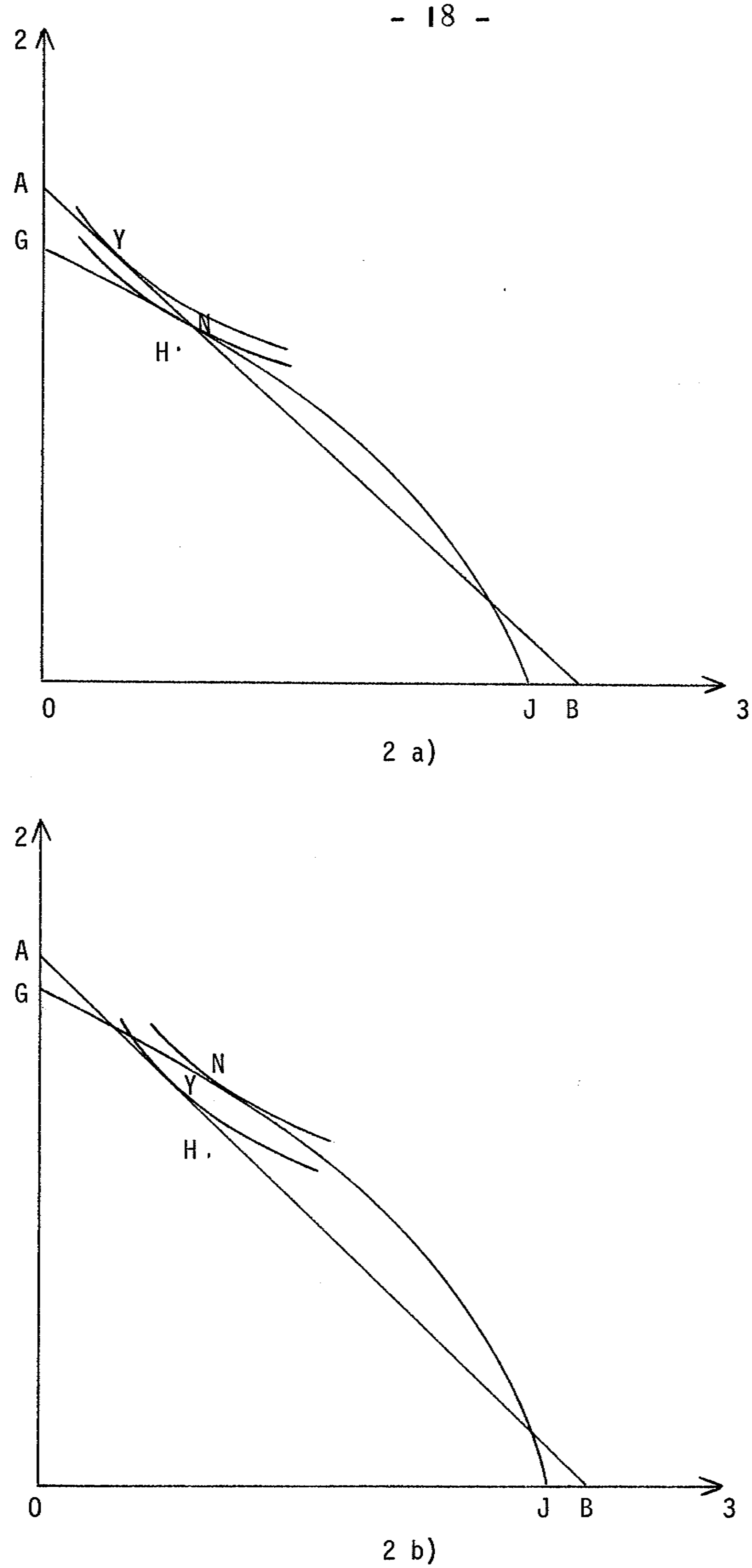

Figure 2.

In Figure 2, the labels indicate the elements of the diagram which correspond with those described for Figure 1. For example AB is the price line indicating the relative prices for commodities 2 and 3 in a trade equilibrium. In Figure 2a the home country's social indifference curve is tangent to the price line at Y. GJ indicates the transformation surface when no transportation goods are being produced. In autarky no transportation goods need be produced and the indifference curve would be tangent to the transformation surface at N. Since the indifference curve through Y lies above that through N, the society is better off at the trade equilibrium than in autarky. In Figure 2b, we have a situation in which the indifference curve through Y for a trade equilibrium lies below that for a situation of autarky, the curve through N. Thus the home country is worse off in a trading situation. The behavioral foundations of economic theory are such as to have the economy driven to the trade equilibrium as if by an invisible hand.

In a centrally directed economy, situations of trade with welfare losses relative to situations of autarky would not arise. A central planner is always free to rank alternative equilibria before these equilibria actually obtain. A planner would evaluate the welfare consequences of opening his country to trade at world market prices and compare that relative welfare level with the level which would obtain in autarky. If welfare was less with trade than in autarky, the planner would decree autarky shadow prices to prevail on factors and let the economy move to the autarky equilibrium.

4. The Samuelson-Mundell Evaporation Model

Samuelson [1954] made use of the following simplifying assumption in order to investigate the effects of transportation costs on trade: "To carry each good across the ocean you must pay some of the good itself...just as only a fraction of ice exported reaches its destination as unmelted ice, so will a_x and a_y be the fractions of exports X and Y that respectively reach the other country as imports. Of course, $a_x < 1$ and $a_y < 1$, except in the costless model, where they were each unity". [1954; p. 1016] I shall refer to this assumption as the evaporation hypothesis. Mundell [1957] subsequently utilized the same assumption in his geometric treatment of transportation cost in trade. It is implicit in Pearce's analysis of transportation costs [1970; p. 356].

In the price space this assumption implies that the equilibrium condition for the traded good and the transportation good associated with that traded good are identical. That is for commodity 2 for example

$$(4.1) \quad wa_2 + rb_2 = p_2^* + p_1t_2 = p_2$$

and

$$(4.2) \quad wa_2 + rb_2 = p_1 = p_2$$

for its transportation good.

Hence,

$$(4.3) \quad p_2^* = p_2(1-t_2)$$

We will not get distinct changes in the prices of commodity 2 and its associated transportation good when transportation costs exist. Recall that under the evaporation hypothesis, different desired, traded goods (commodities numbered 2 and 3 in Section 2) have distinct transportation goods associated with them.

The model with four commodities (two commodities each with its own transportation good) and two factors under the conditions of the evaporation hypothesis readily reduces to a two good model. This is the simplification Samuelson sought to exploit. The price equations for transportation goods are redundant and we end up with a two factor, two commodity model with a particular kind of incomplete commodity and factor price equalization indicated in (4.1), (4.2), and (4.3).

In price space, the equilibrium conditions are identical to those obtaining if t_i, the transportation cost coefficient for commodity i, were actually a tariff on commodity i. However in commodity space, the equilibrium conditions differ in the analysis of tariffs or produced transportation goods with evaporation following Samuelson. This is because a fraction of goods produced is evaporated in transit in model with transportation goods and no physical evaporation of commodities takes place in a trade model with tariffs. Tariffs impose costs via shifts in the terms of trade whereas transportation costs impose costs via shifts in the terms of trade and in terms of commodities consumed in the process of transit.

The possibility of welfare losses being observed in trade equilibria compared with situations of autarky carries over to this evaporation model. That is, there is no limit on the total amounts of commodities and hence factors which will be evaporated in transportation if trade takes place. However, the possibility of trading transportation goods is not present in this evaporation model and thus there is no possibility of observing transportation goods produced where the comparative advantage in their production lines.

5. Factor Prices and Changes in Transport Costs

In this section I attempt to interpret statements in the international trade literature concerning factor prices moving closer together between two countries or farther apart but not in fact being equal. In particular I pose the question do factor prices move closer together when transportation costs become less between countries? The approach taken to answer this question is first to define a situation when factor prices converge and then to conduct numerical comparative static tests with Cobb-Douglas production functions.

In the textbooks, we do in fact have statements concerning movements of factor prices when there are distortions in trade. Consider the following two quotations:

(1) "We can say that with tariffs and transportation costs, there is still [given trade] a tendency toward factor price equalization without its being complete" Heller [1968, p. 101].

(2) "Since factor prices can never be equal where transport costs exist, it follows that, if the set of all weighted averages of techniques in use abroad in the production of all commodities...overlaps the set of all weighted averages of techniques used in the home country ...then there must exist at least one commodity in production abroad satisfying $p_j^A + t_j \leqq p_j^B$ and/or one commodity in production abroad satisfying $p_i^B + t_i \leqq p_i^A$. The situation cannot be one of equilibrium. Production of the 'maverick' commodities must be falling....Hence factor

prices will become more equal". Pearce [1970; p. 356] The question is, what constitutes a tendency toward factor price equalization and if such a tendency has a meaning, does it obtain when transportation costs are reduced? Neither Heller nor Pearce define their change in factor prices.

We will use the following definition of convergency of factor prices. It will serve both for two factor and three factor cases. If the sum of the absolute values of the deviations of factor prices between two countries is smaller in situation A than in situation B then we say factor prices have converged in the transition from situation A to situation B. The opposite result will be divergence of factor prices and when the sum of the absolute values of the deviations of factor prices is zero then factor price equalization has obtained. This definition is consistent with the following notion:[13] If vectors x,y, and z are related as follows $x < y < z$ we might describe y as being "closer to" x than z or "more equal to" x than z. And if y' changes to y'' from a perturbation in such a way that $x < y'' < y'$ then we might describe the change as one in which y' tends to x. We must normalize prices by the same procedure in the two situations under comparison.

We will define an increase in transportation costs to have occurred in situation B when the sum of the values of the transportation requirement coefficients in situation B exceeds the sum in situation A.[14] In other words, when more physical units of transportation good are required in a position of trade than in another, we say transportation costs have increased in the former relative to the latter situation.

Our observation is the following: As transportation costs decline, factor prices do not necessarily move closer together in countries A and B with three commodities and three factors when one commodity is a transportation good.[15] The transportation good will be number 1 in these experiments.

We set up the price equilibrium conditions as in (2.11) for the case of Cobb-Douglas production functions and alter the values of the transportation costs. The familiar price equilibrium conditions are presented below.

$$w^{\alpha_1} T^{\beta_1} s^{\gamma_1} k_1 = p_1$$

$$w^{\alpha_2} r^{\beta_2} s^{\gamma_2} k_2 = p_2$$

$$w^{\alpha_3} r^{\beta_3} s^{\gamma_3} k_3 = p_3$$

for the home country and

$$w^{*\alpha_1} r^{*\beta_1} s^{*\gamma_1} k_1 = p_1 + t_1 p_3$$

$$w^{*\alpha_2} r^{*\beta_2} s^{*\gamma_2} k_2 = p_2 - t_2 p_3$$

$$w^{*\alpha_3} r^{*\beta_3} s^{*\gamma_3} k_3 = p_3$$

for the foreign country.

$$\text{where} \quad k_i = \left(\frac{\gamma_i}{\beta_i}\right)^{\beta}\left(\frac{\alpha_i}{\gamma_i}\right)^{1-\alpha_i} + \left(\frac{\gamma_i}{\alpha_i}\right)^{\alpha_i}\left(\frac{\beta_i}{\gamma_i}\right)^{1-\beta_i} + \left(\frac{\gamma_i}{\alpha_i}\right)^{\alpha_i}\left(\frac{\gamma_i}{\beta_i}\right)^{\beta_i}$$

and $\alpha_i + \beta_i + \gamma_i = 1,\ 0 < \alpha_i,\ \beta_i,\ \gamma_i < 1.$

In Table I, we have three examples illustrating the proposition.

TABLE I

Production Coefficients for the Two Countries

$\alpha_1 = 0.5$	$\beta_1 = 0.15$	$\gamma_1 = 0.35$	
$\alpha_2 = 0.85$	$\beta_2 = 0.10$	$\gamma_2 = 0.05$	(TIA)
$\alpha_3 = 0.6$	$\beta_3 = 0.10$	$\gamma_3 = 0.30$	

Prices in the Home Country

$p_1 = 2.71$	$w = 1.0$	
$p_2 = 1.68$	$r = 1.0$	(TIB)
$p_3 = 2.45$	$s = 1.0$	

(TIC)

	Prices in the Foreign Country			Transport Coefficients		Factor Price Deviations	Transport Costs
	w^*	r^*	s^*	t_1	t_2		t_1+t_2
I	1.3	0.95	0.6019	0.0583	0.1452	0.7481	0.2035
	1.2	1.40	0.6208	0.0276	0.1225	0.9792	0.1501
II	1.8	0.95	0.3140	0.1243	0.3744	1.5360	0.4988
	1.6	1.40	0.3402	0.0879	0.3168	1.6508	0.4046
III	1.6	0.95	0.3974	0.1009	0.2849	1.2526	0.3859
	1.5	1.25	0.4126	0.0785	0.2605	1.3374	0.3390

We observe in Table I for each example in part TIC we see from the last two columns that Transport Costs decline and Factor Price Deviations increase. Factor price deviations are defined as the absolute value of $(w^*-w)+(r^*-r)+(s^*-s)$.

6. Conclusions

By not abstracting from transportation costs in an examination of a simple general equilibrium trade model, we have discovered some important anomalies. The activities of people in markets can lead them to be made collectively worse off. More potential output can be consumed in effecting trade than can be gained from trade in many cases and there are no signals to direct people away from this outcome. The model used to derive this result is quite general judged by the standards of modern economic theory. The question presenting itself is whether the model is implausible or whether the results in this paper are relevant to actual trading situations. There is a bulky literature on trade issues suggesting that transportation costs severely circumscribe the gains from trade for underdeveloped countries exporting primary commodities and importing manufactured commodities. Arguments of this genre emphasize the terms of trade effects of transportation costs. But where the real burden of transportation costs falls (or the incidence of evaporation) must also be investigated. The model in this paper is relevant to these issues.

However, to the extent that trade takes place between regions, countries, or cities which produce localized commodities, then the model is less relevant. For example consider an agricultural region exporting wheat to a city which in turn exports manufactured goods to the rural area. Because there may be substantial real losses from transportation, the city cannot change to an autarky position. Wheat cannot be grown in town. There is really no way in which the costs of trade, the transportation goods consumed, can be avoided. A trade equilibrium will obtain but we cannot say that there are welfare losses relative to an autarky situation since the autarky situation is not a possible equilibrium. Product cycle theories of trade are variants of this conception of a world with trade. Technically speaking, technology may not be smooth in the way in which we have assumed it to be in this model.

We should note that there is a fundamental difference between introducing endogenous transport costs on the movement of commodities compared with costs on the movement of factors. In the absence of transport costs, the world production possibility set is enlarged with commodity trade over the situation in autarky. However, with or without factor mobility and commodity trade, the world production possibility set remains the same. Endogenous transportation costs on factors can only shrink the world production possibility set whereas endogenous transport costs on commodities still allows for the possibility of an enlargement of the world production possibility set with trade than the world production possibility set in autarky.

The introduction of transportation costs to trade models implies a variety of mobility assumptions concerning flows of goods and possibly factors. Samuelson [1953; p. 12] has made explicit the usually unwritten assumptions in Heckscher-Ohlin analysis, factors are assumed to have infinite transportation costs and commodities to have zero transport costs. Pearce [1970; p. 321] has in fact elevated mobility assumptions to a pivotal position distinguishing interregional from international trade. He defines interregional trade to exist when factors are mobile and international trade to exist when factors are immobile. In both cases of course commodities are assumed to be mobile.

Where trade theory has been under-utilized is in explaining the nature of economic landscapes within countries. It is in this realm of focus that mobility assumptions in a Heckscher-Ohlin model become crucial in explaining economic topography. In international economic relations the abstraction from transport costs may not have distorted the basic analytics of the phenomenon of trade. The above analysis can be viewed as a contribution to international trade theory and also as a model for location analysis.

FOOTNOTES

1. The Viner and Mundell treatments are summarized in Herberg [1968]. Herberg also develops a new framework for analyzing the effects of transport costs on commodity prices.

2. Hadley and Kemp deal only with the question of existence and the results in this paper were not adumbrated in their contribution. Pearce's analysis was conducted in terms of one representative country in trade and details of how transportation costs were introduced or of balance of payments equilibrium were not discussed. Pearce's analysis of transportation costs precluding trade was conducted in an offer curve framework in a modified Edgeworth-Bowley box.

3. The original result on efficient points is due to Koopmans [1951].

4. See Chipman [1970b]. This condition is the familiar one stating that in equilibrium, no industry will be earning excess profits in either country. Inequalities would obtain in (2.11) if specialization (non-diversification) were permitted. We are thus restricting ourselves to "interior solutions".

5. This is a question of having factor endowments which permit an "interior solution" to obtain.

6. This form of aggregate utility function can be derived from individual utility functions of a like form and thus desirable aggregation properties are preserved. See Chipman [1970a; pp. 236-37].

7. We are making the following assumption about institutional arrangements: namely that the transportation good is linked with the good transported at the point of export. Alternate institutional arrangements are conceivable and such alternate specifications will change the quantitative aspects of a trade equilibrium but not the qualitative ones.

8. Debreu [1959; p. 19] presents the result. We have admissable vectors in S, the environment of a certain agent, in this case the production system. T is the set of output vectors a priori available

to him. $\emptyset(p)$ (assumed to be closed for every p in S) is the subset of T to which his choice is actually restricted by his environment p. $f(p,y)$, a continuous real-valued function on SxT, is the gain or value of net output for that agent when his environment is p and his action is y. Given $\emptyset$, one is interested in the elements of $\emptyset(p)$ which maximize f (now a function of y alone) on $\emptyset(p)$; they from a set $\mu(p)$. One is also interested in $g(p)$, the value of the maximum of f on $\emptyset(p)$. Theorem: If f is continuous on SxT, and if $\emptyset$ is continuous at $p \in S$, then μ is upper semi-continuous at p, and g is continuous at p. Kakutani's fixed point theorem can now be invoked and the existence of a trade equilibrium is assured.

9. Since a balance in international payments does not obtain in this model, a monetary mechanism must develop in order that international credits and debits can be introduced. This should not be surprising although the inherent absence of balance in international payments is I believe a new result.

10. See footnote 7.

11. Not all conceivable technologies permit this result to obtain. I am restricting the discussion to well-behaved or what might be termed orthodox cases. These propositions hold of course in many cases with more than two countries, more than two factors, and more than two commodities.

12. We have been proceeding as if the trade equilibrium were unique. This is correct if as we have been assuming an equilibrium with diversification exists. If there were two factors in each country then an equilibrium with specialization would obtain. Only one country would produce transportation goods. This can be seen readily by examining the zero profit conditions of (2.13) when production functions are Cobb-Douglas and each country produces three goods with two factors. Note prices between countries for the two goods traded at positive transportation costs must differ and prices between countries for the transportation good must be the same.

13. Consistent is used here to mean that the vector ordering $x < y < z$ satisfies the definition used in the paper but that the definition used in the paper

does not necessarily satisfy the stated vector ordering $x < y < z$ for y close to x.

14. The following examples indicate the results is also true if we define a change in transportation costs in value terms, $p_i t_i$, rather than in only physical terms.

15. This result holds for two countries, three commodities and two factors also. Analogous simulations have demonstrated this.

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