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Risk Aversion Heterogeneity, Risky Jobs and Wealth Inequality

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Abstract

This paper considers the macroeconomic implications of a set of empirical studies finding a high degree of dispersion in preferences for risk. It develops a model with risk aversion heterogeneity, uninsurable idiosyncratic income risk, and (with or without) self-selection into risky jobs to quantify their effects on the distribution of wealth. The results show that the role of risk aversion heterogeneity is quantitatively important. When estimating the risk aversion distribution with the appropriate PSID data on income lotteries, the model matches the observed degree of wealth inequality in the U.S., accounting for both the wealth Gini index and other key features of the wealth distribution. It is also shown that neglecting risk preference heterogeneity has a first order effect on the aggregate allocations.

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1 Introduction

How important are observable differences in risk aversion coupled with labor income risk in accounting for the U.S. distribution of wealth? This paper addresses this question within a macroeconomic model with incomplete markets, heterogeneous agents and (with or without) self-selection into risky jobs.

It is a well known fact that wealth is highly concentrated in the U.S., its Gini index being estimated in the $0.78-0.82$ range for the period $1992-2007$. At the same time, income and labor earnings are less unequally distributed, as discussed by Wolff (1998), Cagetti and De Nardi (2008), and Diaz-Gimenez, Glover, and Rios-Rull (2011).

The wealth distribution and its determinants have important implications for capital accumulation and growth, the design of optimal taxation schemes and their welfare consequences. These issues have been studied, for example, by Imrohoroglu (1998), Ventura (1999), Heathcote (2005), and Conesa, Kitao and Krueger (2009).

The role of entrepreneurship, Quadrini (2000) and Cagetti and De Nardi (2006), uninsurable income risk, Castaneda, Diaz-Gimenez, and Rios-Rull (2003), and intergenerational links, De Nardi (2004), have been found to be key in explaining the high concentration of wealth.[1] One aspect that has not been fully explored in accounting for wealth inequality is the role of preference heterogeneity. This is interesting, particularly in light of the findings of a few recent studies trying to elicit individuals' preferences and determine some measures of their dispersion.

This paper takes these empirical results seriously and studies the importance of risk aversion heterogeneity as a determinant of saving behavior. The workhorse heterogeneous agents macroeconomic model with incomplete markets and uninsurable idiosyncratic income risk is extended to allow for risk aversion heterogeneity. The framework is used to compute the effects of the latter on measures of wealth inequality and, more generally, on the overall wealth distribution, with or without endogenous sorting into jobs that differ in their implied labor income risk.

A recent body of empirical research has aimed at estimating individuals' risk aversion. These contributions have relied on either special modules included in large and well established surveys, or on experimental set-ups. Surveys such as the Health and Retirement Study (HRS) and the Panel Study of Income Dynamics (PSID) for the U.S. (Barsky, Juster, Kimball, and Shapiro (1997), Kimball, Sahm, and Shapiro (2008), and Kimball, Sahm, and Shapiro (2009)), and the Survey of Household Income and Wealth (SHIW) for Italy (Chiappori and Paiella (2011)), have been used to provide estimates of the risk aversion distributions for the underlying populations. In the HRS and, to a less

[1] There is a vast literature developing models that try to account for the dispersion of wealth, reviewed in Quadrini and Rios-Rull (1997), and more recently in Cagetti and De Nardi (2008), and Heathcote, Storesletten and Violante (2009).

extent, in the PSID the respondents have been asked to answer a sequence of questions describing hypothetical lotteries on their *lifetime* income. Under appropriate assumptions, these answers provide a direct measure of the respondent's risk aversion. Differently, in the SHIW it is possible to exploit the households' portfolio composition and, crucially, its change over time (which is a rare feature for large surveys) to identify the risk aversion distribution. In this case the empirical methodology does not measure preferences directly, rather it backs them out from actual intrinsically risky economic choices. The alternative experimental set-ups have been conducted either in a lab environment with a limited number of Danish individuals, Harrison, Lau, and Rutstrom (2007), or with an on-line interface targeting a large set of Dutch households, von Gaudecker, van Soest, and Wengstrom (2011). These experiments were designed for the participants to win or lose some *small* monetary stakes, by selecting the lottery they preferred from a menu of options that differed in their degree of risk.

The possible studies' limitations notwithstanding, a common and seemingly robust finding is the high dispersion of people's attitude towards risk. Both the survey and the experimental approaches to measuring preference dispersion have found considerable heterogeneity in the estimated risk aversion parameter. Figure (1) displays the risk aversion distributions resulting from the estimation procedures implemented by Kimball, Sahm, and Shapiro (2009) and Chiappori and Paiella (2011). I think it is fair to say that this plot provides compelling evidence on the dispersion of risk preferences.

[Figure (1) about here]

Identifying and estimating the risk aversion distribution is a challenging procedure. Pervasive measurement error and limited stakes in the experimental lotteries are unavoidable obstacles that make it a very hard empirical problem. However, in order to avoid some potential drawbacks of the analysis that will be discussed below, it is preferable to rely on sources of risk aversion heterogeneity that are empirically grounded, rather than treating it as unobserved heterogeneity.

First, I am going to consider a model where all individuals face the same stochastic income process. In this framework, I will follow the methodology proposed by Kimball, Sahm, and Shapiro (2009) to estimate the distribution of risk tolerance from PSID data.[2]

Not only the individuals' attitude towards risk is an important ingredient in shaping the consumption and saving decisions, but also the amount of risk faced in the labor market will affect their

[2] In a previous version of the paper I was also considering the distribution of risk preferences derived from the results reported in Chiappori and Paiella (2011). Their study is based on a panel dataset that tracks a representative sample of Italian households. As shown in Figure 1, this distribution differs in some essential features from the one derived from the PSID data. However, the results were found to be quantitatively quite similar for both specifications. Moreover, one should embed this alternative specification in a model calibrated for the Italian economy. This is quite challenging, because of the short SHIW panel dimension, that makes the estimation of stochastic income processes fairly unreliable.

behavior. Labor income risk can be either considered as purely exogenous, or it can be partially driven by self-selection into jobs that differ in their earnings risk.

In the first case, given the wide variety of estimates available in the literature, a number of assumptions on the parameters representing the stochastic process for labor earnings are going to be considered.

The results show that the role of risk aversion heterogeneity is quantitatively important. Compared to a model without preference heterogeneity, the degree of wealth concentration increases substantially. For a standard model with homogeneous preferences the wealth Gini index lies in the $0.50-0.62$ range (depending on the assumed stochastic process for income risk), while for the heterogeneous preferences model the range changes to $0.76-0.85$, which is consistent with what is observed in the data.

When the risk aversion distribution is estimated on the PSID data on attitudes towards risky labor income, the model can match some salient features of the U.S. wealth distribution, such as the bottom and top quintiles. The share of wealth held by the people at the very top almost doubles with heterogeneity in risk preferences, with the share of the top 10% moving from $32.6\% - 43.5\%$ to $57.4\% - 72.8\%$. However, the share of wealth held by the top 1% is still underestimated in all specifications: preference heterogeneity alone does not fully account for the determination of the very top of the wealth distribution.

Although convenient, the assumption that individuals do not sort themselves into jobs that differ in their income volatility according to their preference for risk seems to be strong, and at odds with some recent evidence discussed in Guiso, Jappelli and Pistaferri (2002), Fuchs-Schundeln and Schundeln (2005), Bonin, Dohmen, Falk, Huffman, and Sunde (2007), and Schulhofer-Wohl (2011). The PSID data can also be used to analyze how people's actual earnings instability changes with their risk aversion. This paper also provides a first attempt to tackle this complex empirical issue. It will be assumed that only two different careers are available for every worker to embark on, whose entailed risks are perfectly known to the workers before entering the labor market. The workers are going to sort themselves into the two different jobs, by choosing one of the two different stochastic processes.

A calibrated version of the model with endogenous sorting generally confirms the findings obtained in the simpler version of the model. Moreover, the results related to both wealth and consumption inequality are found to be closer to the data. With this framework the wealth Gini index is 0.81, while the consumption Gini index is 0.27.

Some additional results show that precautionary savings are underestimated in models without preference heterogeneity, with the bias being up to 40 percentage points. Furthermore, considering the homogeneous preferences counterpart of the heterogeneous preferences economy does lead to equilibria and allocations that are very different. The interest rate in the heterogeneous preferences economies is between 1.61 and 3.74 percentage points lower, always leading to substantially larger capital stocks, output, consumption and wages. Neglecting risk aversion heterogeneity has a first order effect also on the aggregate allocations and on the macroeconomic outcomes.

1.1 Related literature

In a seminal paper, Becker (1980) showed that in a deterministic growth model with infinitely lived agents that differ in their discount factors, the whole capital stock is held by the most patient dynasty. However, this result is mainly driven by the absence of uncertainty and by the assumption of complete markets.

There is limited quantitative work on the aggregate effects of preference heterogeneity in stochastic growth models. Notable exceptions are Krusell and Smith (1998), Cagetti (2003), Coen-Pirani (2004), Guvenen (2006), and Hendricks (2007). However, there are a few differences between my framework and theirs.

Krusell and Smith (1998) consider an RBC model with heterogeneous agents. They propose two versions of their model, one with preference heterogeneity and one without. Limited ad-hoc heterogeneity in the discount factors across generations is shown to have a major impact on wealth inequality, with the model being able to account for the observed Gini index only in this case.

Cagetti (2003) and Hendricks (2007) propose life-cycle models with incomplete markets, stochastic incomes, but without aggregate uncertainty. They focus on preference heterogeneity arising from differences in discount factors, and Cagetti (2003) allows for (some) heterogeneity in the risk aversion parameter as well. A structural estimation/calibration procedure is implemented to match a set of moments computed from the PSID and SCF samples. Cagetti (2003) estimates the preference parameters by matching the median age/wealth profile for three educational groups. However, he considers a partial equilibrium model. Hendricks (2007) picks the discount factors and their distribution to match the wealth Gini index profiles over the life-cycle. Their findings show that this type of preference heterogeneity accounts for the dispersion in wealth holdings of observationally equivalent households, and matches the high concentration of wealth.

Coen-Pirani (2004) and Guvenen (2006) study economies with aggregate fluctuations, multiple assets (a risk free asset and a risky one) and with a recursive utility framework. Their agents can potentially differ in both their risk aversions and in their Elasticities of Intertemporal Substitution (EIS). Although interesting, I don't follow this approach, because there are no reliable estimates for the EIS distribution for the overall U.S. population.[3] Coen-Pirani (2004) considers an endowment

[3] The results discussed in Barsky, Juster, Kimball, and Shapiro (1997) are based on a very small sample (less than 200 observations), and apply to a selected sample, namely people in the later stages of their lives.

economy with preference heterogeneity only in the risk aversion parameter: his findings show that, contrary to the results obtained using standard expected utility, for some parameter values the long run distribution of wealth is dominated by the more risk averse agents. Guvenen (2006) proposes an RBC model with heterogenous EIS reconciling the findings of empirical studies using aggregate consumption data that estimate a low EIS, and those of calibrated models designed to match growth and fluctuations facts that require a higher EIS. His results arise because of limited participation in stock markets together with an EIS increasing in wealth. The EIS estimated on simulated aggregate consumption data is small, because it reflects the EIS of the majority of the households, that are asset poor.

Compared to the literature, I allow for a form of preference heterogeneity which is empirically grounded, for GE effects, and for endogenously chosen risky careers. However, for tractability, I do not consider the effects of aggregate shocks.

In the empirical literature, Lawrance (1991) and Vissing-Jorgensen (2002) provide estimates of the consumption Euler equations, finding heterogeneity in time preferences and in the EIS. Furthermore, the results in Chiappori and Paiella (2011) support the notion that individuals' relative risk aversion is constant as they find no significant response of the portfolio structure to changes in financial wealth.

The rest of the paper is organized as follows. Section 2 presents the theoretical model. Section 3 is devoted to the definition of the equilibrium concept used in the model. Section 4 presents the calibration procedure. Section 5 provides the main results and predictions of the baseline model, while Section 6 is devoted to the extension with endogenous sorting into risky jobs. Section 7 discussed the macroeconomic outcomes and Section 8 concludes. Three Appendices discuss the details of the numerical methods and provide some additional results.

2 The Economy

2.1 Risk Aversion Heterogeneity and Inequality: the Mechanics

The intuition behind this paper is easily understood with a simple graph.

[Figure 2 about here]

Figure (2) compares the hypothetical endogenous wealth distributions that would arise in two different economies. Both economies share the same features, namely incomplete markets, an occasionally binding exogenous borrowing constraint and uninsurable labor income risk, as in Aiyagari

(1994). However, they differ in one aspect: in the first economy all agents share the same risk aversion parameter γ, while in the second economy there are both high risk aversion types (γ_h), and low risk aversion ones (γ_l).[4] Figure 3 displays utility functions of the CRRA class for different degrees of constant relative risk aversion γ: the higher γ, the more concave the utility function and the stronger the precautionary savings motive (Huggett and Ospina (2001)).

[Figure 3 about here]

For the sake of the argument, let's assume that both types have the same mass $\mu_{\gamma_h} = \mu_{\gamma_l} = \frac{1}{2}$, and that the types are permanent, meaning that there is no evolution in their risk aversion.

In the first economy the wealth distribution is non-degenerate because agents want to self-insure against the future risk represented by income fluctuations. This leads to a wealth distribution which could be represented as the one in the middle of Figure (2). Asset rich agents are the ones who experienced a long sequence of good income shocks, unlike the asset poor ones, allowing them to pile up a large stock of wealth. In the second economy, agents face the same uncertainty, i.e. the same stochastic process for labor income. However, the endogenous wealth distribution is going to change from the first economy: low risk averse types have a lower desire to smooth consumption across states of the world, hence they are going to accumulate less assets for self-insurance purposes. Differently, the more risk averse agents are going to save more, in order to achieve a better consumption smoothing. If the two agent types were to live in isolation, their wealth distributions would look like the ones in Figure (2). The wealth distributions of the two types are still non-degenerate, because labor income risk is still present and with concave utility functions some self-insurance is going to take place. In the absence of General Equilibrium (GE) effects, the economy would display a wealth distribution represented by the mixture of the two underlying ones. An outcome is clear: moving from the first economy to the second one, wealth inequality is going to increase, because the supports of the heterogenous types wealth distributions will start diverging. The higher the difference in their risk aversion, the stronger the tendency for the two groups to accumulate different wealth levels.

However, the economy with heterogeneous types doesn't aggregate into its homogeneous types counterpart, with the average (or median) risk aversion: risk aversion heterogeneity leads to different

[4] The analysis will rely on utility functions of the CRRA class. This choice is dictated by the set of results discussed in Kimball, Sahm, and Shapiro (2008) and Kimball, Sahm, and Shapiro (2009), obtained assuming this type of preferences, and in Chiappori and Paiella (2011), who directly test this assumption. Notice that the first two papers provide estimates for the risk tolerance parameter τ, not the risk aversion γ. However, there is a simple relationship between the two: $\gamma = \frac{1}{\tau}$. Consequently, with CRRA utility, risk tolerance and elasticity of intertemporal substitution coincide. Moreover, recall that when a random variable is log-normally distributed, its reciprocal is log-normally distributed as well, with the same parameter σ_γ^2 but opposite μ_γ: $\gamma \sim LN\left(\mu_\gamma, \sigma_\gamma^2\right) \rightarrow \tau = \frac{1}{\gamma} \sim LN\left(-\mu_\gamma, \sigma_\gamma^2\right)$.

aggregate capital supplies, triggering GE effects. A strong enough precautionary saving motive of the high risk aversion types leads to an increase in the aggregate capital supply, driving down the equilibrium interest rate. As a consequence, the saving motive for intertemporal reasons is reduced, because of the decreased rate of return: this GE effect leads to a response of saving decisions and to a complex determination of the wealth distribution.

This simple example explains the mechanics of how wealth inequality responds when preference heterogeneity in risk aversion is included in the framework. At the same time, this also raises a potential problem. With enough flexibility, virtually any degree of wealth inequality (far enough from perfect equality) can be achieved. By picking appropriately the degree of uncertainty and the risk aversion distribution any degree of wealth concentration can be obtained, a point made by Quadrini and Rios-Rull (1997).[5]

Given these considerations, the computational experiment carried out in this paper could be flawed. Treating preferences as unobserved heterogeneity can potentially lead to identification issues. Different preference distributions, with different implied equilibria, could match the very same moments. Moreover, considering preference heterogeneity as parametric unobserved heterogeneity, whose parameters are pinned down by a set of moments in the wealth distribution, can lead to some unpleasant implications. According to such theories, trends and fluctuations in the wealth distribution could be explained by changes in preferences. A by-product of this approach is that it could lead to virtually impossible relative assessments of different policy interventions, and their welfare implications. Unless the researcher knows how preferences evolve over time, and what triggers their change, welfare analysis is not a viable option.

In order to avoid the potential drawbacks of this framework, a lot of discipline is imposed in the model's parameterization, in order to avoid issues of the type mentioned above.

As for the amount of uncertainty that the agents are going to face, several estimates for wage processes that are routinely used in the quantitative literature on macroeconomics with heterogeneous agents are going to be considered. If the wage processes are truly exogenous with respect to the agents' degree of risk aversion, this step is a valid one. Since this assumption could be violated, an extension considers a model with endogenous sorting into risky jobs.

[5] By way of an example, let's consider the case of maximum degree of inequality, that is a wealth Gini coefficient that is arbitrarily close to 1. In order to achieve this outcome, it is sufficient to impute a high degree of risk aversion for the high types, say $\gamma_h = 10$, and assign them just an almost negligible positive mass $\mu_{\gamma_h} = \epsilon_\mu > 0$. Likewise, the low types are assigned a small degree of risk aversion, say $\gamma_l = \epsilon_\gamma > 0$, and a mass $\mu_{\gamma_l} = 1 - \epsilon_\mu \approx 1$. This economy would display a wealth Gini coefficient close to 1, as the high degree of risk aversion for the high types would lead them to accumulate a lot of assets and at the same time with little dispersion, while the low types with preferences close to linear would not engage in extensive precautionary savings.

As explained above, the analysis will rely on sources of risk aversion heterogeneity that are based on a set of empirical findings trying to measure it. Although feasible, a calibration of the preference distribution by matching some features of the saving behavior (hence some moments of the wealth distribution) will not be attempted, to impose a lot of discipline in the quantitative implementation of the model.

2.2 The Baseline Model

First, a model with only one exogenous stochastic income process is going to be proposed. A simple extension with endogenous sorting into two jobs that differ in their degree of risk will follow.

2.3 Demographics

Time is discrete. The economy is populated by a measure one of infinitely lived agents (workers) denoted by i.[6]

2.4 Preferences

Agents' utility function is defined over stochastic consumption sequences c_t

$$\begin{aligned} E_0 U(c_0, c_1, ...; \gamma_i) &= E_0 \sum_{t=0}^{\infty} \beta^t u(c_t; \gamma_i) \qquad (1) \\ u(c_t; \gamma_i) &= \frac{c_t^{1-\gamma_i} - 1}{1 - \gamma_i}, i \in [0, 1] \end{aligned}$$

the future is discounted at rate $\beta \in (0, 1)$, which is the common discount factor. The per-period utility $u(.)$ is strictly increasing, and strictly concave. Differently from models without preference heterogeneity, it depends explicitly on the risk aversion parameter γ_i. Every agent i is born with an innate attitude towards risk, as captured by their CRRA parameter, which is a permanent feature.[7]

[6] Data limitation suggests to consider infinitely lived agents. In the PSID the risk aversion question was asked only in 1996, making it hard to know if the higher risk aversion observed for parents vs. their offspring is due to a cohort effect, or if it is an intrinsic demographic trait, possibly due to risk aversion increasing with age instead. Furthermore, Kimball, Sahm, and Shapiro (2009) find a positive correlation between the risk aversion of parents and that of their children.

[7] In the HRS, it is possible to keep track of how an individual answers the same risk aversion question over time. As shown by Kimball, Sahm, and Shapiro (2008), a non-trivial number of respondents change their answers across waves, and they propose an econometric methodology addressing the survey response error. Treating the switches as measurement error is a way of rationalizing this outcome.

2.5 Endowments

There is a stochastic process for the effective units of labor ε a worker is going to supply in the labor market. This process is assumed to be an exogenous continuous first order Markov process.[8]

2.6 Technology

The production side of the model is represented by a constant returns to scale technology of the Cobb-Douglas form, which relies on aggregate capital K and labor L to produce the final output Y:

$$Y = F(K, L) = K^{\alpha} L^{1-\alpha}.$$

Capital depreciates at the exogenous rate δ and firms hire capital and labor every period from competitive markets. The first order conditions of the firm provide the expressions for the net real return to capital r and the wage rate per efficiency unit w:

$$r = \alpha \left(\frac{L}{K}\right)^{1-\alpha} - \delta, \tag{2}$$

$$w = (1-\alpha) \left(\frac{K}{L}\right)^{\alpha}. \tag{3}$$

Notice that the marginal productivity of labor is always positive, hence firms will rely on the total sum of the efficiency units of labor. It follows that in the steady-state:

$$L = \int \varepsilon d\mu_L (\varepsilon)$$

where $\mu_L (\varepsilon)$ is the stationary distribution over the labor endowments implied by the Markov process.

2.7 Other market arrangements

The final good market is competitive. Firms hire capital every period from a competitive market. There are no state-contingent markets to insure against income risk, but workers can self-insure by saving into the risk-free asset. The agents also face a borrowing limit, denoted as $b \geq 0$.

3 Stationary Equilibrium

This Section first defines the problem of the agents in their recursive representation, then it provides a formal definition of the recursive competitive equilibrium.

[8] The analysis will focus on steady states, hence from now on time subscripts will be supressed.

3.1 Problem of the agents

The individual state variables are the labor endowment $\varepsilon \in \mathcal{E} = [0, \overline{\varepsilon}]$, and asset holdings $a \in \mathcal{A} = [-b, \overline{a}]$. Moreover, risk aversion $\gamma \in \Gamma = [\gamma_{\min}, \gamma_{\max}]$ represents a permanent state for the agents. The stationary distribution is denoted by $\mu(\varepsilon, a; \gamma)$. The value function of an agent whose current asset holdings are equal to a, whose current labor endowment is ε, and with innate risk aversion γ is denoted with $V(\varepsilon, a; \gamma)$. The problem of these agents can be represented as follows:

$$V(\varepsilon, a; \gamma) = \max_{c, a'} \left\{ u(c; \gamma) + \beta E_{\varepsilon' | \varepsilon} V(\varepsilon', a'; \gamma) \right\} \tag{4}$$

$$s.t.$$

$$\begin{aligned} c + a' &= (1 + r)\, a + w\varepsilon \\ \log \varepsilon' &= \rho_y \log \varepsilon + \eta', \eta \sim iid\ N(0, \sigma_y^2) \\ a_0\ &given, \quad c \geq 0, \quad a' > -b \end{aligned}$$

Agents have to set optimally their consumption/savings plans. They enjoy utility from consumption, and face some uncertain events in the future. In the next period they will still have the same risk aversion parameter, but their labor income can increase or decrease, depending on the future realizations of the earnings shock η.

3.2 Recursive Stationary Equilibrium

Definition 1 *A recursive stationary equilibrium is a set of decision rules* $\left\{ c(\varepsilon, a; \gamma), a'(\varepsilon, a; \gamma), k = \frac{K}{L} \right\}$, *value functions* $V(\varepsilon, a; \gamma)$, *prices* $\{r, w\}$ *and a set of stationary distributions* $\mu(\varepsilon, a; \gamma)$ *such that:*

- Given relative prices $\{r, w\}$, the individual policy functions $\{c(\varepsilon, a; \gamma), a'(\varepsilon, a; \gamma)\}$ solve the household problem (4) and $V(\varepsilon, a; \gamma)$ is the associated value function.
- Given relative prices $\{r, w\}$, k solves the firm's problem (2)-(3).
- The asset market clears
$$K = \int_{\mathcal{E} \times \mathcal{A} \times \Gamma} a d\mu(\varepsilon, a; \gamma)$$
- The goods market clears
$$F(K, L) = \int_{\mathcal{E} \times \mathcal{A} \times \Gamma} c(\varepsilon, a; \gamma) d\mu(\varepsilon, a; \gamma) + \delta K$$

- The stationary distributions $\mu(\varepsilon, a; \gamma)$ satisfy

$$\mu(\varepsilon', a'; \gamma) = \sum_{\varepsilon} \int_{a:a'(\varepsilon,a;\gamma)=a'} \pi(\varepsilon', \varepsilon)\, d\mu(\varepsilon, a; \gamma), \forall \gamma \in \Gamma \tag{5}$$

In equilibrium the measure of agents in each state is time invariant and consistent with individual decisions, as given by equation (5) above.[9]

3.3 Discussion

As with many other dynamic problems, the Euler equations help getting an intuition of the main intertemporal trade-offs that the agents in this economy are facing. They offer a more formal argument underlying the intuition provided in Figure (2). The optimal consumption functions $c(\varepsilon, a; \gamma)$ satisfy the following optimality condition, where the equality holds whenever the agents are not borrowing constrained:

$$c(\varepsilon, a; \gamma)^{-\gamma} \geq \beta\,(1+r)\, E_{\varepsilon'|\varepsilon}\left[c(\varepsilon', a'; \gamma)^{-\gamma}\right]$$

$c(.)^{-\gamma}$ is a convex function, hence $E_{\varepsilon'|\varepsilon}\left[c(\varepsilon', a'; \gamma)^{-\gamma}\right] \geq \left\{E_{\varepsilon'|\varepsilon}\left[c(\varepsilon', a'; \gamma)\right]\right\}^{-\gamma}$ because of Jensen's inequality. In order to take care of this property, it is always possible to define a positive number $\eta(\gamma)$ such that $E_{\varepsilon'|\varepsilon}\left[c(\varepsilon', a'; \gamma)^{-\gamma}\right] = \left\{E_{\varepsilon'|\varepsilon}\left[c(\varepsilon', a'; \gamma)\eta(\gamma)\right]\right\}^{-\gamma}$. More precisely, the CRRA parameter affects the strength of Jensen's inequality, and for any γ there is an associated $\eta(\gamma)$ satisfying $0 < \eta(\gamma) < 1$ and $\frac{d\eta(\gamma)}{d\gamma} < 0$. Substituting this expression in the optimality condition and rearranging gets:

$$\frac{E_{\varepsilon'|\varepsilon} c(\varepsilon', a'; \gamma)}{c(\varepsilon, a; \gamma)} \geq \frac{\left[\beta\,(1+r)\right]^{\frac{1}{\gamma}}}{\eta(\gamma)}.$$

The LHS represents the expected consumption growth, which depends on β, γ and r. It is easy to show that, for a given interest rate, the expected consumption growth is increasing in both the discount factor and the risk aversion.[10] The higher γ, the more convex the function $c(.)^{-\gamma}$, and the lower the number $\eta(\gamma)$, which increases the expected consumption growth because of the increased

[9] Notice that the equation already exploits the Markov Chain representation of the continuous process for ε.

[10] Notice that $\frac{dLHS}{d\beta} = \frac{[\beta(1+r)]^{\frac{1}{\gamma}}}{\beta\gamma\eta(\gamma)} > 0$ and $\frac{dLHS}{d\gamma} = -\frac{[\beta(1+r)]^{\frac{1}{\gamma}} \ln[\beta(1+r)]}{\gamma^2\eta(\gamma)} - \frac{[\beta(1+r)]^{\frac{1}{\gamma}}\eta'(\gamma)}{[\eta(\gamma)]^2} > 0$. The sign of the last expression is obtained by recognizing that in incomplete markets economies $\beta(1+r) < 1$, and $\ln[\beta(1+r)] < 0$. Moreover $\frac{d\eta(\gamma)}{d\gamma} < 0$, given that $\eta(\gamma) = \frac{\left\{E_{\varepsilon'|\varepsilon}\left[c(\varepsilon',a';\gamma)^{-\gamma}\right]\right\}^{-\frac{1}{\gamma}}}{E_{\varepsilon'|\varepsilon}\left[c(\varepsilon',a';\gamma)\right]}$ and thanks to the properties of generalized means $M_{(.)}$ for non-negative random variables. The numerator of $\eta(\gamma)$ is the generalized mean of order $-\gamma$ while the denominator is the mean of order 1, and generalized means are such that $M_{(p)} < M_{(q)}$, for $p < q$. Finally, $\eta(\gamma)$ is bounded above by 1 because $\gamma > 0$, and it is bounded below by 0 because $M_{-\infty} = \min c(.) \geq 0$.

precautionary savings. At the same time, the RHS is highly non-linear in γ: this property rationalizes a result that is previewed now, and discussed in more detail in Section 7. The aggregate allocations of the heterogeneous preferences economy are going to be quantitatively very different from the ones of its homogeneous preferences counterpart. This is going to be true both for the model with only one risky job, and for the model with multiple risky jobs.

4 Parameterization

This section describes the calibration strategy. The length of the model period is set to one year. The calibration starts by taking as given the two elements characterizing the uncertainty in this economy: how volatile and persistent agents' labor income is and how agents relate to the uncertainty arising from the postulated stochastic income process. The remaining parameters are set such that in the steady state equilibrium the incomplete markets economy matches some characteristics of aggregate level data. It is worth stressing that no parameters are chosen to match directly selected features of the wealth distribution. The complete parameterization of the model is reported in Table 1.

[Table 1 about here]

4.1 Parameterizing Uncertainty

Uncertainty plays a double role in the model. On the one hand the agents are going to face stochastic income sequences with a given persistence and variance, on the other hand they are going to react to these possible income histories differently, according to their innate CRRA parameter.

4.1.1 The Risk Aversion Distribution

The distribution for the risk aversion parameter is estimated following the procedure outlined by Kimball, Sahm, and Shapiro (2009), and this case will be denoted as KSS. Under the assumption that the risk preference parameter is log-normally distributed in the population, $\gamma \sim LN\left(\mu_\gamma, \sigma_\gamma^2\right)$, it is possible to exploit the answers to the survey gambles in the PSID, and estimate with Maximum Likelihood (MLE) an ordered probit model with known thresholds. These thresholds represent the bounds bracketing each respondent's risk aversion, which must be between two values that depend on the answers to the sequence of gambles. Table 2 reports the estimation results.

[Table 2 about here]

In the actual estimation, it is easier to work with the risk tolerance $\tau = 1/\gamma$. The assumption of a CRRA utility function is particularly convenient, because the above mentioned thresholds are easily computed in this case. Given the answers to the sequence of gambles, each individual in the PSID who was administered the labor income gambles falls into one out of six possible risk categories, depending on the actual largest downside risk accepted and the smallest risk rejected across gambles.[11] In the PSID, when focusing on the household heads, the six risk tolerance categories have the following number of observations: 1765 with $0 < \tau \leq 0.13$, 1024 with $0.13 < \tau \leq 0.27$, 869 with $0.27 < \tau \leq 0.50$, 835 with $0.50 < \tau \leq 1.00$, 761 with $1.00 < \tau \leq 3.27$, and 368 with $3.27 < \tau < \infty$. It is apparent that a large fraction of the PSID sample shows very little tolerance to labor income risk. The MLE procedure finds the parameters of the log-normal distribution that maximize the probability of having observed the actual choice categories, namely the share of individuals included in each sub-interval of τ. Once the two parameters' point estimates for the log-normal distribution of τ are computed, it is immediate to retrieve the related parameters for the CRRA distribution, which are $\mu_\gamma = 1.07$ and $\sigma^2_\gamma = 0.76$.[12]

Although Kimball, Sahm, and Shapiro (2008) and Kimball, Sahm, and Shapiro (2009) proposed and implemented a sophisticated econometric technique, inferring a continuous preference distribution from a limited sequence of lotteries and implied outcomes is a challenging task. Many problems can potentially bias the results. For example, the log-normal parametric assumption could give too little flexibility, possibly forcing the tails to be too fat or too thin. Or, more fundamentally, the survey's respondents might not be able to assess accurately the amount of risk involved by the lifetime lotteries they are facing in the questionnaires.

With these caveats in mind, a finding based on the PSID (and HRS) data is that people show very high risk aversions. According to the parameters' estimates, there are few individuals whose γ is around 1, with the average CRRA being 4.28. More in detail, 10.77% of the population has $\gamma \leq 1$, 22.0% has $\gamma \leq 1.5$, 33.0% has $\gamma \leq 2$, and 72.9% has $\gamma \leq 5$. With the class of preferences assumed in this paper, this result could lead to a counterfactual implication. The implied elasticity of

[11] The actual wording and the sequence of questions included in the 1996 wave of the PSID are reported in the Appendix.

[12] Kimball, Sahm, and Shapiro (2009) find $\mu_\gamma = 1.05$, because they do not restrict the sample to the household heads. The results are almost identical when using their estimate. Moreover, Tables 3 and 4 in Kimball, Sahm, and Shapiro (2008) show the estimates for the two parameters with HRS data: σ_γ is drastically reduced (from 1.76 to 0.73) when correcting the estimator for response error. In the PSID it is possible to apply the same correction procedure only by using some information from the HRS data, because the risk aversion question was asked only once. As for the other parameter, the value of $\mu_\gamma = 1.98$ is substantially larger in the HRS. These alternative estimates, in turn, lead to first order effects on the wealth Gini index. However, the HRS respondents are not a representative sample of the overall economically active U.S. population. Exploiting these estimates for the whole U.S. economy could lead to large biases.

intertemporal substitution could be at odds with the values typically estimated in the literature, and surveyed by Attanasio and Weber (2010). Even though the average CRRA is extremely high, due to a manifestation of Jensen inequality, the average EIS is equal to $0.498 \gg \frac{1}{4.28}$, a value consistent with the empirical findings.

4.1.2 Income Uncertainty

The stochastic process for labor earnings is another key element in the analysis. Different degrees of uncertainty, both in terms of the shocks size and how persistent these shocks are, matter for the incentives to save and for the degree of wealth inequality. Given the wide range of estimates available in the literature on stochastic income processes, different assumptions on the exogenous process for the labor efficiency endowments are going to be considered. The econometric specification is a simple AR(1) process, which has only two parameters. The persistence parameter is denoted by ρ_y, while the variance of the innovations by σ_y^2. In order to perform some robustness checks and to gauge how much earnings uncertainty matters for the results, three different sets of estimates for the AR(1) process are considered. Table 3 outlines the complete parameterization of the earnings processes.[13]

[Table 3 about here]

The label *FLIN* in Table 3 refers to the specification estimated on PSID data by Floden and Linde (2001), the label *FREN* refers to the specification estimated by French (2005), while *GRIP* is the estimate provided by Guvenen (2009) referring to the Restricted Income Profiles, in his terminology. These processes were selected because they imply quite a different persistence and variance of the innovations: ρ_y ranges from 0.92 in the *FLIN* case, to 0.988 in the *GRIP* case, while σ_y ranges from 0.12 in the *FREN* case, to 0.21 in the *FLIN* case.[14]

4.2 Calibration

The remaining parameters are calibrated as follows. I assume a Cobb-Douglas production function, hence the capital share is captured by the parameter $\alpha = 0.36$, a common value for the U.S. economy.

[13] Table 10 in Appendix C provides the details on the three processes approximated with the Rouwenhurst method. When approximating highly persistent AR(1) processes this method is considered more reliable and less arbitrary than some of the alternatives, such as the popular Tauchen's procedure that requires a choice for the coverage parameter.

[14] In a previous version of the paper I also considered a fourth case, based on the estimates for the HIP model in Guvenen (2009). The results were somewhat misleading, because I was just considering the AR(1) component of the econometric specification for labor income. Consequently, the model's wealth distribution was not directly comparable with the data.

The capital depreciation rate is set to replicate an investment/output ratio of approximately 25%. This is achieved with $\delta = 0.08$.

The calibration of the rate of time preference β is obtained by targeting an equilibrium interest rate equal to 3.5%.[15] It follows that every specification of the stochastic income process implies a different discount factor: $\beta = 0.8787$ for the *FLIN* case, $\beta = 0.8914$ for the *FREN* case, and $\beta = 0.8721$ for the *GRIP* case. A first consequence of working with heterogeneous risk preferences entailing a high average risk aversion is apparent: in order to obtain a plausible equilibrium interest rate, the discount factor is found to be lower than what is typically assumed in this class of models.[16] The economic implication of this result is that there is scope for a substantial departure from permanent-income behavior. In Bewley's classical contribution, the marginal propensity to save approaches one as β tends to one (and δ tends to zero). In this economy, however, agents are quite impatient because high risk aversion agents accumulate a lot of assets, when facing good earnings shocks. For a given interest rate, this leads to a supply of savings that is much higher than in its homogeneous preference parameters counterpart. With a relatively low value of β agents give less weight to the future, and consume more out of their current income, decreasing their savings. This prevents the interest rate to become implausibly small.

The borrowing limit b is set at its most stringent value, namely $b = 0$. This choice was mainly dictated to facilitate the comparison with previous contributions in the literature that used such a value, e.g. Aiyagari (1994) and Castaneda, Diaz-Gimenez, and Rios-Rull (2003).[17]

5 Results

This Section presents the main results for the baseline model. It is shown how introducing preference heterogeneity, in the form of a non-degenerate distribution of innate risk aversion, does alter considerably typical measures of wealth inequality.

[Table 4 about here]

[15] From (2), for a given labor share and capital depreciation, this is in fact equivalent to matching the capital/output ratio. This is the only wealth related target that is used in the parameterization of the model.

[16] With the KSS preference distribution, a more common value of $\beta = 0.96$ delivers an equilibrium interest rate that is very close to zero.

[17] It is not clear if this value is fully satisfactory. In the data, between 6% and 15% of U.S. households have a negative net worth, as reported by Wolff (1998) and Cagetti and De Nardi (2008). However, in this model the effect of b on the wealth Gini index was found to be small, and in the interest of space the results with different calibrations of the borrowing limit are not reported.

Table 4 compares the Gini index and the Coefficient of Variation obtained in the model with preference heterogeneity (*KSS*) to the inequality measures computed in the corresponding economies without preference heterogeneity (*CRRA*= 4.28). In particular, the latter cases rely on a value of γ that coincides with the average value found in the KSS distribution, while all the other parameters are the same as in the corresponding heterogeneous preferences economy.

Overall, the wealth Gini index moves from the $0.50 - 0.62$ range (for the homogeneous preferences case) to the $0.76 - 0.85$ range, depending on the actual stochastic income process considered. Interestingly, the model almost matches the observed degree of wealth inequality in the U.S.

As expected, when moving from the homogeneous risk aversion set-up to the heterogeneous one, also the (gross) income inequality measures do increase. The concentration of labor earnings is identical in the two economies, because the stochastic income processes are the same. The equilibrium wages do differ considerably, but these changes only induce a level effect and do not alter the relative labor earnings. Overall, the income Gini index moves from the $0.33 - 0.47$ range to the $0.39 - 0.51$ one. Given that the capital share of income is only 36%, the increase in income inequality is way less pronounced than the increase in the concentration of wealth.

[Table 5 about here]

A general finding is that several features of the wealth distribution are replicated well. To see this in more detail, Table 5 reports a set of statistics related to the wealth distribution. The five quintiles (*Q1*-*Q5*) and a breakdown of the top quintile are shown for both the U.S. data, and the various specifications of the model. The first two rows report the quintiles obtained from the SCF in 1998 and the PSID in 1999, respectively.[18] When focusing on the same income process, e.g. *FLIN*, the model with risk aversion heterogeneity shows results in terms of wealth concentration and shares of wealth held by a set of quantiles that are much closer to the data compared to the model without risk aversion heterogeneity. The U.S. data show that the bottom three quintiles hold very little wealth. The first quintile has negative asset holdings, the second quintile has between 0.8% and 1.3% of the total wealth, while the third quintile has between 4.2% and 5.0%. In the model, the left tail of the distributions display some disparities with the data. However, these differences tend to be small. The bottom quintile is prevented to be in debt, which explains the 0% figure Vs. the negative one in the data. A similar pattern is observed for the second and third quintiles. With the *FLIN* income

[18] The same statistics for the SCF 2002, SCF 2007, PSID 1994, and PSID 2007 data are quite similar, and are reported in Appendix C. For a systematic study on how the PSID and SCF wealth data differ because of the different survey designs, see Bosworth and Anders (2008). Their analysis shows that, apart from the top 5%, the two wealth distributions are very close.

process, the bottom 60% holds as little as 3.5% of the total wealth. The model predicts that wealth-poor agents hold very little wealth, although slightly less than what is found in the data, because the bottom quintiles are dominated by the risk prone agents. This outcome is evident when considering the average risk aversions in the five quintiles. For the agents in the first two wealth quintiles these figures are 1.32 (*Q1*) and 1.88 (*Q2*), while the average risk aversions for the other quintiles are 3.18 (*Q3*), 4.76 (*Q4*) and 9.12 (*Q5*). In this framework poor agents are poor because of two different reasons. As usual, in the bottom of the wealth distribution there are the individuals that kept on drawing bad realizations of the income shock. However, most of the poor individuals here save little because of a different reason: their willingness to face consumption profiles that are not very smooth, which drives their precautionary savings down. At the same time, the more risk averse agents are accumulating a lot of assets, driving up the capital supply in the economy, and reducing the rate of return of savings, making future consumption relatively more expensive than current consumption. This GE effect triggers a reduction in the interest rate that drives further down the incentives to save for all agents, the ones with a lower risk aversion in particular. These two effects combined explain why in this economy there are several agents that have little or no assets at all, as the Table shows.[19]

The model does generate much higher wealth holdings for the agents at the top of the wealth distribution, and the share of the top 10% is approximately twice as much as the corresponding figure for the homogeneous preferences economy. This statistic moves from the $32.6\% - 43.5\%$ range to the $57.4\% - 72.8\%$ one, with the value in the data being 69.1% in the SCF and the 68.5% in the PSID.

Overall, the *FLIN* specification is the one showing the least discrepancies with the data. However, this is also the model with the lowest share held by the top 1%. The different income processes lead to a clear pattern in how the quintiles change. The higher the persistence of the shock, the higher the share of wealth held by the top quintile, and this happens mostly at the expenses of *Q3* and *Q4*. The models solved with the three income processes have almost identical bottom quintiles, *Q1* and *Q2*, while a larger difference is observed for the share of wealth held by the top three quintiles, *Q3*, *Q4* and *Q5*. The *FREN* and *GRIP* wealth distributions are very similar, and differ at most by 3 percentage points in the share held by the top 1%. In these two cases, the share of wealth held by the fourth quintile (9.7% and 7.7%) is below the data ($12.2\% - 12.6\%$), while for the top quintile both cases substantially overestimate the actual value (90.1% and 92.2% Vs. $81.7\% - 83.2\%$). The *FLIN* case is somewhat less extreme, as the share of wealth held by the fifth (fourth) quintile is 79.6% (16.8%).

[19] The calibration forces the equilibrium interest rate to be the same in all the KSS economies, masking the interest rate drop. This is instead reflected in different discount factors. Although the average risk aversion is high, which would lead the agents to have very smooth consumption profiles, their impatience dominates in quantitative terms, leading to a departure from permanent-income behavior.

As for the other measure of wealth inequality, the Coefficient of Variation, the model does not match the actual figure of 6.02 found by Diaz-Gimenez, Glover, and Rios-Rull (2011) in the SCF. The values implied by the model are in the $1.87 - 2.62$ range. The reason behind this result is that the model misses the share of wealth held by the richest households, showing that preference heterogeneity alone does not fully account for the determination of the very top of the wealth distribution. In the KSS model economy, the share of wealth held by the top 1% is never higher than 17.6%.

While both wealth and income are more concentrated in the heterogeneous preferences economies, consumption is not. The consumption Gini index is between 0.25 and 0.43 with homogeneous preferences, and between 0.28 and 0.43 with heterogeneous risk aversions. As for the data, Heathcote, Perri and Violante (2010) report that in the CEX, since 1990, the Gini index of consumption of non-durable goods has been above 0.3, with a maximum value of 0.32, reached in 2004 and 2005. In this dimension, the *GRIP* model performs poorly: because of the very persistent income process, consumption inequality is well above the corresponding statistic in the data.

The reason behind the stable concentration of consumption lies in the spectacular differences in the capital stock, equilibrium interest rates and wages between the model with heterogeneous preferences and its homogeneous preferences counterpart. The higher capital stock in the KSS economy increases wages, leading to a higher expected discounted value of labor earnings. For this reason, the borrowing constraint becomes less binding and consumption inequality does not increase by a considerable amount. Another way of understanding this result is by inspecting the results when solving the KSS model in partial equilibrium, fixing the interest rate at the value found in the corresponding homogeneous preferences economy. With a considerably higher interest rate, the consumption Gini index increases by a further 3 to 4 points, now being 0.31 in the *FLIN* case, 0.37 in the *FREN* case, and 0.46 in the *GRIP* case.

5.1 Lorenz Curves

Another way of analysing wealth inequality is by considering the Lorenz curves for the models with and without risk aversion heterogeneity.

[Figure (4) about here]

Figure (4) plots three Lorenz curves. One curve, drawn with dots, refers to the KSS economy for the *FLIN* income process. The second curve, drawn with a dashed line, displays another economy, which has the same amount of uncertainty (*FLIN*), the same trading opportunities, the same baseline parameters, but it is without risk aversion heterogeneity (i.e., with $\gamma = 4.28$). Finally, the third Lorenz curve represents the PSID data on net worth in 1999.[20]

By comparing the two models' Lorenz curves it is clear that there is a large shift to the right. The heterogeneous preferences economy displays a wealth distribution that is substantially more unequal, and that until the 70^{th} percentile it tracks well the empirical one. This is an alternative way of seeing the change in the Gini coefficient, which in this case increases from 0.50 in the homogeneous CRRA case to 0.76 in the heterogeneous one. A similar set of comments and plots apply when considering the other income processes, although the fit with the PSID data is somewhat poorer.

5.2 Discussion

At least three aspects of the analysis call for a further discussion. The first one is related to the assumption of risk aversion being a permanent feature. The second is why I consider preference heterogeneity in only one parameter, the risk aversion. The final aspect refers to risky jobs, preference for risk and sorting into careers, which is dealt with in the next section.

An infinitely lived agents model refers to dynasties. In the PSID, Kimball, Sahm, and Shapiro (2009) find that risk aversion is positively correlated among different generations of the same family. However, preferences could have a life-cycle element. It is conceivable that individuals change their attitude towards risk after having gone through some specific stages in their life. Unfortunately, data limitations do not allow to capture if such dynamics are indeed taking place. Neither the HRS nor the PSID allow to single out the life-cycle component vs. a cohort one. As Kimball, Sahm, and Shapiro (2008) point out, some individuals in the HRS are asked to assess their attitudes towards risk more than once. In quite a few cases these individuals change their answers. With the available information it is hard to say whether this is a form of measurement error, or whether this reflects the fact that individuals change their attitudes towards risk in response to some economically relevant events (e.g., new information on the future income streams, or aggregate and idiosyncratic shocks). However, all the people in the HRS sample are in the later stages of their lives, that is they are in a situation where there is little uncertainty about lifetime income, at least in the form of labor earnings. This is why the measurement error approach seems to be a valid one.

As for the second aspect, the modeling choice was mainly driven by data limitations. Barsky, Juster, Kimball, and Shapiro (1997) find heterogeneity not only as far as risk aversion is concerned, but also in the elasticity of intertemporal substitution, and in the rate of time preference. However, their sample size is really limited, making the estimation of the distributions for the last two parameters

[20] Comparing the model to the PSID wealth data seems appropriate, as the stochastic income processes are all estimated with this dataset.

too unreliable. It goes without saying that, with CRRA preferences, once heterogeneity in risk aversion is allowed for, heterogeneity in the elasticity of intertemporal substitution mechanically follows. The distributions of the two parameters are just a monotone transformation.

In the benchmark model, the amount of income uncertainty from the point of view of a single agent is exogenously determined by the stochastic process for labor income. However, Guiso, Jappelli and Pistaferri (2002), Fuchs-Schundeln and Schundeln (2005), Bonin, Dohmen, Falk, Huffman, and Sunde (2007), and Schulhofer-Wohl (2011), among others, pointed out that different attitudes towards risk can also imply different career choices. High risk averse individuals can self-select into safer jobs, while low risk averse ones can be willing to take jobs with more unstable earnings. This endogenous sorting could have profound implications for the degree of inequality. However, such a research avenue poses some challenges on how to model the career specific stochastic processes for earnings. An attempt along these lines is proposed in the next section.

6 Endogenous Sorting into Risky Jobs

The results obtained above could be partially driven by the absence of self-selection into jobs differing in their income risk. Schulhofer-Wohl (2011) exploits the question about risk tolerance in the HRS to document that less risk averse individuals have the largest earnings fluctuations during their working life, suggesting that preference heterogeneity may be an important factor in occupational choices and the allocation of risk. This section proposes a simple extension to the benchmark model dealing with this issue.[21]

It is now assumed that there are two mutually exclusive careers, and that all the workers have perfect knowledge on the degree of income risk associated with these options. The workers can now self-select into different jobs, according to their characteristics. For simplicity, it is assumed that the workers make a once-in-a-lifetime decision on their career. Upon becoming economically active, workers choose which career $j = h, l$ they want to undertake (the riskier job being denoted by h), by solving the following problem:

$$\max \left\{ V^h(\varepsilon_0^h, a_0; \gamma), V^l(\varepsilon_0^l, a_0; \gamma) \right\}$$

[21] It goes without saying that this specification neglects considerations related to shopping for a career in the early stages of a worker's labor market experience, and learning about the relative riskiness of a job. Moreover, data limitation suggests to restrict the number of jobs to two. An alternative approach could be to use the career definitions available in the PSID. However: 1) for most respondents, it is not possible to observe their labor market entry, making the comparison between the theory and the data somewhat problematic, 2) using job definitions instead of individuals as the unit of observation would not allow to estimate the variance and persistence of the income shocks by considering the conventional dynamic panel data random effects model used here.

where the $V^j(\varepsilon_0^j, a_0; \gamma)$ represent the expected lifetime utility derived when choosing career j, whose formulation resembles equation (4) when a specific income process is considered. Notice how the choice is made after having observed the initial realization of the income shocks for both careers ε_0^j, which are assumed to be drawn from the Markov processes' invariant distributions. Moreover, different agents start their life with different wealth levels. Intuitively, agents are "parachuted" in an economy whose wealth distribution coincides with the long-run one, and make their career choice only once.[22]

Associated with this choice there is an optimal policy function $\Phi(\varepsilon, a; \gamma)$, which is an indicator function equal to one whenever $V^h(\varepsilon_0^h, a; \gamma) \geq V^l(\varepsilon_0^l, a; \gamma)$, namely when an agent decides to work in the riskier job.[23] Hence, in equilibrium the endogenous share of workers choosing to work in the riskier job (ϕ) is represented by:

$$\phi = \int_{\mathcal{E}\times\mathcal{A}\times\Gamma} \Phi(\varepsilon, a; \gamma) d\mu(\varepsilon, a; \gamma)$$

Since there are two Markov processes for the effective units of labor, the equilibrium conditions related to the labor input have to be changed. In the steady-state the expression for L becomes:

$$L = \int_{\mathcal{E}\times\mathcal{A}\times\Gamma} \varepsilon d\mu(\varepsilon, a; \gamma).$$

6.1 Two Labor Endowment Processes

The two postulated careers are represented by two different stochastic processes, which govern the dynamics of the efficiency units of labor ε a worker is going to supply in the labor market. It follows that

$$\log \varepsilon_j' = m_j + \rho_{y,j} \log \varepsilon_j + \eta_j', \eta \sim iid\ N(0, \sigma_{y,j}^2),\ j = h, l$$

which highlights how the parameters $\rho_{y,j}, \sigma_{y,j}^2$ and m_j can now differ in the two stochastic processes. Needless to say, agents in the model care about the level of the process, not its log. In principle one could be tempted to assert that the riskier job should be such that $\sigma_{y,h}^2 \geq \sigma_{y,l}^2$, and that it should

[22] An alternative set-up with agents facing mortality risk (with probability p) and leaving their offspring bequests could be seen as more appealing, because it would allow each generation to re-optimize over the career choice. In the simplest case, with perfect altruism $\chi = 1$ and accidental bequests, the Bellman equation would read $V^j(\varepsilon, a; \gamma) = \max_{c,a'} \left\{ u(c; \gamma) + \beta(1-p) E_{\varepsilon'|\varepsilon} V^j(\varepsilon', a'; \gamma) + \beta p \chi E_{\varepsilon'} \widetilde{V}(\varepsilon', a'; \gamma) \right\}$, where $\widetilde{V}(\varepsilon, a; \gamma) = \max \left\{ V^h(\varepsilon^h, a; \gamma), V^l(\varepsilon^l, a; \gamma) \right\}$. Unfortunately, despite some effort, this alternative framework shows numerical instability, making the calibration of the model infeasible. Obviously, imposing $\chi = 0$ does mitigate the problem. In particular, setting p to match an average (model) life-span of 50 years and recalibrating the remaining parameters leads to a wealth Gini index of 0.76.

[23] Notice that the income shocks are career dependent, and they would evolve independently for the two careers $j = h, l$, with only one of the two histories of shocks being actually experienced by the worker.

also command a risk premium with $m_h > m_l$, as the labor endowment processes can have two career-dependent averages (in the following I impose the normalization $m_l = 0$). One important implication of allowing for different persistences and/or variances in the processes for the log efficiency is that the processes in levels will mechanically have different means, because the unconditional average is a function of all the parameters, not just m_j:

$$E(\varepsilon_j) = \exp\left[m_j/(1-\rho_{y,j}) + 0.5 * \sigma^2_{y,j}/(1-\rho^2_{y,j})\right].$$

This suggests that even very risk averse individuals could be willing to choose the riskier job, possibly without imposing $m_h > m_l$ or $\sigma^2_{y,h} > \sigma^2_{y,l}$. As a matter of fact, just imposing $\rho_{y,h} > \rho_{y,l}$ could induce a sufficient increase in the mean of the process in levels to make the riskier career an attractive option for many risk aversion types. It is understood that, according to the previous inequality for the persistence parameters, the riskier job is the one with the higher persistence. The lower the persistence parameter, the more transitory the income shocks will be, because their effect dies out faster. Differently, more persistent processes imply that a shock of a given size will have a more long lasting effect. More risk averse individuals prefer smoother consumption profiles, hence they consider riskier a stochastic income process whose shocks have a higher variance and that are more persistent, namely shocks that cannot be easily offset by their saving behavior. Whether this assumption has an empirical foundation is an open issue. However, an appropriately designed empirical analysis on PSID data can be used to support it. Below, I also provide some additional evidence favoring the view that agents do not sort only on the basis of their risk aversion. Instead, there seems to be some "mixing" taking place, with also some very risk averse individuals selecting the riskier job.

6.2 Empirical Analysis

With the PSID data it is possible to compute the autocorrelation of labor income shocks together with their variance for individuals belonging to different risk aversion categories. As costumary in this literature, I estimate stochastic income processes with a minimum distance estimator that also allows for purely transitory shocks and permanent heterogeneity in the productivity level. A novelty consists of estimating two type-dependent processes, splitting the sample into two risk aversion categories. Starting from the SRC sample I follow a set of standard steps, outlined for example in Guvenen (2009) or Heathcote, Perri and Violante (2010). First, I select the PSID respondents according to the following criteria: I consider only the household heads that have at least 15 years worth of labor income data, I drop the individuals who have reported to have worked for more than 5096 or less than 520 hours, and I drop some wage outliers. For each year $t = 1968, ..., 1993$, I run a first stage regression of log labor earnings on a set of controls, $\log y_{it} = \theta_t X_{it} + e_{it}$, with the controls X_{it} being a cubic polynomial in age. With the estimated parameters $\widehat{\theta}_t$ I compute the earnings residuals $\widehat{e}_{it}$ and estimate with a minimum distance estimator the following panel data model with random effects (f_i), persistent shocks ($\log \varepsilon_{it}$) and transitory shocks (v_{it}). All shocks are assumed to be normally and independently distributed, with $f_i \sim N(0, \sigma_f^2), \eta_{it} \sim N(0, \sigma_y^2)$ and $v_{it} \sim N(0, \sigma_v^2)$. The model is estimated separately for two risk aversion categories, represented by $RA = \{\underline{\gamma}, \overline{\gamma}\}$, with $\overline{\gamma} > \underline{\gamma}$. The breakdown in the data is chosen such that each group has approximately 50% of the respondents. The econometric model for the labor endowment is:

$$
\begin{aligned}
e_{it}^{RA} &= f_i^{RA} + \log \varepsilon_{it}^{RA} + v_{it}^{RA} \\
\log \varepsilon_{it}^{RA} &= \rho_{RA} \log \varepsilon_{it-1}^{RA} + \eta_{it}^{RA}
\end{aligned}
$$

Before reporting the results, it is worth stressing that this estimation procedure does not recover the structural parameters of the model. From the PSID data it is possible to track the yearly labor earnings of individuals according to their risk category. However, it is not possible to estimate exogenously two (or more) stochastic processes from the data and impose them in the model. The endogeneity of a person's career prevents this estimation procedure from recovering the true processes, and suggests not to use them as an input in the model. An indirect inference method is needed to estimate the structural parameters in equilibrium, because only the actual volatility and persistence of earnings are observed, with agents having the same risk aversion but with different labor income shocks upon labor market entry selecting different jobs. It follows that the estimation results, reported in Table 6, will be used as targets in the calibration of the economic model.

[**Table 6 about here**]

Perhaps surprisingly, the results show that the variances for the high and low risk aversion agents are almost identical, and equal to $\sigma_{y,\overline{\gamma}}^2 = \sigma_{y,\underline{\gamma}}^2 = 0.018$. Hence, in the model I impose the restriction $\sigma_{y,h}^2 = \sigma_{y,l}^2$. Differently, the persistence parameters are as expected, with the less risk averse individuals showing a higher persistence of their labor income shocks $\rho_{\underline{\gamma}} = 0.947 > \rho_{\overline{\gamma}} = 0.935$.

To conclude, although the variances of the innovations are set to be the same, both the variance and the mean for the efficiency units in levels are higher for the risky process, also because m_h will turn out to be positive.

[**Table 7 about here**]

As for the issue of sorting, Table 7 shows selected percentiles of the distribution of (residual) earnings by risk aversion type and potential labor market experience. I constructed only five experience groups, for each category to have a sizable number of observations. A model based on individuals selecting their most preferred job only according to their innate risk aversion would have a clear testable prediction. The top quantiles of the earnings distribution for the high risk aversion group should always be below the corresponding quantiles for the low risk aversion individuals. As the Table shows, this prediction is violated in the data. For some experience levels, the top of the earnings distribution of the high risk aversion types clearly dominates the other one. In particular, for experience levels greater than 14 years, the top 1% and 5% are systematically higher for the more risk averse individuals.[24] The assumption that individuals can observe the first income shock realization of both jobs is a way of rationalizing this outcome. The model I work with is consistent with the empirical findings, because selection is also affected by the first draw of labor market shocks. Upon drawing a relatively high realization of the shock for the riskier job, also some very risk averse individuals will select this career. The calibration of the model is presented below, confirming that a) the riskier job's shocks are considerably more persistent, b) the riskier job has a parameter for its average $m_h > 0$, and c) agents are fairly impatient. These elements, combined with a relatively high shock at labor market entry, act as if the conditional average of earnings for the riskier job was higher.[25] Hence, in this set up, sampling variability explains the crossings of the earnings distributions by risk aversion types.

6.3 Calibration of the Endogenous Sorting Model

The calibration of this version of the model is more challenging.[26] The four parameters calibrated in equilibrium are $\beta, \rho_{y,h}, \rho_{y,l}$ and m_h. These are chosen to match four statistics: an interest rate of 3.5%, the two labor income persistences of the agents above and below the median in the risk aversion distribution (i.e., $\widehat{\rho}_{\underline{\gamma}}$ and $\widehat{\rho}_{\overline{\gamma}}$ estimated above), and the ratio between the average labor incomes of the

[24] It is worth mentioning that this result is robust to different specifications of the first stage regression, to different definitions of the experience categories, to using age instead of potential experience, and to considering the earnings variable directly rather than its residuals.

[25] Think about the extreme case of agents living for only two periods. A large shock ε_0^h together with its high persistence $\rho_{y,h}$ make a high second period income very likely. This can compensate for the worker's innate risk aversion, making the riskier job an attractive option for many risk aversion types.

[26] With a Nelder-Mead optimization algorithm, a highly parallelized code, and searching over four parameters, the calibration takes up to 100 hours to complete. This is due to the very large number of grid points and artificial agents used in the simulations. Given the vast degree of heterogeneity, these are a necessity to achieve a high accuracy of the numerical solution. Hence, the computational burden prevents from having transitory shocks and permanent productivity differences as additional state variables.

same agents.[27]

[Table 8 about here]

Table 8 reports the calibrated parameters together with the calibration targets for the model with the KSS preference distribution and the endogenous sorting into different jobs. This case is going to be labeled as model *ES*. The calibration exercise is successful: the model endogenous statistics are extremely close to their empirical counterparts.

A first outcome of the calibration is that there is a substantial difference in the persistence parameters of the two income processes. The riskier job has $\rho_{y,h} = 0.966$, while the safer job has $\rho_{y,l} = 0.918$. The above-mentioned mixing, such that agents with the same risk aversion make different career decisions, is indeed taking place. Hence, when computing the labor earnings autocorrelations by risk aversion groups (to make the model's measurements consistent with their data counterpart), these statistics are in between the autocorrelations of the two underlying stochastic income processes.

In the data, the ratio of the average earnings by risk aversion groups is 1.009, in favor of the low risk aversion group. A value of $m_h = 0.0012$ matches this target, namely the riskier job has a risk premium of approximately 3.8%.

Finally, the calibrated rate of time preference is $\beta = 0.9094$, implying that agents are less impatient than in the model without the endogenous selection into risky jobs.

6.4 Wealth and Consumption Inequality

In Table 4, the row denoted by *ES* shows the values for the wealth Gini index in the model with endogenous sorting into risky jobs. Also in this case, the model accounts for the observed concentration of wealth, the Gini index being 0.81. As for ϕ, the endogenous share of workers selecting the riskier career, this is found to be 48.7%. As expected, overall there is a strong negative relationship between the share of agents in the riskier job and their risk aversion: 52.3% of the agents whose risk aversion is below the average select the riskier job, as opposed to 42.0% of the agents whose risk aversion is above the average.

Selected quantiles of the wealth distribution are shown at the bottom of Table 5. Comparing the outcomes of the models without endogenous sorting to the model with two risky jobs, there are no major differences, but the *ES* model does a somewhat better job at replicating the share of wealth

[27] The discreteness of the risk aversion categories in the PSID makes it impossible to rely exactly on the median as the threshold value. The actual value is the 52^{nd} percentile, while in the results I refer to it as the median, because it is a simpler mnemonic.

of the top two quintiles, *Q4* and *Q5*. The first two quintiles still have virtually zero wealth holdings, while the top quintile holds 85.7% of the total wealth, which lies in between the different versions of the one-job model, and it is quite close to its empirical counterpart of $81.7\% - 83.2\%$. The richest 10% now hold 65.8% which is remarkably close to the 69.1% of the SCF data and the 68.5% of the PSID data. Just like in the benchmark model, the share of wealth held by the top 1% is still below the data, this statistic now being 15.2%. By way of comparison, in the representative agent counterpart of this economy the wealth Gini index is equal to 0.51, the bottom three quintiles hold 22.2% of the total wealth, while the top 10% and 1% holdings are only 32.7% and 5.2%, respectively.[28] Also with self-selection into risky jobs, preference heterogeneity does have a quantitatively important role in shaping the agents' saving behavior and determining the distribution of wealth.

[Figure (5) about here]

Figure (5) plots three Lorenz curves. As before, the curve drawn with a dashed/dotted line represents the PSID data, the one drawn with a dashed line now refers to the *ES* economy, while the one drawn with a dotted line represents the *FLIN* economy. The fit of the preferred specification of the baseline model (*FLIN*) is comparable to the fit of two risky jobs extension. Until the 50^{th} percentiles they lie on top of each other, while they differ afterwards. The Lorenz curve for the data lies above the *ES* one in the middle portion of the distribution, while it lies above it in the very top part.

An important shortcoming of *GRIP* parameterization of the model without self-selection was represented by the concentration of consumption, which was too high, being 0.43. Differently, the model with endogenous sorting achieves a plausible degree of consumption inequality, with the Gini index now being 0.27.

The model implies that average consumption is increasing in the degree of risk aversion: the higher γ, the higher the stock of accumulated savings, and the higher the capital income used for consumption smoothing purposes. In terms of the data, it is hard to test whether this feature of the model is supported by the PSID, as this dataset provides limited information on consumption, which in the sample period is confined only to the food category. Taking this empirical evidence with caution, however, the data show that the average food consumption is slightly higher for the individuals in the higher risk aversion category. After eliminating some clear outliers (with annual food consumption/expenditure less than \$365 and higher than \$100,000) the average food consumption for

[28] In this case, the income process is the average of the two processes found in the calibration of the heterogeneous preferences economy, weighted by the endogenous share of riskier jobs ϕ.

the high risk aversion group is \$7,661, while the corresponding figure for the low risk aversion group is \$7,457. The former group enjoys a higher consumption, which is 2.74% more than the one of more risk prone individuals. The *ES* model counterparts of these statistics are 1.533 and 1.117, a consumption level which is 37.32% higher for the more risk averse agents. The model is qualitatively consistent with the data. However, it is hard to assess it on a quantitative basis, given that the definition of consumption in the model is much broader than the one in the PSID, and that measurement error in surveys trying to measure consumption is sizable. Although not conclusive, these results serve as prima facie evidence supporting the model's predictions.

7 Comparison of Equilibria: Homogeneous Vs. Heterogeneous Preferences

An interesting feature of both versions of the model is that the allocations in the economies with heterogeneous preferences do not coincide with the allocations of the corresponding economy with homogeneous preferences.[29] A comparison of the equilibria reported in Table 9 shows that the two sets of allocations are quantitatively extremely different.

[Table 9 about here]

Depending on the specification of the income process, the equilibrium interest rate in the heterogeneous preferences case is lower by no less than 1.61 percentage points, and by up to 3.74 percentage points. It follows that the capital stock, total output, and aggregate consumption are always substantially higher in the economies with heterogeneous preferences.[30]

For the *ES* economy, output (consumption) moves from 1.57 to 1.75 (from 1.23 to 1.31), for the *FLIN* specification from 1.67 to 1.79 (from 1.30 to 1.34), for the *FREN* specification from 1.62 to 1.82 (from 1.29 to 1.36), and for the *GRIP* specification from 1.81 to 2.12 (from 1.46 to 1.59). Overall, the output increase is between 7.2% and 17.1%, while the increase in consumption is between 3.1% and 8.9%. These changes are impressive, suggesting that a macroeconomic analysis neglecting risk aversion heterogeneity is likely to get inaccurate quantitative answers to questions where self-insurance plays an important role.

[29] In the latter economy, I set β to the calibrated value in the corresponding heterogeneous preferences case, and γ to the average value of the relevant preference distribution.

[30] It goes without saying that it is possible to find a representative risk preferences economy that has the same equilibrium prices and allocations. However, it is interesting to notice that the outcome of this aggregation exercise does not coincide with the economy with the average risk aversion.

A similar result concerns the change in the capital stock, hence the measurement of the size of precautionary savings. Typically, in standard heterogeneous-agent models, precautionary savings are found to account for a small increase in the aggregate savings, usually below 2%, as discussed in Aiyagari (1994). This is not true in this model, in which the percentage increase of the aggregate capital stock, when moving from the complete markets economy to the incomplete markets one, is extremely large.

In the endogenous sorting model, which appears to be the most robust framework, this downward bias is equal to 39.9 percentage points. Precautionary savings are quantitatively sizable, not only because of the relatively large variance and persistence of the income shocks, but also because of the stronger consumption smoothing motive of the agents with a relatively high risk aversion parameter. A clear implication is that models with homogeneous risk aversion substantially underestimate the size of precautionary savings.

8 Conclusions

This paper contributed to the literature on the determinants of saving behavior. A model of incomplete markets and precautionary savings was extended to allow for both the type of preference heterogeneity found in the PSID data, and self-selection into risky jobs.

A first result of this paper is that a form of empirically grounded preference heterogeneity goes a long way in accounting for several features of the U.S. wealth distribution. Compared to a model without preference heterogeneity, the left tail of the distribution gets closer to the little share of wealth held by the first three quintiles. At the same time, also the top quintiles are replicated relatively well, especially in the model with endogenous sorting into risky careers. A consequence of this result is that the Gini index in the model and in the data are very close to each other. One feature of the wealth distribution is less satisfactory: the share of wealth held by the top 1% is at most 17.6%, which is well below the observed share of more than 30%. Entrepreneurship still seems to be the candidate channel leading to such large estates, although preference heterogeneity can provide a microfoundation of why some workers self-select into entrepreneurial activities while others decide not to work as self-employed. Such a model can be considered as an alternative way of microfounding the models of entrepreneurship and wealth accumulation developed in Quadrini (2000) and Cagetti and De Nardi (2006).

The results of this paper also show that neglecting the risk preference heterogeneity channel can have a first order effect also on the aggregate allocations and on the macroeconomic outcomes. This points to the need for further applied research aimed at eliciting more accurately individuals' risk preferences in large longitudinal and cross-sectional data, also because preference heterogeneity introduces another layer for heterogeneous welfare effects stemming from a policy change.

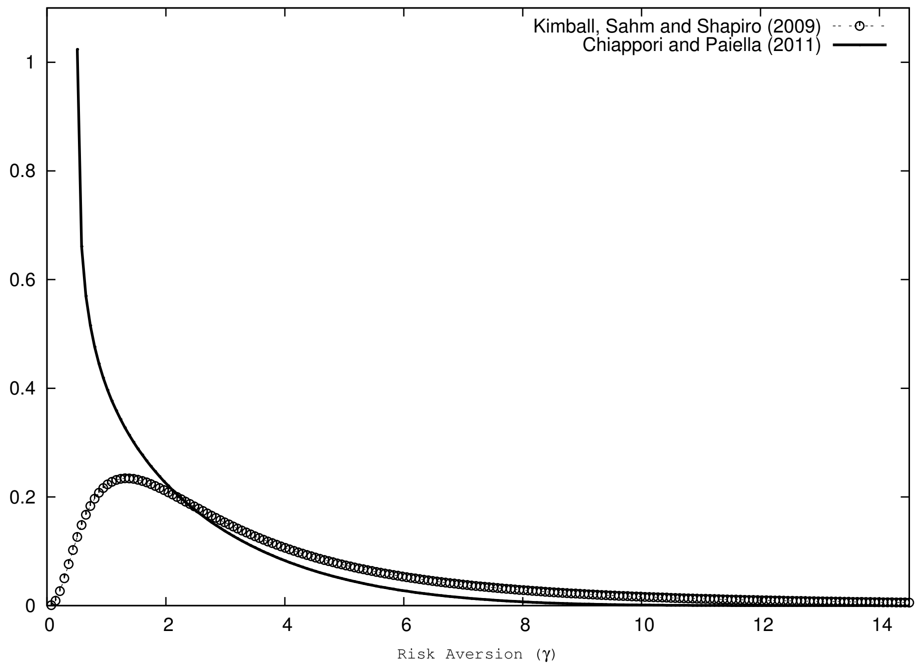

Figure 1: Risk Aversion Heterogeneity, Estimated Densities from PSID and SHIW data.

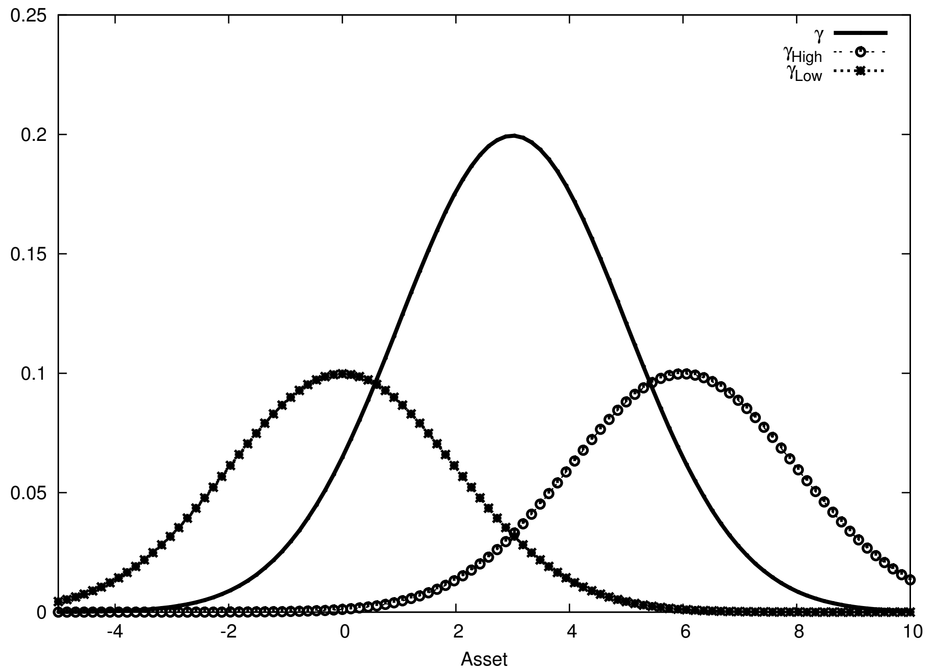

Figure 2: Heterogeneous Risk Aversion and Wealth Inequality.

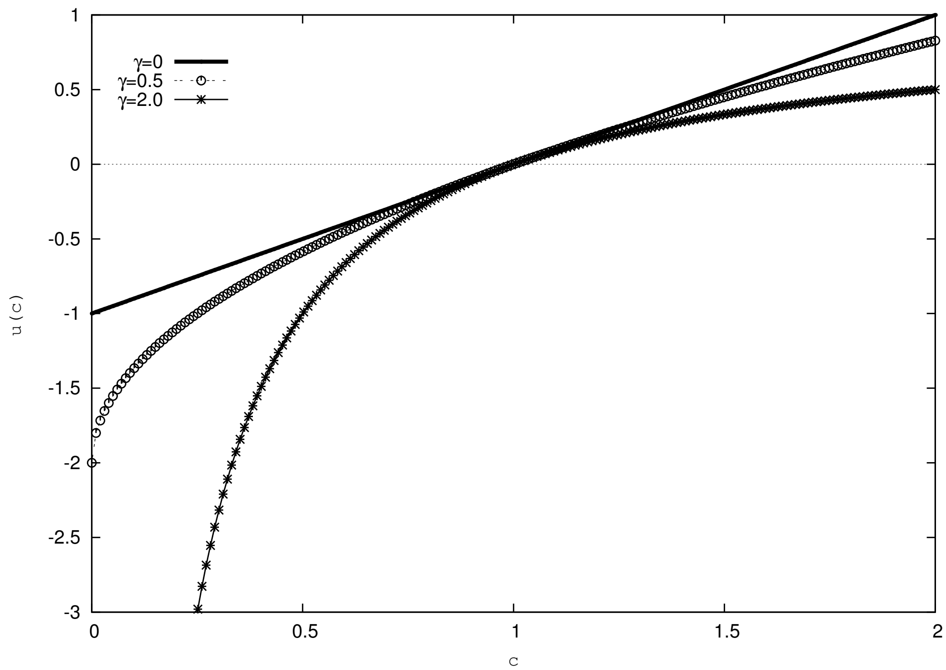

Figure 3: CRRA Utility Functions, $\gamma = \{0, 0.5, 2\}$.

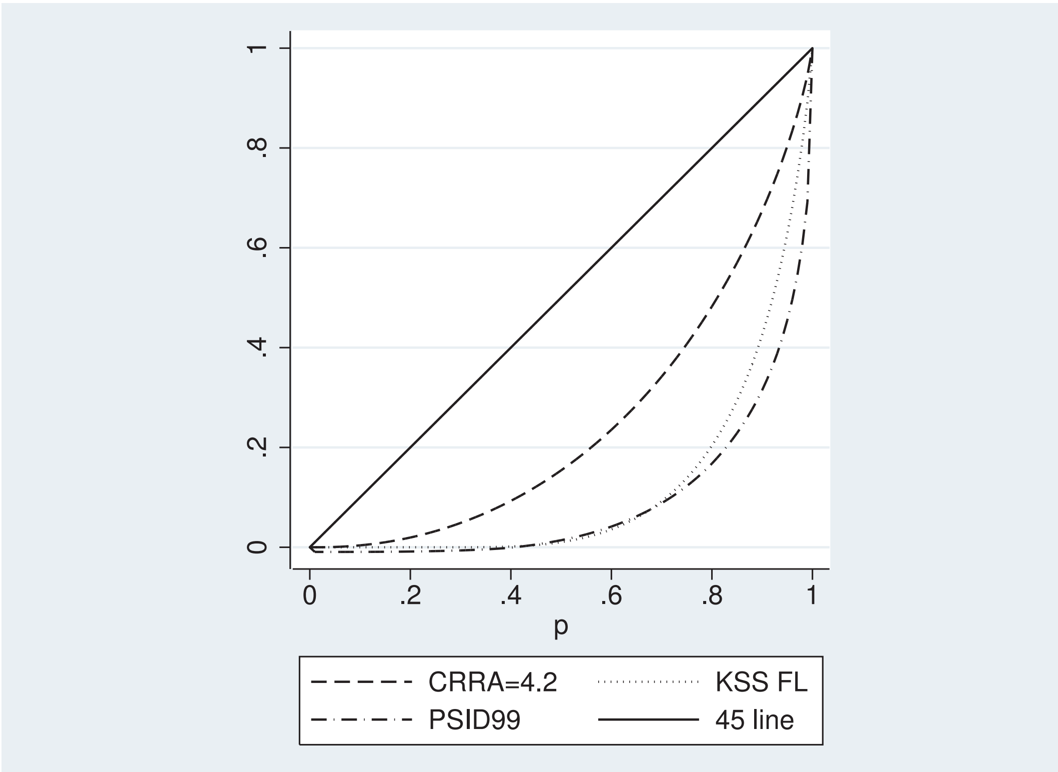

Figure 4: Lorenz Curves for Assets: Homogeneous Preferences, RA Heterogeneity (KSS - FLIN case), and PSID data.

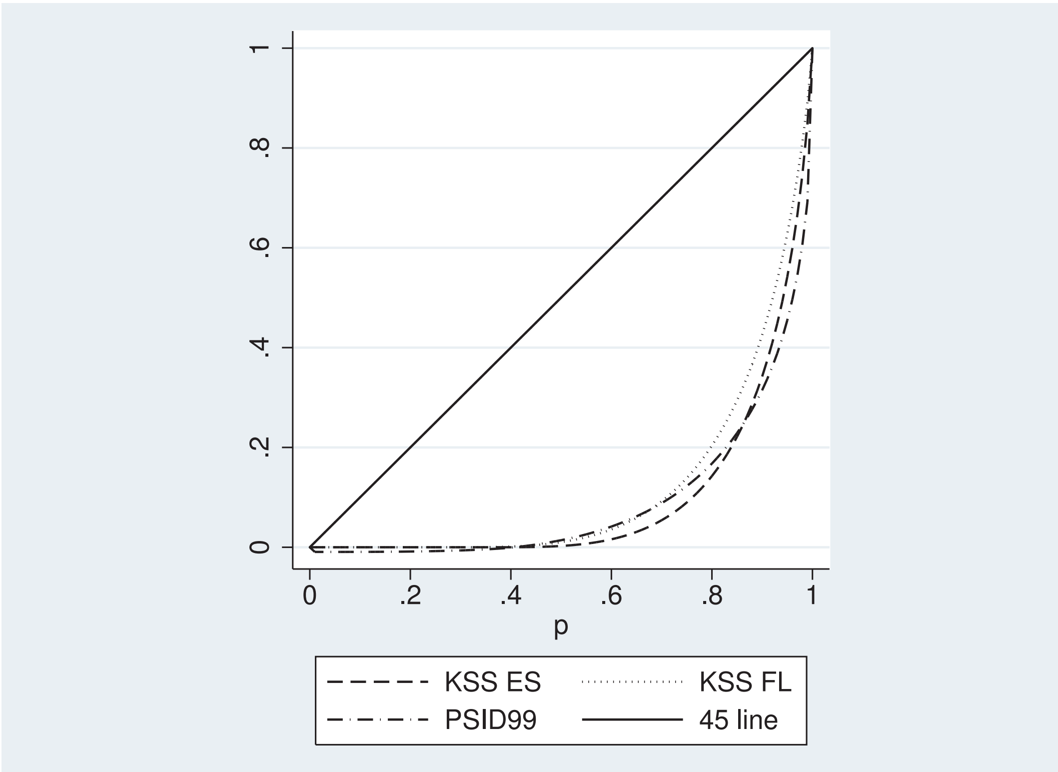

Figure 5: Lorenz Curves for Assets: CRRA Heterogeneity (ES and FLIN cases), and PSID 1999 data.

Parameter	*Value*	*Target*
Model Period	*Yearly*	
γ - *CRRA*	*See Table 2*	*Distribution computed from PSID data*
ρ_y, σ^2_γ - *Persistence and variance of shocks*	*See Table 3*	*Various AR(1) stochastic income processes*
β - *Rate of time preference*	$0.87 - 0.89$	*Annual interest rate* $r = 3.5\%$
δ - *Capital depreciation rate*	0.08	*Investment/Output ratio* $\approx 25\%$
α - *Capital share*	0.36	*Capital Share of Output*
b - *Borrowing limit*	0	*No Borrowing*

Table 1: Benchmark Model, Calibration - U.S.

Case	*Distribution*	μ_γ	σ_γ
KSS - PSID Data	$LN\left(\mu_\gamma, \sigma_\gamma^2\right)$	1.07	0.87

Table 2: Risk Aversion Distribution, author's calculations from PSID data.

Income Process	(1) *FLIN*	(2) *FREN*	(3) *GRIP*
Source	*Floden-Linde* (2001)	*French* (2005)	*Guvenen* (2009)
ρ_y	0.92	0.977	0.988
σ_y	0.21	0.12	0.122

Table 3: Labor Income Stochastic Processes.

RRA Heterogeneity:	*Case*	*Asset*	*Income*	*Consumption*
		Gini, CV	*Gini, CV*	*Gini, CV*
Yes - *KSS*:	(1) *FLIN*	$0.76, 1.87$	$0.39, 0.82$	$0.28, 0.52$
	(2) *FREN*	$0.83, 2.32$	$0.44, 0.98$	$0.33, 0.63$
	(3) *GRIP*	$0.85, 2.62$	$0.51, 1.22$	$0.43, 0.91$
	(4) *ES*	$0.81, 2.28$	$0.39, 0.93$	$0.27, 0.53$
No - $\gamma = 4.28^*$:	(1) *FLIN*	$0.50, 0.99$	$0.33, 0.62$	$0.25, 0.47$
	(2) *FREN*	$0.57, 1.20$	$0.38, 0.75$	$0.32, 0.62$
	(3) *GRIP*	$0.62, 1.42$	$0.47, 1.02$	$0.43, 0.92$
	(4) *ES*	$0.51, 1.00$	$0.29, 0.55$	$0.21, 0.39$

Table 4: Equilibrium - Inequality Measures in the one job model (cases 1-3) and in the two risky jobs model (case 4): Gini Index and Coefficient of Variation. An asterisk denotes that all the parameters are the same as in the corresponding heterogeneous preferences economy.

	$Q1$	$Q2$	$Q3$	$Q4$	$Q5$	$90-95th$	$95-99th$	$100th$
Data:								
U.S. - SCF98	$-.3$	1.3	5.0	12.2	81.7	11.3	23.1	34.7
U.S. - PSID99	$-.8$	0.8	4.2	12.6	83.2	14.0	23.7	30.8
Exogenous Earnings:								
KSS (FLIN)	.00	0.1	3.4	16.8	79.6	20.3	26.4	10.8
KSS (FREN)	.00	.01	0.2	9.7	90.1	22.5	31.7	14.4
KSS (GRIP)	.00	.00	0.01	7.7	92.2	21.5	33.7	17.6
CRRA=4.28 (FLIN)*	2.0	7.3	14.2	24.7	51.8	13.0	14.2	5.5
CRRA=4.28 (FREN)*	0.6	4.9	12.1	24.0	58.4	14.5	16.7	6.9
CRRA=4.28 (GRIP)*	0.6	4.1	10.3	21.7	63.3	15.2	19.2	9.1
Endogenous Sorting:								
KSS (ES)	.00	.01	1.6	12.7	85.7	20.0	30.6	15.2
CRRA=4.28 (ES)*	1.3	6.7	14.2	25.4	52.4	13.3	14.2	5.2

Table 5: Data Vs. Equilibria - Statistics of the Wealth Distribution, author's calculations, the SCF figures are from Diaz-Gimenez, Glover and Rios-Rull (2011). An asterisk denotes that all the parameters are the same as in the corresponding heterogeneous preferences economy. For the ES case, the income process is the weighted average of the two processes found in the calibration of the heterogeneous preferences economy.

RA group	$\widehat{\rho}$	$\widehat{\sigma}_y^2$	*Individuals*
$\underline{\gamma}$	.947	.018	560
$\overline{\gamma}$	.935	.018	569

Table 6: Estimates of the Labor Earnings Shocks Variance and Persistence by two Risk Aversion Categories in the PSID (1968-1993), author's calculations.

Percentile	$75th$	$90th$	$95th$	$99th$	*Obs*
$Exp. \leq 8$					
$\overline{\gamma}$:	.347	.553	.683	.985	2193
$\underline{\gamma}$:	.391	.593	.732	1.02	2724
$8 < Exp. \leq 14$					
$\overline{\gamma}$:	.291	.514	.684	1.07	2697
$\underline{\gamma}$:	.323	.544	.697	1.20	3032
$14 < Exp. \leq 19$					
$\overline{\gamma}$:	.258	.497	.683	1.26	2320
$\underline{\gamma}$:	.282	.568	.738	1.18	2283
$19 < Exp. \leq 25$					
$\overline{\gamma}$:	.273	.527	.749	1.48	2235
$\underline{\gamma}$:	.308	.580	.741	1.16	1760
$Exp. > 26^{+}$					
$\overline{\gamma}$:	.400	.713	1.00	1.64	2306
$\underline{\gamma}$:	.433	.758	.917	1.42	1172

Table 7: Selected Statistics of Residual Earnings by Potential Labor Market Experience (Exp.), author's calculations from PSID data.

Parameter	*Value*	*Targets*	*Data*	*Model*
$\rho_{y,h}$	.9662	*Earnings autocorrelation for types* $\gamma \geq \gamma^{Median}$	.9475	.9440
$\rho_{y,l}$	.9177	*Earnings autocorrelation for types* $\gamma < \gamma^{Median}$	.9353	.9384
β	.9094	*Annual interest rate*	.0350	.0351
m_h	.0012	*Ratio of average earnings by risk aversion groups*	1.009	1.009
$\sigma_{y,l} = \sigma_{y,h}$	.1363	*S.d. of persistent shocks estimated on PSID data*	-	-

Table 8: Two Risky Jobs Model with Endogenous Sorting (ES), Calibration - U.S.

Case	*RRA Heterogeneity*	r (%)	w	Y	I	C	K	K/Y
(1) *FLIN*	*No - CRRA=4.28**	5.12	0.92	1.67	0.37	1.30	4.57	2.74
	Yes - KSS	3.51	1.00	1.79	0.45	1.34	5.61	3.13
(2) *FREN*	*No - CRRA=4.28**	6.16	0.89	1.62	0.33	1.29	4.12	2.54
	Yes - KSS	3.51	0.99	1.82	0.46	1.36	5.70	3.13
(3) *GRIP*	*No - CRRA=4.28**	7.24	0.85	1.81	0.35	1.46	4.27	2.36
	Yes - KSS	3.50	1.00	2.12	0.53	1.59	6.64	3.13
(4) *ES*	*No - CRRA=4.28**	5.26	0.92	1.57	0.34	1.23	4.27	2.71
	Yes - KSS	3.51	1.00	1.75	0.44	1.31	5.48	3.13

Table 9: Homogeneous Vs. Heterogeneous Preferences: Equilibrium Allocations and Prices. An asterisk denotes that all the parameters are the same as in the corresponding heterogeneous preferences economy.

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Appendix A - Computation

- All codes solving the economies were written in the FORTRAN 2003 language, relying on the Intel Fortran Compiler, build 11.1.048 (with the IMSL library). They were compiled selecting the O3 option (maximize speed), and without automatic parallelization. Both the simulations of a sample of agents and the computation of the career dependent value functions were parallelized with the OpenMP directives. They were executed on a 64-bit PC platform, running Windows 7 Professional Edition, with an Intel $i7-2600k$ Quad Core processor clocked at 4.4 Ghz.

- Depending on the parameters (essentially on the discount factor β, and the precision of the initial guess) without endogenous sorting the model solution took between 30 minutes and 3 hours to complete (between 4 and 20 iterations on the interest rate are needed to find each equilibrium). With endogenous sorting, thanks to the parallelization of both the decision problem and the simulations, the model solution took on average one hour. The moment matching procedure required up to 100 hours to complete.

- In the actual solution of the model, I need to discretize the three continuous state variables ε, a, and γ. As for ε, I rely on Rouwenhurst's method, which approximates the $AR(1)$ process for the efficiency units with a Markov chain. I use an eleven-state chain. As for a, I rely on an unevenly spaced grid, with the distance between two consecutive points increasing geometrically. In order to keep the computational burden manageable, I use 151 grid points on the asset space, the lowest value being the borrowing constraint and the highest one being a value $a_{\max} > \overline{a}$ high enough for the saving functions to cut the 45 degree line ($a_{\max} = 150$). This is done to allow for a high precision of the policy rules at low values of a, where the change in curvature is more pronounced. As for γ, I discretize its support using 100 unevenly spaced points, the lowest value being close to zero ($\gamma_{\min} = 0.001$) and the highest one being 11 ($\gamma_{\max} = 11$). Overall, I need to solve a functional equation at $11 \times 151 \times 100 = 166,100$ points in the state space.

- The model is solved with a 'successive approximation' procedure on the set of value functions. I start from a set of guesses $V(\varepsilon, a; \gamma)_0$. I compute the vector of parameters Ω representing the Schumaker spline approximations of the value functions. I solve the constrained maximization problems and retrieve the policy functions, $a'(\varepsilon, a; \gamma)$. Notice that I do not restrict the agents' asset holdings to belong to a discrete set. As for the approximation method, I rely on the quadratic spline approximations for the future value functions, when evaluated at the chosen saving level. I keep on iterating until a fixed point is reached, i.e. until two successive iterations satisfy:

$$\underset{a}{Sup}\,|V(\varepsilon,a;\gamma)_{n+1}-V(\varepsilon,a;\gamma)_n|<\epsilon,\forall\varepsilon \text{ and } \forall\gamma$$

where ϵ is a small convergence criterion. In a previous version of the paper I solved the model with a time iteration scheme on the policy functions. For several parameters, this method showed numerical instability for high values of the risk aversion parameter. With value function iteration, the model can be solved more reliably with larger upper bounds for γ.

- In the model without endogenous sorting, the stationary distributions $\mu(\varepsilon,a;\gamma)$ are computed by simulating a large sample of $100,000$ individuals for $3,000$ periods, which ensure that the statistics of interest are stationary processes. With endogenous sorting, the model needs longer simulations of $10,000$ periods, with the $100,000$ agents re-optimizing their career choices every $1,500$ periods.

- The first step of the simulation is a random draw from the risk aversion distribution for each agent, whose risk aversion type is permanent, hence it will not evolve over time. Once the risk aversion is assigned, I compute the fixed weights for the interpolation in the γ dimension. I then start simulating the sample of agents by drawing sequences of efficiency units from the Markov chain, and compute the capital supply together with the inequality measures.

- In the model with endogenous sorting, I apply Rouwenhurst's method twice, once for each exogenous earnings process. The value functions become job dependent, $V^j(\varepsilon,a;\gamma)$ $j=h,l$, but the solution method is similar to the one outlined above. In this case I need to solve a functional equation at $2\times 11\times 151\times 100=332,200$ points in the state space. I also need to simulate two independent sequences of shocks for each individual.

- As for the approximation method, I rely on a bi-linear approximation scheme for the saving and consumption functions, for values of a and γ falling outside the grid. Notice that I interpolate also in the γ dimension. Some experimenting showed that a linear interpolation implied relatively large errors for lower values of γ, suggesting to have several points for $\gamma<1$. Numerically, it seems more appropriate to have a large number of gridpoints, with a slowly growing distance between gridpoints, rather than a coarser grid with an aggressive interpolation scheme. It goes without saying that this impacts quite substantially the computational burden. While I make sure that the asset grid is not binding, I extrapolate the saving choices in the risk aversion dimension up to $\overline{\gamma}=14.5$, so that there is a small number of constrained individuals in the KSS case.

Appendix B - Solution Algorithm

The computational procedure used to solve the baseline model can be represented by the following algorithm:

- Generate a discrete grid over the CRRA space $[\gamma_{\min}, ..., \gamma_{\max}]$.
- Generate a discrete grid over the asset space $[-b, ..., a_{\max}]$.
- Generate a discrete grid over the efficiency units space $[\varepsilon_{\min}, ..., \varepsilon_{\max}]$.
- Get the aggregate labor supply L.
- Guess the interest rate r_0.
- Get the capital demand k.
- Get the wage rate per efficiency units w.
- Get the saving functions $a'(\varepsilon, a; \gamma)$.
- Get the stationary distributions $\mu(\varepsilon, a; \gamma)$.
- Get the aggregate capital supply.
- Check the asset market clearing; Get r_1.
- Update $r_0' = \varpi r_0 + (1 - \varpi) r_1$ (with ϖ an arbitrary weight).
- Iterate until market clearing.
- Get the consumption functions $c(\varepsilon, a; \gamma)$.
- Check the final good market clearing.

The computational procedure used to solve the two risky jobs model needs three additional steps: the computation of two job-dependent value functions $V^j(\varepsilon, a; \gamma)$, the computation of the optimal job policy functions $\Phi(\varepsilon, a; \gamma)$, and the related supply of efficiency units of labor L.

- Generate a discrete grid over the CRRA space $[\gamma_{\min}, ..., \gamma_{\max}]$.
- Generate a discrete grid over the asset space $[-b, ..., a_{\max}]$.
- Generate two discrete grids over the efficiency units space $\left[\varepsilon^j_{\min}, ..., \varepsilon^j_{\max}\right], \ j = h, l$.
- Guess the aggregate labor supply L_0.
- Guess the interest rate r_0.
- Get the capital demand k.
- Get the wage rate per efficiency units w.
- Get the saving functions $a'_j(\varepsilon, a; \gamma), \ j = h, l$.
- Get the job-dependent value functions $V^j(\varepsilon, a; \gamma)$.
- Get the optimal job decisions $\Phi(\varepsilon, a; \gamma)$.
- Get the stationary distributions $\mu(\varepsilon, a; \gamma)$.
- Get the aggregate capital supply.
- Check the asset market clearing; Get r_1 and L_1.
- Update $r'_0 = \varpi_r r_0 + (1 - \varpi_r) r_1$ (with ϖ_r an arbitrary weight).
- Update $L'_0 = \varpi_L L_0 + (1 - \varpi_L) L_1$ (with ϖ_L an arbitrary weight).
- Iterate until both market clearing and $L'_0 \approx L_0$.
- Get the consumption functions $c(\varepsilon, a; \gamma)$.
- Check the final good market clearing.

Appendix C - Additional Results

Wage Process	ρ_y	σ_y	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7	ε_8	ε_9	ε_{10}	ε_{11}
(1) *FLIN*	.92	.21	0.18	.26	.36	.51	.71	1.00	1.40	1.97	2.76	3.88	5.44
Floden-Linde													
(2) *FREN*	.977	.12	0.17	.24	.34	.49	.70	1.00	1.43	2.04	2.91	4.15	5.93
French									.	.	.	.	.
(3) *GRIP*	.988	.122	0.08	.14	.22	.37	.61	1.00	1.65	2.72	4.48	7.38	12.2
Guvenen													

Table 10: Labor Efficiency Units - AR(1) Discretization with the Rouwenhurst Method.

ES Processes	ρ_y	σ_y	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7	ε_8	ε_9	ε_{10}	ε_{11}
High Risk	.966	.136	0.19	.27	.38	.53	.74	1.04	1.45	2.03	2.83	3.96	5.53
Low Risk	.917	.136	0.34	.42	.52	.65	.80	1.00	1.24	1.54	1.92	2.38	2.96
*Weighted Average**	.941	.136	0.28	.36	.47	.61	.78	1.01	1.30	1.69	2.18	2.81	3.63

Table 11: Labor Efficiency Units with Endogenous Sorting - AR(1) Discretization with the Rouwenhurst Method. An asterisk denotes the average process, used in the corresponding one risky job economy.

RA group	$\widehat{\rho}$	$\widehat{\sigma}_y^2$	$\widehat{\sigma}_f^2$	$\widehat{\sigma}_v^2$	*Individuals*
γ_l	.947	.018	.061	.099	560
γ_h	.935	.018	.074	.079	569

Table 12: Estimates of the Labor Earnings Shocks Variance and Persistence by two Risk Aversion Categories in the PSID (1968-1993), author's calculations.

	$Q1$	$Q2$	$Q3$	$Q4$	$Q5$	$90-95th$	$95-99th$	$100th$
U.S. - SCF07	$-.2$	1.1	4.5	11.2	83.4	11.1	26.7	33.6
U.S. - SCF92	$-.4$	1.7	5.7	13.4	79.5	12.6	23.9	29.5
U.S. - PSID01	-0.9	0.7	4.2	13.2	82.8	14.3	24.1	28.6
U.S. - PSID94	-1.2	0.8	5.0	14.7	80.7	15.2	24.8	23.9

Table 13: Data Vs. Equilibria - Statistics of the Wealth Distribution.

Appendix D - PSID Labor Income Gambles

The wording of the initial question related to the risk tolerance is as follows (the bold type is mine):

> Suppose you had a job that guaranteed you income for life equal to your current, total income. And that job was (your/your family's) only source of income. Then you are given the opportunity to take a new, and equally good, job with a 50-50 chance that it will **double** your income and spending power. But there is a 50-50 chance that it will cut your income and spending power by a **third**. Would you take the new job?

The subsequent questions branch out in different directions, depending on whether the respondent accepted or rejected the initial gamble.

Individuals accepting the initial risky job are then asked about another job gamble with a higher downside risk:

> Now, suppose the chances were 50-50 that the new job would double your (family) income, and 50-50 that it would cut it in **half**. Would you still take the new job?

Individuals rejecting the initial risky job are then asked about another job gamble with a lower downside risk:

> Now, suppose the chances were 50-50 that the new job would double your (family) income, and 50-50 that it would cut it by **20 percent**. Then, would you take the new job?

In a similar vein, another final question is asked to the individuals accepting both the first and the second gambles:

> Now, suppose that the chances were 50-50 that the new job would double your (family) income, and 50-50 that it would cut it by **75 percent**. Would you still take the new job?

Finally, another question is asked to the individuals rejecting both the first and the second gambles:

> Now, suppose that the chances were 50-50 that the new job would double your (family) income, and 50-50 that it would cut it by **10 percent**. Then, would you take the new job?