Preferences

- 3 axioms of preference (basics of consumer theory)
  - Completeness
  - Transitivity
  - Monotonicity

- Ordinal ranking VS Cardinal ranking

- Utility Functions: math expressions of preferences
  - e.g. \( U(x) = x^{1/2} \)
  - e.g. \( U(x, y) = x^{1/2} y^{1/2} \)

- Hard to draw on blackboard (3-D)
Indifference Curves

- 2-D representation of 3-D Utility surface.
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"well-behaved" ICs have 4 properties:

1. Slope is negative (when you like both goods)
2. ICs cannot intersect
3. Every bundle lies on *one & only one* IC
4. ICs are not thick (nonsatiation)
**Figure:** Utility Surface in 3D
Figure: Utility Surface (IC Skeleton)
Figure: Indifference Map
Marginal rate of substitution (MRS)

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  - \(MRS_{x,y} = -\frac{\Delta Y}{\Delta X}\)

Diminishing MRS

- \(MRS\) decreases as consumption of \(x\) increases.
  - direct consequence of Diminishing Marginal Utility
  - Example, see class notes
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Several Utility Functions

- Cobb-Douglas utility function

Cobb-Douglas utility function

$$U(X, Y) = AX^a Y^b$$

where $A$, $a$, $b$ are positive constants

$$\text{MRS}_{x, y} = \frac{aY}{bX}$$

and

$$a \times b$$

measures relative importance of $X$ and $Y$

(I) Constant returns to scale when $a + b = 1$

**Perfect Substitutes**

$$U(X, Y) = aX + bY$$

where $a$, $b$ are positive constants

$$\text{MRS}_{x, y} = \frac{a}{b} = \text{CONSTANT}$$

(I) always substitute at the same rate (perfect subs).

**Perfect Complements**

$$U(X, Y) = \min\{aX, bY\}$$

How to get ICs:

1. equate two arguments to find the cut-off line
2. start at a point on the line, then increase $X$ while holding $Y$ constant and/or increase $Y$ while holding $X$ constant.
3. bundles have the same level of utility as the initial bundle

(I) see class notes for another way.

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Tianyi Wang  (Queen’s University)  Lecture 5  Winter 2013  8 / 32
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Several Utility Functions (Cont’d)

- Quasi-Linear

\[ I(U(X, Y)) = v(X) + bY, \]

where \( b \) is a constant, and \( v(X) \) is any increasing function.

\[ IMRS_x, y = v_0(x)b, \]

independent of \( Y \).

I have just enough curvature to get interior solution (more later)

I (no income effect)

One Good, One Bad

\[ I(U(X, Y)) = aX^bY, \]

where \( a, b \) are positive constants.

\[ IMRS_x, y = \frac{a}{b}, \]

ICs slope upward!

e.g. return and risk; leisure and labor.

Exercise: Draw IC for the following utility function

\[ I(U(X, Y)) = \frac{X}{2} \frac{Y}{2}. \]

I How to get ICs:

1. level of utility, say \( U = 9 \)
2. find 3 pairs of \((X, Y)\) on this IC.
3. connect these points to get the indifference curve.
Several Utility Functions (Cont’d)

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  - \( U(X, Y) = v(X) + bY \), where \( b \) constant, \( v(.) \) any increasing function
Several Utility Functions (Cont’d)

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  - \( U(X, Y) = \nu(X) + bY \), where \( b \) constant, \( \nu(.) \) any increasing function
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  - \( U(X, Y) = aX - bY \), where \( a, b \) are positive constants
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  - \( U(X, Y) = v(X) + bY \), where \( b \) constant, \( v(.) \) any increasing function
  - \( MRS_x,y = \frac{v'(x)}{b} \), independent of \( Y \)
  - have just enough curvature to get interior solution (more later)
  - (no income effect)

- **One Good, One Bad**
  - \( U(X, Y) = aX - bY \), where \( a, b \) are positive constants
  - \( MRS_x,y = -\frac{a}{b} \), ICs slope upward!
  - e.g. return and risk; leisure and labor.

- **Exercise:** Draw IC for the following utility function
  - \( U(X, Y) = X^{1/2} Y^{1/2} \)
  - How to get ICs:
    - fix level of utility, say \( U = 9 \)
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    3. connect
Budget Constraint

- Cannot purchase infinite goods due to budget constraint.

Assumptions:

I. A1: Two goods: \( X_1 \) and \( X_2 \), example: bananas and apples.

I. A2: Your income: \( m \).

I. A3: \( P_1 \) is the price for good \( X_1 \) and \( P_2 \) is the price for good \( X_2 \).

F. Take as given for now, can be determined in equilibrium (Chapter 2).

I. A4: spend all your income on the two goods.

BC expresses the combination of \( X_1 \) and \( X_2 \) the consumer can buy given her/his income written as:

\[
P_1 X_1 + P_2 X_2 = m
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I. What are the values of the intercepts?

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  - What does the shaded area represent?
  - What is the slope of the line?
Budget Constraint (Cont’d)

- Vertical intercept = \( m/p_2 \)
- Budget line; slope = \( -p_1/p_2 \)
- Budget set

Horizontal intercept = \( m/p_1 \)
Budget Constraint (Income Increase)

The graph illustrates a budget constraint with two goods, $x_1$ and $x_2$, and two prices, $p_1$ and $p_2$. The budget constraint is given by the equation:

$$ m/p_2 \leq x_2 \leq m/p_1 $$

The slope of the budget lines is $-p_1/p_2$. The budget constraint limits the consumption possibilities based on income and prices.
Budget Constraint (Price Change)

![Graph showing budget constraints with slopes and intercepts]

- **Budget lines**
- **Slope**:
  - $m/p_2$
  - $m/p_1$
  - $-p_1/p_2$
  - $-p_1/p_2$
Budget Constraint (summary)

- Changes in income cause BC shift.
- Changes in price cause BC rotate about intercepts.
- Application 4.4
  - Suppose consumer chooses between two phone plans.
  - Plan 1: $40 per month for call up to 450min, additional minute costs $0.40
  - Plan 2: $60 per month for call up to 900min, additional minute costs $0.40
  - Income $500, price of other goods $1.
  - draw BC.
Optimal Choices: Interior Solution

- Consumer maximizes utility while facing his/her budget constraint.
- Let consumer choices over $x$ and $y$.
- Let $U(x, y)$ denotes her utility function and $I$ denotes her income.
- Consumer’s problem can be formally expressed as:
  \[
  \max_{(x,y)} U(x,y) \quad \text{subject to} \quad P_x x + P_y y \leq I
  \]
- Note in our case budget constraint holds at equality, since consumer does not derive utility from holding income.
- Interior: At optimum, consume both $x$ and $y$.
- How to choose?
Graph Summary

1. Start at arbitrary bundle A, know there is an IC goes through A.
2. Ask if can increase utility by shifting IC to the right.
3. Stop if IC detaches from BC (non-affordable).
4. Utility maximized when IC "just touches" BC.
Optimality Condition

- Notice at optimum slope of BC equals slope of IC (tangency).
- Slope of IC $= \frac{\Delta Y}{\Delta X}$ and $MRS_{x,y} = -\frac{\Delta Y}{\Delta X} = \frac{MU_x}{MU_y}$
- Therefore, Slope of IC $= -\frac{MU_x}{MU_y}$
- Slope of BC $= -\frac{P_x}{P_y}$
- Then we have the following Optimality Condition:
  - $\frac{MU_x}{MU_y} = \frac{P_x}{P_y}$
- To understand, rewrite is as:
  - $\frac{MU_x}{P_x} = \frac{MU_y}{P_y}$
- Marginal Utility derived from last dollar equals.
- "bang for the buck"
Optimality Condition (Con’t)

- Why bundles like A and B are not optimal?
- For instance, at A slope of IC is steeper than slope of BC:

\[
- \frac{MU_x}{MU_y} \leq - \frac{P_x}{P_y}, \text{ or }
\]

\[
\frac{MU_x}{P_x} \geq \frac{MU_y}{P_y}
\]

- Last dollar on X gives more utility than on Y.
- How to solve Consumer’s Problem? In 3 steps:
  1. Calculate \(MU_x\) and \(MU_y\).
  2. Use Optimality Condition to express \(X\) in terms of \(Y\).
  3. Sub into BC to solve for \(Y\). Then get \(X\) from step 2.

- See class notes for example.
- See class notes for opportunity cost argument.
Optimal Choices: Corner Solution

- At optimum, some goods are not consumed (when chooses among multiple goods).
- In our case, only one good is consumed.
- Graphically, for any interior points on BC, slope of IC is steeper/flatter than slope of BC.
- See class notes for example (short sales).
Example: Savings Decision

- A simple two-period model: present (1) and future (2).
- Consumer consumes in both period
  \[ U(C_1, C_2) = U(C_1) + \frac{U(C_2)}{1 + \rho} \]
- \( \rho \) rate of time preference, usually a small positive number (0.05).
- Income endowment in each period \( I_1 \) and \( I_2 \).
- Can borrow or save at given interest rate \( r \).
- Write down BC, draw Budget Line.
- Present consumer’s problem and solve for optimum.
Consumer’s problem:

\[ \max_{(C_1, C_2)} U(C_1, C_2) = U(C_1) + \frac{U(C_2)}{1+\rho} \]

subject to: \( C_1 + \frac{C_2}{1+r} \leq I_1 + \frac{I_2}{1+r} \)

\[ \frac{MRS_{C_1, C_2}}{MU_{C_2}} = (1 + \rho) \frac{U'(C_1)}{U'(C_2)} \]

Slope of BC = \(- (1 + r)\)

Then at optimum \((1 + \rho) \frac{U'(C_1)}{U'(C_2)} = 1 + r\)

When will she save/borrow?

\[ (1 + \rho) \frac{U'(I_1)}{U'(I_2)} \leq 1 + r \]

See class notes for example.
Changing Interest Rate
Changing Interest Rate

Graph showing the relationship between C1 and C2 with point E as the intersection.
Changing Interest Rate
A rise in interest rates will make saving more attractive ...
Different Interest Rate for Borrowing and Saving