Can Stock Indices Be Made Better At Predicting Financial Contagion - A Network Model

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Preliminary

Abstract

When distressed traders are subject to regulatory solvency or leverage constraints, they have to liquidate their positions quickly, which may depress the asset prices. The shareholders of these assets will incur mark-to-market losses, which may be forced to liquidate as well, and the liquidation can spread out and become contagious over the financial markets. This paper explores the process of contagion of this type across asset markets through the price effect and shows that prices of thinly-held assets (assets with only a few shareholders) tend to drop earlier and faster than thickly-held assets at the early stages of a contagion (if a contagion were to happen). If this is treated as a signal, then this signal can predict an extensive contagion of fire sale fairly accurately. This suggests that a new type of stock indices which include only low degree assets should be composed, and it may help policymakers and investors respond to financial crises in a timely manner. The complex financial markets are depicted by random graph-based bipartite networks, which allow for multiple assets and multiple agents with arbitrary portfolios, and the results are derived through a analytical/numerical hybrid method, as well as simulations. We also examine the price dynamics of thinly/thickly-held assets in the formation of bubbles and contagion of boom, which is the reverse process of contagion of fire sales.

Keywords: Networks models, financial contagion, systemic risk, crisis forecast, liquidity risk, fire sale

JEL classification: D85; G01; G23; G24; G28

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1 Introduction

When financial institutions are in distress, forced sales of assets, either prescribed by regulatory constraints or forced by margin calls, may depress the price of the asset being liquidated and adversely affect other shareholders. This adverse effect can induce further round of forced sales by these shareholders in other asset markets, and the rapid price declines can be contagious and spread extensively across markets. In the US subprime crisis of 2008, the Dow Jones Industrial Average declined by 18% as of October 10, the largest weekly decline ever. Since various US stocks and derivatives were purchased by both American and European investors, and when their capital were eroded by US portfolio plunges, they had to liquidate their positions on European assets, either prescribed by VaR constraints or simply to show solvency. Thus crisis transmitted quickly from US to Europe, and in the same week, the FTSE100 (share index of the 100 top UK companies) declined by 20%, again the worst ever.

In August 2011, Standard & Poor's downgraded U.S. Treasury debt. Trillions worth of US Treasuries are pledged as collateral by borrowers such as banks and derivatives traders. The debtors will be required to append more cash or securities to reassure the lenders if their collateral isn't considered as safe by creditors. That could force debtors to sell off other assets to come up with the money. In the 30 days that follows, the three major US stock indices indeed declined by more than 10%.

In modern financial systems, institutions are interlinked by debts as well as common assets in their portfolios, and face a system risk of contagion of default and fire sale when the initial shock is large enough, as demonstrated by the recent crises. These financial contagions generally spread through two channels - direct credit exposures and indirect linkages through holding same assets and their prices changes. While the former has been extensively studied in the literature, the latter has received much less attention. This paper thus focuses on the contagion through indirect price channel.

In network literature the number of links of a node is termed its *degree*, so that if an asset has only a few shareholders, we say it is a thinly-held asset, or low degree asset; conversely, it is a thickly-held assets, or high degree asset. Similarly, if a trader has many assets in her portfolios (well-diversified), she is a high degree trader; otherwise, she is a low degree trader.

The simulation results from this paper which concerns about liquidity risks, as well as those from previous literature on credit risks, show that, as the financial system/networks are getting better and better connected, i.e. banks/agents' portfolios are more diversified, the probability of widespread contagion (termed the contagion frequency) and extent of contagion develop in a fashion depicted in Figure 1. It is worthy of note that for networks with moderate to

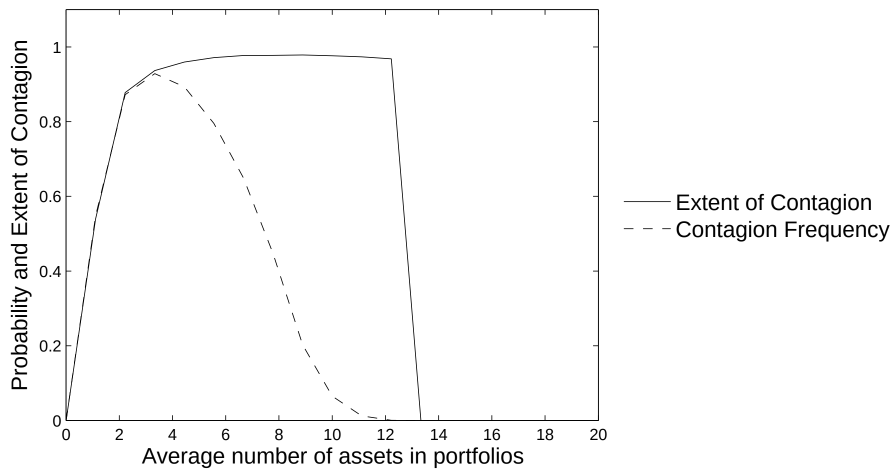


Figure 1: Simulation results of contagion frequency and extent in financial networks

moderate-high connectedness (i.e. between number of assets in portfolios of 8 and 12, which are reasonably good substitute of the real world financial networks) in the Figure 1, the contagion frequency is modest and decreasing. But once a contagion does happen, it is almost certain that 100% agents go bankrupt. "The distribution of cascade sizes observed in single-seed GK[1] simulations is thus typically bimodal: only a certain fraction of cascades reach a network-spanning size, the remainder remain small (typically only a few nodes)"(Gleeson et al. (2011)). This "robust-yet-fragile tendency ... highlights that a priori indistinguishable shocks can have very different consequences for the financial system, depending on the particular point in the network structure that the shock hits the resilience of the financial system to fairly large shocks prior to 2007 was not a reliable guide to its future robustness."(Gai and Kapadia (2010)). These concerns motivated the development of this paper.

This paper aims to identify signals of extensive contagions of fire sale across asset markets at their early stages, and try to distinguish extensive contagion from those contained within its origin, due to priori indistinguishable shocks. The main result is that prices of thinly-held

[1] Gai and Kapadia (2010)

assets (assets with only a few shareholders) are more sensitive to contagion than average assets, i.e. their prices decline faster than others at the beginning. In addition, the larger the price drop of thinly-held assets is (compared with thickly-held assets), the more likely a system wide contagion will materialize. If this is treated as a signal, then it predicts an extensive fire sale contagion fairly accurately, e.g. after observing this signal, 96% the chance that an extensive contagion will happen, while not observing this signal, only 6% the chance a contagion happens (See Table 1). So it suggests that a new type of stock indices which include only low degree assets should be composed, and this may help investors respond early to reduce loss and policymakers react to prevent a pandemic contagion from happening. To my knowledge, this paper might be the first one which investigates the relationship between asset degree and its capability to predict contagion. We also show that under some conditions, traders exhibit the same trend: at the beginning of a contagion, less diversified (low degree) traders go bankrupt faster than better diversified (high degree) ones.

Compared to the existing literature, which considers defaults only, or at most a single common asset, this paper deals with multiple assets and multiple traders, where we resort to bipartite networks[2]. The bipartite networks allow multiple assets explicitly represented by nodes that are distinct from agents (Figure 2 shows an example), in contrast to most financial network models where only agents/banks are modeled. With multiple assets distinguished from agents, price of each asset can be individually identified, and the contagion through price effect from one asset to another becomes self-evident. To highlight the price effects, I abstract away the counterparty risks/direct exposures and study networks of pure asset portfolio linkages.

The bipartite network structure can also be used to model the emergence and contagion of bubbles (see Section 6) in the asset market, which is the reverse process of fire sale.

In this paper we focus on random graph/network structures only. A random graph/network is a graph generated by some random process. In particular, how many links each node has is determined by a given probability distribution. In addition, who is connected to who is also determined by this random process. This type of networks highlights the topological complexity, and bears some key features of the real world financial networks, which makes it a good

[2]Its counterparty, unipartite network, is thus the network widely used in the existing literature which primarily concerns about interbank loans

substitute in system risk research. This paper extends Gleeson's framework (see next section) by applying it to bipartite random networks. By using both Gleeson's analytical/numerical hybrid method as well as simulations, we will exam in greater details about what happens in the contagion process, which has not been explored in previous literature.

2 Relationship to existing literature

Allen and Gale (2000) is the first paper that studied the financial contagion of counterparty risk over a network. With a simple network of four banks, they showed the advantage of diversification in a network sense: when banks only have exposures to a few others, the counterparty risk is not well diversified and the system is more vulnerable. On the other extreme, where every bank has exposures to all other banks, the risk is diversified across more counterparties, and default may be absorbed, so that contagion is less likely. This intuition turns out to be fundamental, and most models in financial networks demonstrate and depend on it.

Although the insights from simple and rigid network structures are seminal, its generality to the real world financial systems is doubtful. As indicated by Cifuentes et al. (2004) in more complicated networks, there is a non-monotonic relationship between connectedness and financial stability (depicted in Figure 1). Gai and Kapadia (2010) use random graph-based network structures to accommodate arbitrary and complex networks, so the results of this paper are compatible to and apply to all possible networks. They introduce the generating function techniques from the literature of physics on complex networks (Newman et al. (2001)) and derive an elegant analytical solution of the probability of system wide contagion.

While both Cifuentes et al. (2004) and Gai and Kapadia (2010) incorporate the asset price effects, there is only one generic illiquid asset in their models. In most of the literature on financial networks, including the above two papers, asset (if any) is the source instead of the medium/channel of contagion, and contagion still spreads through the credit channel, i.e. without default, there will be no contagion. The last three decades have seen a transition from bank-based financial system to a market-based financial system. As demonstrated in recent crises, contagion across assets markets via price effects plays an important role in market-based systemic events. Following Newman et al. (2001) and Gai and Kapadia (2010), Shen (2010) investigates the contagion of fire sale across multiple assets markets on a random graph network, evaluates the systemic risks by using generating functions and obtains results similar to those of Gai and Kapadia (2010). Established in an almost identical setting to Shen (2010), this paper identifies early signals of contagion of fire sale and provides a novel method to predict financial crisis on asset markets.

A parallel methodology to characterize the contagion process on random networks is provided by Gleeson and Cahalane (2007), Gleeson (2008) and applied to banking networks by Gleeson et al. (2011). Their methods calculate, period by period, the expected infection rate of nodes of each and all degrees, and then compute the infection rate of links emitted from these nodes, and accumulate current and past distressed links by using binomial distribution, and then enter the next period, until the results converge. This method crucially depends on the assumptions of networks being random. I call this model the Gleeson's method hereafter. In this paper, this recursive method is extended to bipartite networks and used to calculate the price drop and trader bankruptcy in the contagion process.

The rest of the paper is organized as follows: in Section 3, the model is specified and the vulnerability of assets are identified. Section 4 outlines two parallel and complementary methods used to calculate the contagion: Gleeson's method and monte carlo simulations, and the main results of this paper are given. To examine the robustness and generality of the these results, Section 5 modifies the original model and compare the results of the two models. As an extension, I discuss contagion of bubble, the reverse process of fire sale contagion in Section 6. A final section concludes.

3 The model

3.1 Network representation

We use nodes and links to represent the relationship among multiple traders and multiple assets (see Figure 2). Nodes are divided into two groups - traders and assets (or securities, I use them interchangeably). A link between a trader and an asset means that this trader holds some share of this asset. As shown in Figure 2, one trader may hold shares of some assets but not others, and one asset may be held by some traders but not every trader.

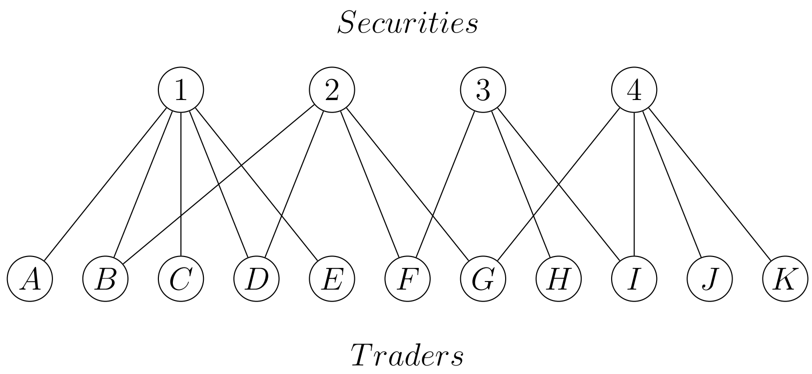


Figure 2: Asset markets and shareholders represented by a bipartite network

We assume that there are in total S assets, T traders in the network. In network literature, the number of links of a node is called its *degree*. Let d_s denote security s's degree and d_t trader t's degree.

3.2 Asset and pricing

A trader does not have any other asset than those in the network. Assets are assumed to be illiquid. Government bonds and treasury bills are sometimes considered perfectly liquid assets so they might not apply to this model. Each trader is assumed to be a large player, so that their behavior will affect prices.

Let V denote mark-to-market initial value of a security held by traders in the network. "initial" means before any trader is hit by any shock and forced to liquidate. V is constant across all assets means that the initial value of each security held in the network is identical[3]. We also require that, for each security, V is equally distributed among its shareholders, with the value of each link for the security being $\frac{V}{d_s}$.

It is assumed that there are also long term investors who are outside the network. When trading, traders in the network do not trade with each other, instead they trade with only those outside the network: as traders in the networks buy the security, the outside market has a limited supply and the price is pushed up; and as the traders sell the security, the outside market has a limited capacity to absorb and the price is depressed. Let P_s be the price of

[3]Alternatively we can make V a random variable. See Footnote 6.

security s[4]. Price P_s is thus a strictly increasing function of x_s:

$$P_s = \rho(x_s) \tag{1}$$

where x_s is the fraction of security s held in the network, and thus $1 - x_s$ is the fraction held outside the network by those long term investors. By this pricing formula, assets are assumed to be illiquid, and investors outside the network are assumed to be passive to the short term price fluctuations. These long term investors are willing to take the assets being liquidated by traders in the networks because the prices are depressed low enough. To simplify the analysis, we impose that $x_s = x$, $\forall s$ (before any contagion happens).

Short selling is not allowed.

3.3 Contagion

Time is discrete. At time 0, a randomly chosen trader is hit by a shock and is forced to liquidate all her portfolio, which depresses the prices of those assets so that shareholders may be in distress. Assume that all traders in the network have an identical capital buffer k. If a trader's mark-to-market loss is larger than k, it is assumed that this trader will be forced to liquidate all the assets she holds[5]. This is sometimes referred to as zero recovery assumption. These liquidations may induce yet further round of liquidations of other traders. In the real world a constrained trader forced to to reduce holdings of assets does not always end up totally liquidating. But large share sale often depress the price and may induce other shareholders to sell. When others are forced to sell as well, the price will be depressed further and this triggers a downward spiral in price. This process is similar to a system with a high but unstable equilibrium disturbed and evolving into a lower but stable equilibrium. Cifuentes et al. (2004) discuss this procedure in detail and have a similar result. When a crisis is fermented and investors are highly uncertain, this type of processes are more likely than often to occur.

The contagion continues until a period where no trader is liquidating, and then the contagion ends.

[4]Therefore prices may differ from security to security, and shares may differ from link to link

[5]A capital buffer that is proportional to a trader's total asset looks more natural, and this is discussed in Section 5.

3.4 Network structure

The network is a random graph with securities' degree distribution p_j and traders' degree distribution q_k exogenously given (j and $k = 0, 1, 2, ...$). The distributions prescribe that, of the S securities, a fraction of p_j has exactly degree of j; and of the T traders, a fraction of q_k has exactly degree of k. But exactly who is connected to who is determined by a stochastic process that complies with the degree distributions. Let $\mu = \sum_j jp_j$ and $\nu = \sum_k kq_k$, so the average degree of security is μ and the average degree of trader is ν, and we have $S\mu = T\nu$ =total number of links in the network.

3.5 Vulnerability of security

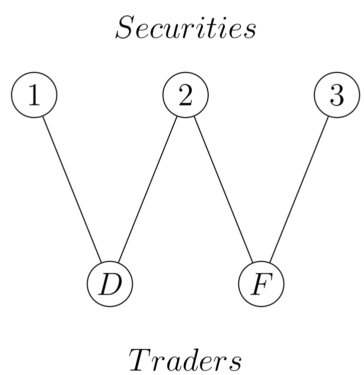


Figure 3: Contagion

In Figure 3, when trader F is forced to liquidate her positions of security 2 and 3, x_2 (the fraction) decreases from x to $x - \frac{x}{d_2}$, and price P_2 decreases from $\rho(x)$ to $\rho(x - \frac{x}{d_2})$. Trader D's holding of security 2 is initially worth of $\frac{V}{d_2}$. Security 2's price decline leads to a loss of $A(d_2) \equiv \frac{V}{d_2}\frac{\rho(x)-\rho(x-\frac{x}{d_2})}{\rho(x)}$. If $A(d_2) > k$, trader D will go bankrupt and be forced to liquidate all her positions of securities 1 and 2, otherwise D survives. Since ρ is a strictly increasing function, $A(d_2)$ is strictly decreasing in d_2, and the equation $A(d_2) = k$ has exactly one solution, and denote it d^*. If any asset's degree $d < d^*$, we have $A(d) > k$, otherwise, $A(d) < k$.

Since V, x and k are constant parameters, it turns out that if a security's degree is less than d^*, as long as there is one shareholder liquidates, the price decline will be so large that it will induce the bankruptcy of all other shareholders, which force them to liquidate all their holdings of other securities; contrarily, if a security's degree is equal or larger than d^*, one shareholder's liquidation does not generate a price decline large enough to trigger other shareholders' bankruptcy by itself. The bankruptcy must then be the due to accumulated price declines in the portfolios later. In short, if a security is held by only a few traders ($< d^*$), it is vulnerable; otherwise, we say it is (relatively) safe. We thus define the vulnerability of a security as follows:

Definition 3.1 *A security s is vulnerable if*

$$d_s < d^* \tag{2}$$

Define an indicator function $v(d_s)$[6],

$$v(d_s) = \begin{cases} 1 & \text{if } d_s < d^*; \\ 0 & \text{if } d_s \geq d^*. \end{cases}$$

A particularly simple example is when $\rho(x) = \gamma x$, where γ is a constant, i.e. the price is linear in x. Again in Figure 3, when trader F is forced to liquidate all her positions of security 2 and 3, x_2 decreases by $\frac{1}{d_2}$, and since price is linear in x_2, P_2 also decreases by $\frac{1}{d_2}$. Trader D's holding of security 2 is initially worth of $\frac{V_2}{d_2}$. This price decline leads to a loss of $\frac{1}{d_2}\frac{V_2}{d_2} = \frac{V}{d_2^2}$ for trader D. If $\frac{V}{d_2^2} > k$, i.e. $d_2 < \sqrt{\frac{V}{k}}$, trader D will be forced to liquidate all her positions of securities 1 and 2, otherwise D survives. This shows that, the smaller the ratio $\frac{V}{k}$, the less likely a security is vulnerable to contagion, which implies that higher capital buffer may reduce the chance of contagion.

4 Numerical methods

In the numerical methods, we assume the simple linear pricing formula:

$$P_s = \gamma X_s$$

where γ is a constant.

4.1 Gleeson's method applied to bipartite networks

Gleeson's methods is an analytical/numerical hybrid method. It calculates the expected infection rate for nodes of each and all possible degree, and then compute the infection rate of links

[6] Alternatively, we can make both V and k random variables, so that the rigid assumptions on constant v and k are gone and assets and traders becomes heterogenous, and this indicator function will become a probability. But we will lose the tractability of the model and the analytical result will not be available.

emitted from these nodes, and accumulate current and past distressed links by using binomial distribution, and then enter the next period, until the results converge. This done period by period. This binomial assumption depends on the assumptions of networks being random. In this paper, I extend this recursive method to bipartite networks and calculate the price drop and trader bankruptcy in the contagion process. See Appendix A for details.

4.2 Monte carlo simulations

In simulations we relax the no-loop restriction on the network structures and allow multiple impacts occur to a trader consecutively and simultaneously. We consider networks with 1000 securities and 2000 traders. We assume a random graph in which each possible link between a trader and a security is present with independent and identical probability p (binomial distribution). The binomial distribution is chosen for simplicity. The distribution is implemented by Configuration Model[7].

I draw 500 realizations of network for each p[8], and in each of these realized networks, I randomly choose a trader and force her to liquidate all her portfolio. This whole procedure is repeated 10 times. Any trader whose accumulated loss on all affected securities is larger than k must liquidate all their assets as well.

I assume the price is linear in the fraction of the asset held in network, i.e. $\rho(x) = \gamma x$, where γ is a constant. By varying the ratio V to k, we examine the effect of capital buffer on the contagions. By varying the value of p, we have networks with different degree distributions and thus different average degrees of traders and securities. The average degree of a network is an indicator of how well or poor a network is connected. We would like to see when system-wide contagion is more likely and when it is rare. When more than 5% of the total traders are infected, we consider it a system-wide contagion or crisis. Conditional on there is a system-wide contagion, we also examine the extent of that contagion.

4.3 Results

The simulation results of the model with $k/V = 0.025$ is shown in Figure 1. We can see that when the average degree is either very low or very high, system-wide contagion is not likely to

[7] See Jackson (2008), Section 4.1.4

[8] Traders and securities have independent degree distributions.

happen. Whereas within a certain window where the average degree is moderate, an extensive contagion is more likely, but is non-monotonic in average degree. This confirms the results in Section ??. The extent of contagion, conditional on that it has infected more that 5% of the trader population, is approximately the same as the frequency of contagion at the range of lower average degrees. But at the higher end of average degree, although system-wide contagion is rare, but once it happens, it will be a severe one. So this model has the same robust-yet-fragile feature as Gai and Kapadia (2010).

Gleeson's method turns out to agree with simulation result quite well (see Figure 4).

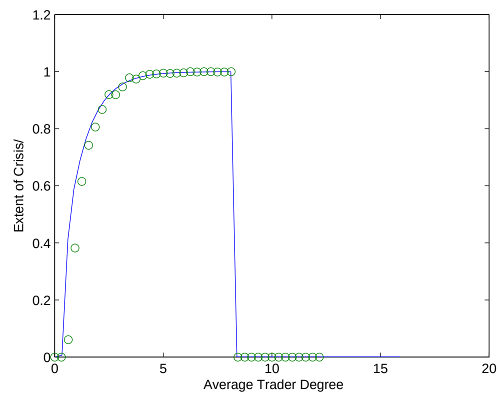
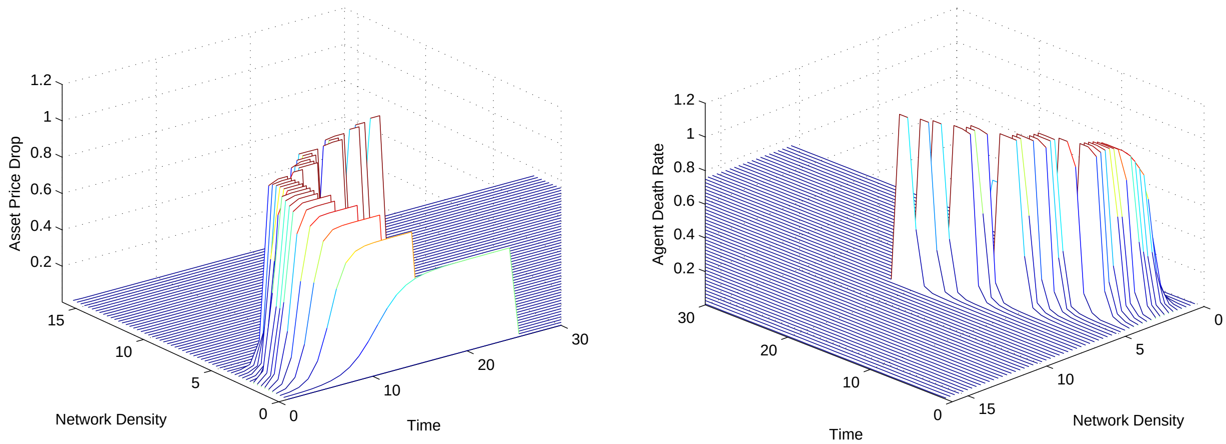


Figure 4: Agreement between simulation results and Gleeson's method

By using Gleeson's method, Figure 5 gives a clearer image of how trader's bankruptcy rate develops as contagion proceeds. Notice that, in low density networks, trader's bankruptcy rate develops slowly and then increases steadily, and then converges at a moderate level. In intermediate-dense network, it develops rapidly at the very beginning and reach the top almost in a blink of an eye. As networks become even denser, the bankruptcy rate increases again slowly and the big jumps will not happen until the last minute, while the jumps are more dramatically and suddenly. Above certain network density, no further contagion happens. This observation agrees with that from Figure 1: the contagion happens only with certain window of network density; within that window, the denser the network is, the more abruptly the bankruptcy rate changes. As network is getting denser, the initial contagion develops with more resistance, because now agents are more diversified, and price drop becomes smaller since asset degrees also increase. These result suggest that, in the real world financial networks, where agents are usually diversified, contagion will happen either very fast (intermediate connectedness), or



(a) Front view

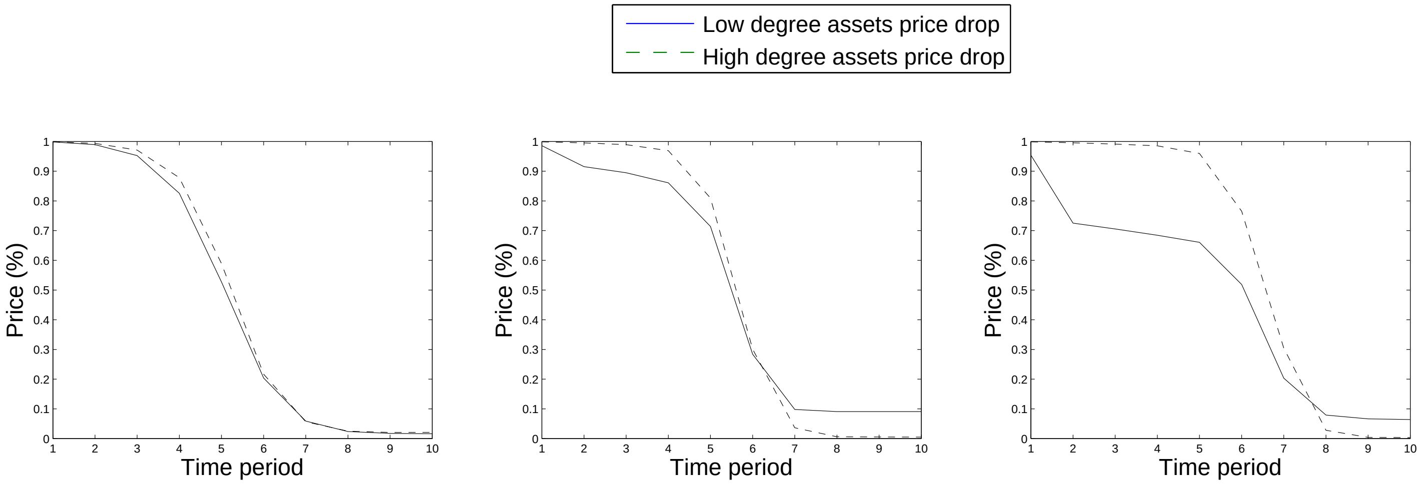
(b) View from the back

Figure 5: Dynamics of average asset price drop

develops sneakily and then abruptly spreads extensively (high connectedness). In these cases, especially in the latter, it is important that the policymakers could detect and predict at an early stage of the contagion.

In simulations, we recorded price drops of different assets - low degree assets and high degree assets - in the process of contagion. The cutoff between the two group is their vulnerabilities: vulnerable assets are in low degree group and others in high degree group. Figure 6a shows that in low density networks, the difference of price drop speed between the two asset groups do not differ much, though we can still observe that, at the beginning, low degree assets decrease faster than high degree group. This difference becomes much more apparent when networks are getting denser, which is show in Figure 6b and 6c. Based on these observations, especially those in the first several periods, it is suggested that, the low degree assets are more sensitive to the initial contagion. The intuition is simple: if any vulnerable asset gets infected, all its shareholders will go bankrupt in the next period and will be forced to liquidate all their portfolios. This makes price of vulnerable assets drop significantly. While for safe assets, their prices are only eroded slowly by the liquidation of its shareholder every now and then, induced either by other assets or this asset itself. To verify the robustness of the observation that low degree assets are more sensitive, we will exam another model without this obvious degree dependency in Section 5.

Figure 7 shows a typical price drop in the second period in simulations, in which the price



(a) Ave. trader degr.=1.667 (b) Ave. trader degr.=6.667 (c) Ave. trader degr.=8.333

Figure 6: Price dynamics of vulnerable and safe assets in materialized contagions

drop of vulnerable assets (degree$\leq$ 6) are much larger than that of other assets. The vertical line is the average asset degree in this network. It is now tempted to set up a signal to see

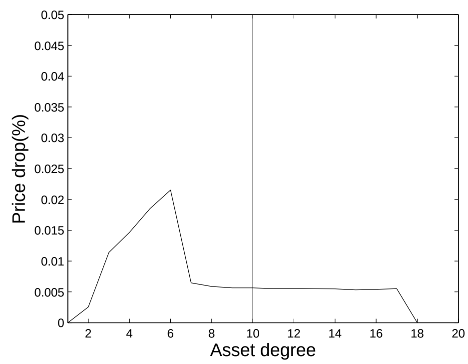


Figure 7: A snapshot in 2nd period of price drop of assets of all degrees

whether low degree assets' sensitivity can be used to help predict contagion. The signal is defined as follows: In the simulations, focusing on second period[9] only, when vulnerable assets' average price drop is as twice or larger than that of safe assets, the signal is true; otherwise it is false. We then examine whether this signal helps to predict the contagion or not. Table 1 shows the results.

Each row corresponds to a certain density of network. The first three columns are average trader degrees, contagion frequency and contagion extent, which are quite similar to data presented in Figure 1. Signal frequency is the probability that the signal is true. Fifth column

[9]The first period in simulation has the initial shock only, and there is no contagion yet. So the second period is when contagion first develops.

Average trader degree	Unconditional crisis frequency F_c	Contagion extent	Signal frequency F_s	Crisis freq. conditional on signal F_{st}	Crisis freq. conditional on no signal F_{sf}
0.556	0.00032	0.05781	0.3322	0.00096	0
1.111	0.51526	0.52276	0.60034	0.84932	0.01346
1.667	0.73086	0.78284	0.74004	0.97319	0.04101
2.222	0.76254	0.87937	0.75742	0.98746	0.06027
2.778	0.78422	0.94621	0.75982	0.99413	0.12016
3.333	0.7072	0.96783	0.67736	0.99135	0.11065
3.889	0.62646	0.98099	0.57956	0.99217	0.12235
4.444	0.51350	0.98600	0.43112	0.98817	0.15378
5.000	0.35404	0.99300	0.30074	0.97679	0.08621
5.556	0.22592	0.99800	0.19274	0.93183	0.05738
6.111	0.10062	0.99900	0.08640	0.80324	0.03417
6.667	0.04844	1	0.04038	0.74443	0.01915
7.222	0.02812	1	0.02400	0.69167	0.01180
7.778	0.00898	0.99900	0.00076	0.50000	0.00861
8.333	0.00520	1	0.01128	0.23936	0.00253
8.889	0.00014	0.99900	0	0	0.00014
9.444	0.00086	1	0.00016	0	0.00086
10.000	0	0	0	0	0
10.556	0	0	0	0	0
11.111	0	0	0	0	0

Table 1: Unconditional crisis frequency and frequency conditional on signal

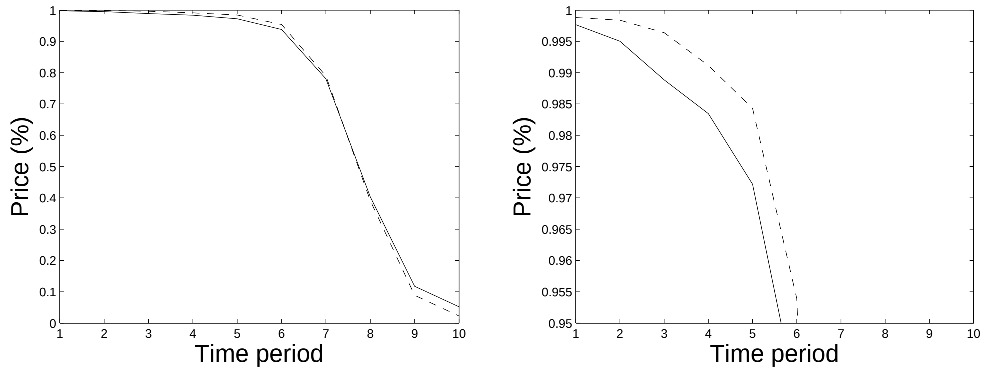
is the probabilities that, conditional on signal is true, there is an extensive contagion. The higher this probability is, the better the signal is. The last column is the probabilities that, conditional on signal is false, there is an extensive contagion. The lower this probability, the better the signal. One can verify that $F_c = F_s \times F_{st} + (1 - F_s) \times F_{sf}$ for each row.

The results show that the signal is very helpful. For example, when average asset degree is 6.667, the probability that an extensive contagion happens is 0.04844, but once it happens, it will all traders will go bankrupt and all prices drop to zero accordingly. While with the signal being true, we are quite confident (0.74443) that this will happen, and with the signal being false, the crisis happens with very small probability (0.01915). So the signal dramatically increases our ability to predict whether or not the crisis happens. Again, in Section 5 we will examine a modified model to see its robustness.

The current used stock indices, e.g. S&P 500, FTSE 100, DJIA or Nasdaq Composite, are usually those of most highly capitalised companies, but not necessarily those with only a few shareholders. To better predict the potential contagion on stock markets, I recommend that stock indices which include, respectively, low degree stocks only and high degree stocks only, and monitor the ratio (or other similar parameters) between the two.

5 Robustness

The signal in the original model is surprisingly good in predicting contagion because of the way the model is specified. To examine the sensitivity of the low degree assets in general, we modify the model as follows: The total value of each asset held in network is no longer assumed to be constant. Instead, the monetary value of each and all links is assumed to be identical. In addition, all traders maintain a constant leverage L, so that a trader's capital buffer is $1/L$ of her total value of portfolios, instead of an absolute constant. An important change is added: if any price decreases by 50% or more, its price plummets to zero in the next period. It can be justified by the observation that, when price plummets, a lot of shareholders want to sell this asset to avoid further loss. After these modification, the vulnerability of assets with a cutoff degree no longer exists. Asset prices and agent's portfolio values are eroded bit by bit as the contagion progresses.



(a) Ave. trader degr.=6.667

(b) Details

Figure 8: Price dynamics of vulnerable and safe assets in materialized contagions (modified model)

From simulations, we have contagion frequency and extent, as well as average price drop are similar to those of the original model and hence are omitted. Figure 8a shows that in well connected networks (average trader degree= 6.667), the difference of price drop speed between low and high degree assets[10] becomes less obvious. Figure 8b gives the details of the first several periods in Figure 8a, where we can still tell that low degree assets' price declines faster than high degree group. Under the same definition of signal, we examine whether this signal can still help to predict contagions. Table 2 shows the results. Now the reliability of the signal is not as good as is in the original model. F_{st} are much lower than their counter parties in Table 1, while F_{sf} are higher. For example, at a average degree of 6.111, the unconditional crisis frequency is 0.13220. With the signal present, its conditional probability is 0.26837, compared with 0.80324 in Table 1. Without the signal, its missing rate also rises to 0.09900, compared with 0.03417 in Table 1. Though not as efficient as before, the signal still put us at a better position in forecasting the realization of a contagion. Considering the serious consequence of such a system-wide disaster, it is still worthwhile to construct such a signal as a reference.

The intuition why, absent of degree cutoffs, prices of low degree assets still drop faster than high degree asset at the early stage of contagion can be seen from Appendix A. From equation 3 and 4 we have $v_k^1(m) = B_m^k(\rho^0 + (1-\rho^0)f_2^1)$, where $B_m^k(p) = \binom{k}{m} p^m (1-p)^{k-m}$. Assuming

[10]Now there is no degree cutoff between low and high degree assets, the separation of the two groups becomes more difficult. In simulations we separate the two groups merely by subjective standards based on past observations

that all initial prices are 1. The average price of degree k assets at time t $\overline{P}_k^t$ is

$$\overline{P}_k^t = \frac{1}{k} \sum_{m=0}^{\lfloor \frac{k}{2} \rfloor} (k-m) v_k^t(m)$$

where $\lfloor \cdot \rfloor$ is the floor function, returning the greatest integer less than or equal to its argument. In period 1, we have

$$\begin{aligned} \overline{P}_k^1 &= \frac{1}{k} \sum_{m=0}^{\lfloor \frac{k}{2} \rfloor} (k-m) v_k^1(m) \\ &= \frac{1}{k} \sum_{m=0}^{\lfloor \frac{k}{2} \rfloor} (k-m) B_m^k(p), \text{ where } p = \rho^0 + (1-\rho^0) f_2^1 \\ &= (1-p) \sum_{m=0}^{\lfloor \frac{k}{2} \rfloor} \binom{k-1}{m} p^m (1-p)^{k-1-m} \end{aligned}$$

Let $X \equiv \sum_{m=0}^{\lfloor \frac{k}{2} \rfloor} \binom{k-1}{m} p^m (1-p)^{k-1-m} = \frac{\overline{P}_k^1}{1-p}$ and $Y = \sum_{m=\lceil \frac{k}{2} \rceil}^{k-1} \binom{k-1}{m} p^m (1-p)^{k-1-m}$, where $\lceil \cdot \rceil$ is the ceiling function. We know that $\sum_{m=0}^{k-1} \binom{k-1}{m} p^m (1-p)^{k-1-m} = X + Y = 1$, so that $Y = (1-p) - X$. By symmetry of $\binom{k-1}{m}$ (i.e. $\binom{k-1}{m} = \binom{k-1}{(k-1)-m}$) we have $\sum_{m=0}^{\frac{k-1}{2}} \binom{k-1}{m} = \frac{1}{2}$, thus $\sum_{m=0}^{\frac{k}{2}} \binom{k-1}{m} \approx \frac{1}{2}$ when $k \gg 1$. Since $\rho^0 \ll 1$ and initial link infection rate $f_2^1 \ll 1$, we know $p \ll 1$, then $p^m (1-p)^{k-1-m}$ decreases with m dramatically. So that the larger k is, any summing entry in X is much more larger than its corresponding (with same combinatory coefficient) entry in Y. Therefore the larger k is, the much more larger X is than Y. This shows that $\overline{P}_k^1$ increases with k roughly, i.e. low degree assets have generally lower prices than high degree assets in period 1.

Average trader degree	Unconditional crisis frequency F_c	Contagion extent	Signal frequency F_s	Crisis freq. conditional on signal F_{st}	Crisis freq. conditional on no signal F_{sf}
5.556	0.28120	0.99800	0.16740	0.56153	0.22484
6.111	0.13220	0.99900	0.19600	0.26837	0.09900
6.667	0.01880	1	0.18120	0.04967	0.01197
7.222	0.00660	1	0.20180	0.01685	0.00401
7.778	0.00220	0.99900	0.17020	0.00705	0.00121
8.333	0.00032	1	0.10636	0.00132	0.00020
8.889	0.00004	0.99900	0.05752	0.00035	0.00002
9.444	0	0	0.05874	0	0
10	0	0	0.00302	0	0

Table 2: Unconditional crisis frequency and frequency conditional on signal - modified model

But in the modified model, not only choice of low degree assets becomes more difficult, the more subtle difference between the prices of the two groups makes it more difficult to identify the initial price drops since the stock market fluctuates constantly. So it would be desirable to evaluate this result by empirical evidence to see if the signal can still be identified.

6 Contagion of boom/bubbles

All the processes we have discussed so far are fire sales and liquidations, which induce price declines and crises. But imagine the reverse process, in which the initial positive shock to a random chosen asset's price could strengthen its shareholders' capital, which induce them to invest on other assets, which in turn will push up prices, and this contagion of boom/bubbles could potentially spread all over the network. In a stock market boom, stocks prices appreciate and people expect that expansion will continue, so they keep buying shares, which in turn push up the prices further. The mechanism that works underlying is a positive feedback and discussed by Adrian and Shin (2010), among others, and now we apply it in a network setting and examine its consequences on complex financial systems. The feedback is shown in the left panel in Figure 9 (taken from Adrian and Shin (2010)). Assume all traders maintain a constant

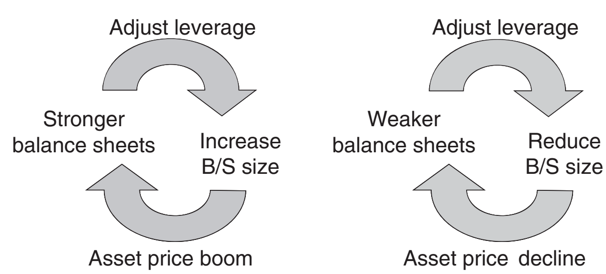


Figure 9: Positive feedbacks in booms and busts

leverage. When one asset's price increases significantly, its shareholders have capital gains, which reduce their leverage. These gains entitle them to borrow more to invest on other assets. According to previous asset pricing formula, these new investment will boost up the prices of those assets, which give further capital gain to shareholders of those assets. This process can potentially go very far, thus an lead to a market-wide boom, which can be thought of as bubbles in the stock market. The right panel in Figure 9 is the reverse procedure, which is the downward process or contagion of fire sale we have discussed in previous sections.

In the same networks used in simulations in Section 4.2, but assume initially all asset prices are identical, which is P. According to our previous asset pricing formula, this implies that 10% of each asset is held by traders in the network. All traders initially maintain a leverage of L. Randomly choose an asset and increases its price from P to rP ($r > 1$). If an shareholder's capital gain decreases her leverage by more than a tolerance (percentage) z, then she will borrow more money and randomly choose an asset from all available assets to invest with the borrowed money, so that her leverage returns to L; otherwise, she does not respond to the capital gain. An asset is available if traders out of network still hold some shares, i.e. not all its shares are already held in the network. The contagion stops when either no trader wants to buy any shares or no asset is available any more (i.e. all assets are already entirely held by traders in the network).

The contagion frequency and extent of boom exhibits a similar patter to contagion of fire sale. In addition, low degree assets tend to get infected early and their price increase faster at the beginning of the contagion, which, again, makes it a potential predictor of such a contagion. (Simulations to be added).

7 Conclusion

In this paper I investigate the contagion of fire sale in complex bipartite networks of multiple assets and traders. The initial idiosyncratic shock that forces an individual to liquidate can potentially transmit across asset markets and spread to a large population, which causes a huge market crash just like the crash in 1987. By compare the price drops of low degree assets and high degree assets at the early stage of contagion, it can be show that the former is more sensitive to contagion, i.e. their prices decline faster than the latter at the beginning of the contagion. This suggests that the comparison of the two may give a signal that could be used to predict the realization of contagion at its early stage. The simulation results show that the signal is very efficient and can improve the accuracy of the prediction dramatically.

This method is then put in a modified model to examine its robustness and generality. The reliability of the signal decreases considerably. The difficulties come from the choice of appropriate low degree assets to include in the low degree group, as well as the small difference between the high/low degree asset groups when the whole market fluctuates. So it would be desirable to evaluate the feasibility with empirical evidence. I further briefly discuss the applicability of the method in the case of contagion of boom/bubbles.

This model provides a preliminary method to forecast the contagion of fire sale across asset markets. It would be useful to test it in more general forms of networks, e.g. networks with geometric/scale-free distributions, which are generally believed to be more consistent with the real world financial systems. It may also be interesting to allow short selling so that the predation would be more fierce, and this would allow a new link (temporarily) created in the network and thus introduce more flexibility on the network structures. In the real world, it is usually not a default on a bilateral loan (as discussed in Gai and Kapadia (2010)) or the knock-on effect of fire sale alone that causes the trouble, but the combination of the two. It might be possible to combine the two models to analyze the contagion on both channels, but it remains to see whether it is technically feasible. I will leave these for future works.

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Appendix A: Gleeson's method extended to bipartite networks

We describe how Gleeson's method can be extended to the modified model, because it is easier to characterize. Its version of the original model will be presented in later version of this paper.

Let us define that, each period of contagion has two sub-phases. A period starts with the "trader phase", in which infection transmits from traders to assets; then "asset phase" follows, in which infection transmits the other way around. Imagine in period 0 a shock hits a randomly chosen trader who is forced to liquidate all her portfolios. The probability that her degree is j is p_j. Total number of links is $S\mu = T\nu$. So in period 0, the rate of new infection for links is just $f_2^0 = \frac{1}{S\mu}$. This is the "trader" phase. Now we enter the "asset phase". Since it is a random network, the probability for any single link of any asset to be infected is f_2^0. The probability that a degree k asset has m links liquidated is in period 0 is

$$v_k^0(m) = B_m^k(\rho^0), \text{ where } B_m^k(p) = \binom{k}{m} p^m (1-p)^{k-m} \tag{3}$$

Suppose now we have already calculated up to period t, and we are entering period $t+1$. In the "trader phase", if an asset has m links liquidated, then its price declines by $\frac{m}{k}$. Among all the existing links (links not liquidated yet), the number of links with $\frac{m}{k}$ loss at period $t+1$ is

$$C_k^{t+1}(m) = Sq_k v_k^{t+1}(m)(k-m)$$

The number of links with any positive loss is

$$C^{t+1} = \sum_k \sum_{m=1}^{k} C_k^{t+1}(m)$$

The number of all existing links (includes both links with zero loss and those with positive loss) is

$$D^{t+1} = \sum_k \sum_{m=0}^{k} C_k^{t+1}(m) = \sum_k \sum_{m=0}^{k} Sq_k v_k^{t+1}(m)(k-m)$$

So among all the existing links, the link infection rate is

$$f_1^{t+1} = \frac{C^{t+1}}{D^{t+1}}$$

again, by virtue of random networks. For links with positive loss, their losses are in varied degree, but their average loss in period $t+1$ is

$$s^{t+1} = \frac{\sum_k \sum_{m=1}^{k} \frac{m}{k} C_k^{t+1}(m)}{C^{t+1}}$$

Now we consider the threshold, M_j^{t+1} of number of infected links for traders of degree j above which the trader will go bankrupt. Approximating the situations by using the average loss s^{t+1}, and assuming the initial value of all links is just 1, the bankruptcy happens when $\frac{ns^{t+1}}{j} > \frac{1}{L}$, where n is the number of infected links. This gives $M_j^{t+1} = n = \frac{j}{s^{t+1}L}$. The probability that a j degree trader survived previous period and now has n infected links is

$$u_j^{t+1}(n) = \sum_{l=0}^{\min(n, M_j^t)} B_{n-l}^{j-l}(f_1^{t+1}) u_j^t(l)$$

Then we have the unconditional bankruptcy rate of j degree traders in period $t+1$ as

$$\rho_j^{t+1} = 1 - \sum_{n=0}^{\min(j, M_j^{t+1})} u_j^{t+1}(n)$$

and the overall bankruptcy rate

$$\rho^{t+1} = \sum_j p_j \rho_j^{t+1}$$

Now we enter the "asset phase" of period $t+1$. Given the above infection, some agents go bankrupt and the total number of newly liquidated links is

$$E^{t+1} = T \sum_j j p_j (\rho_j^{t+1} - \rho_j^t)$$

Right before these liquidations, the total number of all existing links is

$$F^{t+1} = T \sum_j j p_j (1 - \rho_j^t)$$

So the link infection due to new liquidation is

$$f_2^{t+1} = \frac{E^{t+1}}{F^{t+1}}$$

After these liquidations, asset prices decline again, and in particular those whose price depressed more than half will be further depressed to zero. The probability that a k degree asset has (accumulated) m liquidated shareholders is

$$\begin{aligned} v_k^{t+1}(m) &= \sum_{l=0}^{m} B_{m-l}^{k-l}(f_2^{t+1}) v_k^t(l) \ , \ m \leq \frac{k}{2} \\ v_k^{t+1}(k) &= v_k^t(k) + \sum_{\kappa=1}^{k} \tilde{v}_\kappa^{t+1}(m) \\ \text{where } \tilde{v}_k^{t+1}(m) &= \sum_{l=0}^{\lfloor \frac{k}{2} \rfloor} B_{m-l}^{k-l}(f_2^{t+1}) v_k^t(l), \ m > \frac{k}{2} \end{aligned} \qquad (4)$$

Repeat above procedure recursively until the overall bankruptcy rate converges, i.e. the difference between the overall bankruptcy rates of two adjacent time periods, ρ_j^t and ρ_j^{t+1}, is smaller than some threshold, then the contagion process can be thought as having ended.