8 Appendix B - Not for Publication

8.1 Moral Hazard in the Bank-Borrower Relationship

We now relax the assumption that monitoring of the borrower is costless for the bank, thereby introducing the traditional moral hazard problem into our framework. For simplicity, we do away with the asymmetric information problem, or alternatively, assume that the needed parameterization underlying Proposition 1 is satisfied.[31] This will allow us to focus on only one bank loan type. Define M as the amount monitored that takes a value in the compact interval $[0, \overline{M}]$. We introduce a cost of monitoring function for a loan: $c(M)$ with $c'() > 0$, $c''() > 0$ and $c(0) = 0$. For simplicity, we rule out corner solutions by assuming $c(\cdot)$ satisfies the Inada conditions: $c'(0) = 0$ and $c'(\overline{M}) = +\infty$.

We enrich the bank's loan to a continuous return conditional on the amount monitored (M). The expected return on the bank's loan is given by:

$$E(R; M) = \int_0^{\overline{R}} \psi h(\psi; M) d\psi - c(M), \tag{42}$$

where ψ is the total return from the loan and h is a density function with corresponding distribution H. We define the upper bound of the support as $\overline{R} > 1$. To include the possibility default of the bank loan, we assume that the loan fails if the realized value, $\tilde{\psi} \in [0, 1]$.

We assume that $H(\psi; M)$ satisfies the usual Monotone Likelihood Ratio Property (MLRP) so that $\frac{\partial}{\partial \psi}\left(\frac{h_M(\psi;M)}{h(\psi;M)}\right) > 0$. Finally, we make the standard assumption that the distribution satisfies the convexity-of-distribution function (CDFC) assumption (as in Hart and Holmström, 1987).[32] This assumption implies that for any $\lambda \in [0, 1]$, and for any M, M':

$$h(\psi; \lambda M + (1 - \lambda)M') \leq \lambda h(\psi; M) + (1 - \lambda)h(\psi; M'). \tag{43}$$

The MLRP and CDFC assumptions together intuitively say that increasing the monitoring, increases, at a decreasing rate, the probability that the return will be above some level ψ.[33] We begin by analyzing the case in which the bank cannot insure itself to avoid the penalty (Z) if the loan fails.

[31] The analysis of this section carries through if we allow for asymmetric information; there will be expressions for each loan type in both the pooling and separating equilibria yielding the same qualitative results.

[32] This assumption is used in the first order approach to principal agent problems when the monitoring space is continuous. It allows us to write the infinite number of incentive constraints in one equation. We do not wish to weigh in on the debate that began in the late 1970's as to the validity of the first order approach. For those who find the CDFC assumption unpalatable, Jewitt (1988) Theorem 1 shows how it can be relaxed with additional assumptions on the utility function. The alternative approach to the continuous case is to discretize the monitoring space so that there is a finite number of incentive constraints. The qualitative results of this section follow through with such a procedure, provided there are greater than 2 levels of monitoring (for reasons which will soon become apparent). We use the continuous setup for convenience.

[33] See Laffont and Martimort (2002) for a review of the first order approach to principal agent problems.

8.1.1 No Insurance

When the bank does not use insurance, the optimal amount of monitoring is the incentive feasible level as follows:

$$\int_1^{\overline{R}} \psi dH(\psi;M) + (1-\gamma)\int_0^1 dH(\psi;M)\psi + \gamma\int_0^1 (\psi - Z)dH(\psi;M) - c(M)$$
$$\geq \int_1^{\overline{R}} \psi dH(\psi;M') + (1-\gamma)\int_0^1 dH(\psi;M') + \gamma\int_0^1 (\psi - Z)dH(\psi;M') - c(M') \ \ \forall\, M' \neq M. \quad (44)$$

Hart and Holmstrom (1987) showed that given MLRP and CDFC, these constraints can be re-written as:

$$c'(M) = \int_0^{\overline{R}} \psi dH_M(\psi;M) + \gamma\int_0^1 Z dH_M(\psi;M), \quad (45)$$

where H_M is the partial with respect to M. Note that the MLRP assumption implies the weaker condition of First Order Stochastic Dominance (FOSD) which implies that $\int_0^{\overline{R}} \psi dH_M(\psi;M) > 0$. The left hand side represents the marginal cost of increasing monitoring and is given by the marginal increase in the cost of monitoring itself. The right hand side represents its marginal benefit and is comprised of both the increase in the expected value of the loan, and the reduced probability of loan default that monitoring brings.

We now analyze the case in which the bank can perfectly insure (i.e. no counterparty risk) themselves to avoid the possible cost of Z.

8.1.2 Insurance, No Counterparty Risk

When the bank uses insurance with no counterparty risk, the optimal amount of monitoring is as follows:

$$\int_1^{\overline{R}} \psi dH_M(\psi;M) + (1-\gamma)\int_0^1 \psi dH_M(\psi;M) + \gamma\int_0^1 (1+\psi)dH_M(\psi;M) + \gamma P_M^{NCR} - c'(M) = 0, \quad (46)$$

where P_M^{NCR} represents the marginal price with no counterparty risk. Note that since FOSD implies $\int_0^1 dH_M(\psi;M) < 0$ and Lemma 3 implies $\frac{\partial P}{\partial b} > 0$, it follows that $P_M = \frac{\partial P}{\partial M} = \frac{\partial b}{\partial M}\frac{\partial P}{\partial b} = \int_0^1 dH_M(\psi;M)\frac{\partial P}{\partial b} < 0$. Finally, since FOSD implies both $\int_0^1 dH_M(\psi;M) < 0$ and $\int_1^{\overline{R}} dH_M(\psi;M) > 0$, we can rewrite (46) in a more intuitive form:

$$c'(M) + \gamma\int_0^1 dH_M(\psi;M) = \int_0^{\overline{R}} \psi dH_M(\psi;M) + \gamma P_M^{NCR}. \quad (47)$$

The left hand side represents the marginal cost of monitoring. For an increase in monitoring, the bank incurs the monitoring cost itself, plus a decrease in expected payout from the claim (because claims are made less often with more monitoring). The benefits to monitoring are the increase in the expected return of the loan, plus the reduced insurance premium the bank will enjoy by reducing the probability that a claim will be made.

Comparing (45) and (47) we see that insurance reduces the incentive to monitor when the following holds:

$$Z > \frac{P_M^{NCR} - \int_0^1 dH_M(\psi; M)}{\int_0^1 dH_M(\psi; M)}. \tag{48}$$

In other words, when default of the loan without protection is sufficiently costly, the firm will choose to monitor more when it is not insured. Note that the sign of $P_M^{NCR} - \int_0^1 dH_M(\psi; M)$ is ambiguous and depends on the underlying parameters of the model. When $P_M^{NCR} \leq \int_0^1 dH_M(\psi; M)$, the bank will always monitor more when it is not insured (for any $Z > 0$).

We continue by adding counterparty risk to the insurance contract and show that the moral hazard problem may be less severe than in the current case.

8.1.3 Insurance with Counterparty Risk - Double Moral Hazard

When the bank uses insurance with counterparty risk, a double moral hazard problem is present: both the monitoring by the bank, and investment decision by the IFI occur simultaneously. Therefore, there is an optimization problem for both the bank and the IFI. We now write the first order condition for the bank taking β^* as given.

$$\begin{aligned} &\int_0^{\overline{R}} \psi dH_M(\psi; M) + \gamma \int_0^1 dH_M(\psi; M) \int_{C(\gamma - \beta^* P^* \gamma)}^{\overline{R}_f} dF(\theta) \\ -Z\gamma &\int_0^1 dH_M(\psi; M) \int_{\underline{R}_f}^{C(\gamma - \beta^* P^* \gamma)} dF(\theta) - c'(M) - \gamma P_M^{CR} = 0 \end{aligned} \tag{49}$$

Where P_M^{CR} is the marginal price with counterparty risk. Because $P_M^{CR} < 0$, $\int_0^1 dH_M(\psi; M) < 0$ and $\int_0^{\overline{R}} dH_M(\psi; M) > 0$, we can rewrite (49).

$$\begin{aligned} &\underbrace{c'(M) + \gamma \int_0^1 dH_M(\psi; M) \int_{C(\gamma - \beta^* P^* \gamma)}^{\overline{R}_f} dF(\theta) =}_{\text{Marginal Cost of Monitoring}} \\ &\underbrace{\int_0^{\overline{R}} \psi dH_M(\psi; M) + Z\gamma \int_0^1 dH_M(\psi; M) \int_{\underline{R}_f}^{C(\gamma - \beta^* P^* \gamma)} dF(\theta) + \gamma P_M^{CR}}_{\text{Marginal Benefit of Monitoring}} \end{aligned} \tag{50}$$

Altering Lemma 1 to include an optimal choice of monitoring by the bank, we obtain β^* for a given M^*.

$$\begin{cases} \beta^* = 0 & \text{if } b(M^*) \leq b^* \\ 0 = b(M^*) \left[C'(\gamma - \beta P\gamma) \left(\overline{R}_f + P\gamma(1-\beta)R_I - C(\gamma - \beta P\gamma)\right) - R_I \left(\overline{R}_f - C(\gamma - \beta P\gamma)\right)\right] & \\ \quad -(1-b(M^*))(R_I - 1)\left[\overline{R}_f + (\beta + (1-\beta)R_I)\right] & \text{if } b(M^*) \in (b^*, b^{**}) \\ \beta^* = 1 & \text{if } b(M^*) \geq b^{**} \end{cases} \tag{51}$$

where $b^* = \frac{(R_I-1)(\overline{R}_f+R_I)}{(R_I-1)(\overline{R}_f+R_I)+\overline{R}_f C'(\gamma)-\overline{R}_f R_I - C'(\gamma)C(\gamma)+C'(\gamma)P\gamma R_I + C(\gamma)R_I}$,
and $b^{**} = \frac{(R_I-1)(\overline{R}_f+1)}{(R_I-1)(\overline{R}_f+1)+\overline{R}_f C'(\gamma-\beta P\gamma)-\overline{R}_f R_I - C'(\gamma-\beta P\gamma)C(\gamma-\beta P\gamma)+C(\gamma-\beta P\gamma)R_I}$.

Comparing (50) and (47) we derive a condition under which the bank monitors strictly more when counterparty risk is present.

$$Z > \frac{-\int_0^1 dH_M(\psi; M)\left(1 - \int_{C(\gamma-\beta^* P^*\gamma)}^{\overline{R}_f} dF(\theta)\right) + P_M^{NCR} - P_M^{CR}}{\int_{\underline{R}_f}^{C(\gamma-\beta^* P^*\gamma)} dF(\theta)} \tag{52}$$

We conclude that if Z is sufficiently high, the traditional moral hazard problem will be less severe. The intuition is that the parameter Z ties the bank to the loan. If the loan and the IFI default, the bank is not protected and is subject to the cost Z. Therefore, the higher is Z, the more vigorously the bank will monitor the loan.

One of the key elements that emerges from this section is that when we introduce the classical moral hazard into the model, we need only modify the return structure of the bank loan to include a monitoring amount. In other words, the IFI simply adjusts its belief of the probability of a claim given the amount of monitoring the bank will engage in.