

Inflation and Unemployment: The Roles of Goods and Labor Markets Institutions*

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Abstract

Empirical evidence on inflation and unemployment suggests that they can be either positively or negatively related in the long run. This paper studies this relationship in an environment in which inflation has differential effects on employed and unemployed workers. Due to either heterogeneous money holdings or imperfect indexation of unemployment insurance, the unemployed are affected by the inflation tax to a larger extent than the employed. A higher rate of inflation increases workers' incentives to work and generates a negative effect on unemployment. On the other hand, inflation lowers a firm's return from creating job vacancies, thereby raising unemployment. In the steady state the inflation-unemployment relationship is either positive or negative, depending on goods and labor markets institutions. Sales taxes, the degree of heterogeneity in money holdings and the market power of firms are major factors determining the direction of this relationship. Through a comparison of market institutions, the model generates an inflation-unemployment relation that is qualitatively consistent with the empirical evidence.

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1 Introduction

The relationship between inflation and unemployment is one of the classic topics in macroeconomic study. Since 1958 when A.W. Phillips first showed an inverse relationship between wage inflation and unemployment in U.K. from 1861 to 1957, a large literature has been devoted to examine this relationship. Over the past decades, the empirical evidences from various studies seem to suggest that the shape of the long-run Phillips curve varies widely across time and different countries. For example, using U.S. data from 1970 through 1999, Beyer and Farmer (2007) study the low frequency comovements in unemployment and inflation and find a positive long-run relation between these variables. Berentsen, Menzio and Wright (2008) examine the U.S. data from 1955-2005, and also document a positive relation after filtering out the higher-frequency movements. Using a panel of European countries, however, Karanassou, Sala and Snower (2003) provided empirical results which imply a long-run trade-off between inflation and unemployment. Franz (2005) and Schreiber and Wolters (2007) both examine the experience in Germany and provide evidence for a long-run downward sloping relationship.

Despite the divergent evidence on the empirical long-run Phillips curve, a large part of the theoretical literature tends to advocate a positive relationship. Earlier work by Cooley and Hansen (1989) introduced a cash-in-advance constraint into Rogerson's (1988) indivisible labor model. In their study, a rise in anticipated inflation reduces labor supply through a consumption-leisure substitution mechanism, and increases unemployment. In recent years the development of search models has inspired several studies to combine the labor search environment with a monetary search framework¹. For example, Lehmann (2006) extends the labor matching model of Mortensen and Pissarides (1999) through the introduction of frictions in the goods market. Berentsen, Menzio and Wright (2008) integrate the labor search model by Mortensen and Pissarides (1994) into the monetary search environment in Lagos and Wright (2005). Kumar (2008) combines the monetary search model of Shi(1997) with the labor matching model of Mortensen and Pissarides, and examine the inflation-unemployment relation under four different wage-setting mechanisms. All these studies suggest a positively sloped long-run Phillips curve: a higher rate of inflation reduces real money balances and leads to a lower profit in the goods market. This discourages firms' entry and job creation, thereby raising the unemployment rate.

The present paper proposes to complement this literature by introducing asymmetric effects of inflation into the above models, and examining their impact on labor market decisions and outcomes. One major result is that the implied inflation-unemployment relation can be positive or negative, depending on goods and labor market institutions.

In this model the positive effect is generated through the same job posting mechanism as discussed in the previous literature: A higher rate of inflation lowers firms' return from goods market trade and hence discourages job creation. The negative effect, on the other hand, is a

¹There is a separate strand of literature which studies this issue in environments with nominal or real rigidities, for example, Blanchard and Gali(2008) , Gertler and Trigari(2007) and Faia (2008).

result of asymmetric effects of inflation on unemployed and employed workers. In an economy with trading frictions in the decentralized goods market, the unemployed consume more cash-intensive goods than the employed, and therefore are more exposed to the inflation tax. A higher rate of inflation reduces the value of outside option for workers, and lowers the bargained wage. This induces firms to create more job vacancies. At the same time, the reduction in the outside value also decreases the reservation productivity at which workers and firms are willing to match. The same mechanisms exist in an economy where the asymmetric effects of inflation are generated through frictions in the labor market. When Unemployment insurance (UI) benefit is not perfectly indexed, inflation lowers the real value of income for unemployed workers and thereby hurts them more than employed workers. Overall, the heterogeneous effects of inflation reduce the outside values for workers and lead to a lower unemployment rate.

The magnitude of the two opposite effects depends on goods and labor markets institutions. In particular, goods market characteristics are the main factors that determine the direction of the inflation-unemployment relation. According to the results in the quantitative examples, both a high rate of sales tax and a low bargaining power of firms (as a proxy of firm's market power in the goods market), favor the negative effect, and generate a negatively sloped Phillips curve. Secondly, large heterogeneity between employed and unemployed workers' cash holdings also boosts the negative effect and produces an inflation-unemployment trade-off. As for labor market institutions, worker's wage bargaining power only influences the size of the slope, but not the direction of the Phillips curve. The UI benefit, when not perfectly indexed to inflation, may directly affect the sign of the slope. More generous UI benefit makes unemployed workers suffer a larger cost when inflation rises, and therefore leads to a negative inflation-unemployment relation. Overall by comparing the levels of sales tax and UI benefit between U.S. and Europe, the model generates an inflation-unemployment relationship that is qualitatively consistent with the empirical evidence reviewed above.

The crucial element in this framework is the differential effects of inflation on employed and unemployed workers, either through heterogeneous money holdings or imperfect indexation of UI benefit. Several studies have provided evidence on heterogeneity in transaction patterns and currency holdings across individuals with different income levels. For instance, Avery et al. (1987) and Erosa and Ventura (2002) report that low income households in the U.S. tend to use cash for a larger percentage of their total expenditure relative to high income households. Mulligan and Sala-i-Martin (2000) and Attanasio et al. (2002) find that the probability of having an interest bearing bank account is positively related to the level of income and wealth. As unemployed people tend to be low income individuals, these studies indirectly present the evidence of heterogeneous money holdings between the employed and unemployed.

For direct evidence, the estimation results in Duca and Whitesell(1995) show that being unemployed is negatively related to the probability of having a credit card, checking account and other types of banking accounts. Putting it differently, this suggests the unemployed

tend to use cash more often in their transactions. To investigate further the link between employment status and currency holdings, using the data of the Italian Household Survey of Income and Wealth 2004², I estimate the effects of employment status on cash holdings as a percentage of annual income, as well as cash spending as a percentage of total purchases. The estimation results are presented in Table A.1 in Appendix I. On average unemployed households hold a larger percentage of their wealth in cash, and also use money for a greater fraction of their total expenditure. With positive nominal interest rates, the difference in cash holdings implies different exposures to the inflation tax. This seems to indicate that the burden of the inflation tends to be higher for the unemployed than for the employed.

The basic model in this paper follows the environment in Berentsen, Menzio and Wright (2008), which combines monetary search economy with random matching model of labor. The major difference lies in the asymmetric effects of inflation on different types of workers. In my model heterogeneity introduces a negative effect of inflation on unemployment, while in their model unemployed and employed people face the same inflation cost and unemployment is affected by inflation only through firm's vacancy posting decision. In addition, my model introduces match specific productivity into the labor market, which endogenizes the job acceptance and rejection decision for firms and workers.

The results of the paper to some extent are consistent with those of Rocheteau, Rupert and Wright (2008) and Dong (2008). Both studies integrate the idea of Rogerson's indivisible labor into the monetary search environment of Lagos and Wright (2005). The long-run Phillips curve in their economy is also positive or negative, depending on specifications of preference conditions. In contrast, my model suggests that the characteristics of the goods market structure are the main factors determining the direction of the slope. Shi (1998) constructs a model with endogenous search in both goods and labor markets. In his economy, inflation has two opposing effects, and the inflation enhancing effect is a result of an increase in search intensity. In my model, the positive effect on employment is due to the differential effect of inflation, which does not hinge on the positive correlation between inflation and search intensity.

In terms of the mechanism through which inflation encourages employment, Heer(2003) follows a similar line as my paper, although he introduces money to a labor search economy through a cash-in-advance constraint. In his model the reservation wage of household depends on the consumption of cash goods. Inflation, on the one hand, reduces job search and employment. But on the other hand, it lowers consumption and the bargained wage, which encourages job creation. The net effect depends on the exogenous elasticity of labor supply. In contrast, my paper explicitly models the frictions in the goods market, which enables me to compare the shape of the Phillips curve across different markets.

The remainder of the paper is organized as follows. Section 2 describes the economic environment. Section 3 defines a stationary monetary equilibrium, as well as characterizes the conditions under which a unique equilibrium exists. In section 4, the effects of inflation

²The detailed description of the dataset is provided in the Appendix.

on unemployment are examined. Section 5 presents the quantitative results that demonstrate how different goods and labor market institutions affect the long-run inflation-unemployment relationship. Finally, conclusions and the plan for future research are given in Section 6.

2 The Economy

Time is discrete and continues forever. Each period consists of two subperiods. In the first sub-period, a labor market and a decentralized goods market open at the same time. The labor market, or LM, follows the structure of the search economy in Mortensen and Pissarides (1994, 1999), with match specific productivity. The decentralized goods market, or DM, is characterized by a random matching environment, where money is essential as a medium of exchange due to anonymity. In the second sub-period, there is a centralized goods market, or CM, where frictionless Walrasian trades can take place with or without money. The CM/DM structure follows from Lagos and Wright (2005).

The economy is populated by two types of private agents: workers and firms, indexed by w and f , respectively. Workers work in the LM and enjoy utility from consuming in both the DM and CM. The measure of workers is normalized to 1. Firms employ workers to produce in the LM and sell their outputs in the goods market. Assume that the measure of firms is arbitrarily large. When a worker and firm meet in the labor market, they first draw match-specific productivity y from a distribution F . Assume that y is observed by both the worker and the firm. In general, there exists a reservation value, y_R , which is characterized below, such that firms and workers agree to match if and only if $y \geq y_R$. A successful job match with productivity y produces y units of output each period.

As noted above, the LM and the DM open at the same time during the first sub-period. Hence it is reasonable to imagine that workers who are working in the LM have less time to shop than workers who are not working³. With the presence of search frictions³ in the DM, this difference implies that the trading probability for an employed worker, α_1 , is smaller than the probability for an unemployed worker, α_0 . For simplicity, I assume α_0 and α_1 are exogenous and satisfy $\alpha_0 > \alpha_1$ ⁴. An important result from this environment is that employed and unemployed workers generally consume different amounts of goods in the DM and thereby enter into Sub-period 1 with different money balances⁵.

³To justify this assumption, Table A.2 in Appendix I reproduces part of the result in Table 3 of Krueger and Mueller(2008) , which examine the time use of employed and unemployed individuals in 14 countries. Their evidence shows that unemployed people on average spend more time on shopping and services than employed ones in all the sample countries.

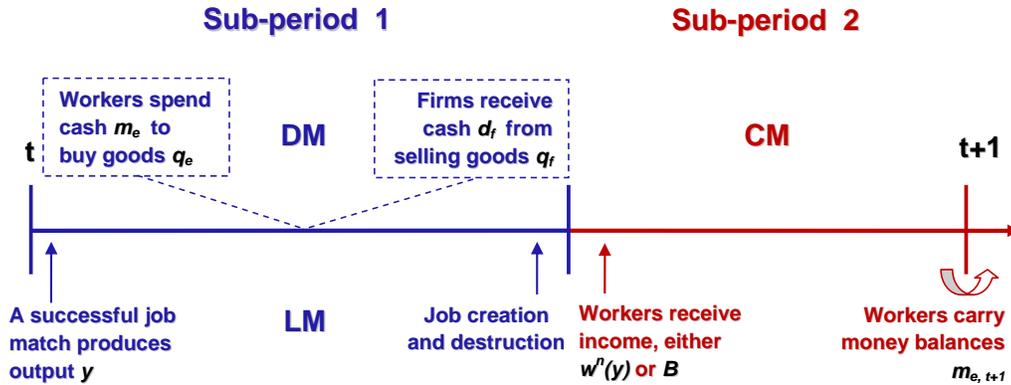
⁴One can easily endogenize α_0 and α_1 by introducing a search intensity function in the DM. However, all the analytical results remain the same as long as unemployed workers hold more cash in the DM than employed workers.

⁵There are certainly many other ways to generate the differential effects of inflation in this model. For instance, one can allow workers to choose different transaction patterns in the DM. The advantage of the current setup is that it captures the implications of the asymmetric effects, but at the same time maintains simplicity in its analysis.

In addition to the private agents, there exists a government who consumes G in each period. To investigate how different policies affect the equilibrium outcomes, the model considers two fiscal policy instruments. In the DM, the government collects a sales tax from each dollar transaction, and in the LM it pays out unemployment insurance (UI) benefits to unemployed workers. The government also conducts monetary policy to increase the stock of money at gross rate γ . New money is injected (or withdrawn if $\gamma < 1$) through a lump-sum transfer (or tax) in the second sub-period.

To understand how consumption and production take place in each market, it is useful to describe first the timing of events within each period t . As shown in Figure 1, agents enter into the first sub-period with different employment status. Employed workers join with firms to produce output y , while unemployed workers start to search for jobs. At the same time, both types of workers may enter into the DM to purchase goods for consumption. New jobs are created and some existing matches are separated at the end of Sub-period 1. In Sub-period 2, employed(unemployed) workers receive their wage payments(UI benefits), as well as the dividend income from firms. Through exchanges in a Walrasian market, workers acquire consumption and currency to carry into period $t + 1$.

Figure 1: Timing of Events



Now consider agents' optimal choices in a representative period t . Let subscript e indicate employment status. With $e = y$, a worker (firm) is matched with a firm (worker) with productivity y . $e = 0$ refers to an unemployed worker or unmatched firm. Let V_e^i and W_e^i denote the expected value of a type $i \in \{w, f\}$ agent during Sub-period 1 and Sub-period 2, respectively, conditional on employment status e . As I will focus on the steady state analysis, the time subscript t is dropped for ease of notation. Denote \hat{X} as the value of any variable X in the next period.

2.1 Workers

We now examine workers' value functions in each of the sub-periods, starting with the second one. A worker that enters the CM with employment status e and money holdings m , chooses consumption goods x and money holdings for next period \hat{m} , to solve

$$\begin{aligned} W_e^w(m) &= \max_{x, \hat{m}} \left[x + \beta \hat{V}_e^w(\hat{m}) \right] \\ \text{s.t.} \quad x &= \phi(m - \hat{m}) + \phi T + \Psi, \end{aligned} \quad (1)$$

where ϕ is the real price of money in the CM, T is the lump-sum transfer of money from the government, and Ψ is the real dividend income. The assumption of linear preferences follows Mortensen and Pissarides (1994), and also keeps the model analytically tractable as in Lagos and Wright (2005).

Using the budget constraint to eliminate x in the objective function in (1), we obtain

$$W_e^w(m) = \phi m + \phi T + \Psi + \max_{\hat{m}} \left[-\phi \hat{m} + \beta \hat{V}_e^w(\hat{m}) \right]. \quad (2)$$

The first order condition with respect to \hat{m} gives

$$\phi = \beta \hat{V}_e^{w'}(\hat{m}), \quad (3)$$

and the envelope condition implies that W_e^w is linear in m : $W_e^{w'}(m) = \phi$. As in Lagos and Wright (2005), with linear preferences the optimal choice of \hat{m} is independent of m , the money balances in the previous sub-period. It depends, however, on workers' employment status e , as long as employed and unemployed workers have different expected values over the next Sub-period. It turns out that money holdings in this economy follows a two-point distribution⁶.

Moving back to the first sub-period, workers with different employment status have different activities. Employed workers work in the LM and purchase consumption goods q_1 with probability α_1 in the DM. At the end of Sub-period 1, they lose their jobs at an exogenous rate δ . With probability $1 - \delta$, they remain employed and earn the same wage next period. The nominal wage $w^n(y)$ ⁷, which depends on match-specific productivity y , is received at the beginning of Sub-period 2. Overall, the expected value for an employed worker entering into Sub-period 1 with m_1 dollars is

$$\begin{aligned} V_y^w(m_1) &= \alpha_1 \left\{ v(q_1) + \delta W_0^w[m_1 - d_1 + w^n(y)] + (1 - \delta) W_y^w[m_1 - d_1 + w^n(y)] \right\} \\ &\quad + (1 - \alpha_1) \left\{ \delta W_0^w[m_1 + w^n(y)] + (1 - \delta) W_y^w[m_1 + w^n(y)] \right\}, \end{aligned} \quad (4)$$

where $v(\cdot)$ is the utility function satisfying $v'(\cdot) > 0$, $v'(+\infty) = 0$ and $v'(0) = \infty$. d_1 is the

⁶Independent of the individual wage payment, all employed workers have the same trading probability in the DM. As shown in Section 3, in equilibrium they hold the same quantity of money.

⁷In fact it makes no difference whether firms pay real or nominal wage in this model, as the wage payment is made in the CM.

quantity of money spent in exchange for q_1 units of goods. The determination of the terms of trade (q_1, d_1) is described in Section 3.

Unemployed workers enjoy leisure l . With probability α_0 , they consume q_0 units of goods in the DM. In the LM, they search for suitable jobs and receive a job offer at rate λ_w . The worker accepts the offer if and only if $y \geq y_R$. Nominal UI benefits B are distributed at the beginning of Sub-period 2. Hence the expected value of an unemployed worker with money balances m_0 is given by

$$V_0^w(m_0) = \alpha_0 \left\{ v(q_0) + \lambda_w \int_{y_R} W_y^w(m_0 - d_0 + B) dF(y) + [1 - \lambda_w + \lambda_w F(y_R)] W_0^w(m_0 - d_0 + B) \right\} \\ + (1 - \alpha_0) \left\{ \lambda_w \int_{y_R} W_y^w(m_0 + B) dF(y) + [1 - \lambda_w + \lambda_w F(y_R)] W_0^w(m_0 + B) \right\} + l, \quad (5)$$

where $[1 - \lambda_w + \lambda_w F(y_R)]$ is the probability that an unemployed worker receives no job offer, or he gets an offer but turns it down. The job arrival rate λ_w is endogenously determined by a matching technology, $\lambda_w = \mathcal{M}(u, v)/u$, where u is the unemployment rate and v is the number of vacancies created by firms. Following the usual assumptions, \mathcal{M} is nonnegative, increasing in both arguments and concave. Moreover, \mathcal{M} is assumed to display constant returns to scale, which implies $\lambda_w = \mathcal{M}(1, \sigma)$, where $\sigma = v/u$ is referred to as a measure of labor market tightness.

Before turning to the analysis of firms, one can simplify expressions (4) and (5) using the linearity property of W_e^w . For the employed worker, (4) is equivalent to

$$V_y^w(m_1) = w(y) + \alpha_1 [v(q_1) - \phi d_1] + \phi m_1 + \delta W_0^w(0) + (1 - \delta) W_y^w(0), \quad (6)$$

where $w(y) = \phi w^n(y)$ is defined as the real wage. Similarly for the unemployed worker, (5) becomes

$$V_0^w(m_0) = b + l + \alpha_0 [v(q_0) - \phi d_0] + \phi m_0 + \lambda_w \int_{y_R} W_y^w(0) dF(y) + [1 - \lambda_w + \lambda_w F(y_R)] W_0^w(0), \quad (7)$$

with $b = \phi B$ represents the UI benefits in real terms.

2.2 Firms

In Sub-period 1, firms with $e = 0$ have no activities, i.e., $V_0^f = 0$. If a firm enters the market with a job match, it can produce y units of output in the LM. Matched firms may immediately sell their products in the DM, with a probability α_f . In general α_f is determined in equilibrium by α_0 , α_1 and the number of people unemployed. When a firm sells q_f units of output, the rest $y - q_f$ is carried into the CM in Sub-period 2 as inventory. Without loss of generality, I assume that each unit of output can be transformed directly into one unit of consumption goods in the next CM market. In other words, the opportunity cost of a sale in the DM is

linear: $c(q_f) = q_f$.

At the end of Sub-period 1, existing job matches end exogenously at rate δ . Once entering into the second sub-period, a firm with a job match productivity y pays wage $w^n(y)$ to its previous employee. Overall, the expected value of a firm with $e = y$ during the first sub-period is

$$V_y^f = \alpha_f \left[\delta W_o^f(y - q_f, d_f - w^n(y)) + (1 - \delta) W_y^f(y - q_f, d_f - w^n(y)) \right] \\ + (1 - \alpha_f) \left[\delta W_o^f(y, -w^n(y)) + (1 - \delta) W_y^f(y, -w^n(y)) \right], \quad (8)$$

where d_f is the amount of money received from selling q_f units of output. Both terms are determined by the equilibrium conditions in the DM.

Moving to the second sub-period, it should be obvious that no firm acquires positive amounts of \hat{m} , since they do not use money during Sub-period 1. A matched firm with x units of inventory and m dollars of cash receipts has a value

$$W_y^f(x, m) = x + \phi m + \beta \hat{V}_y^f. \quad (9)$$

Firms with $e = 0$ may enter into the next LM with a vacancy, if they pay a fix cost k in the current CM. Depending on match-specific productivity, a firm is matched with a worker if and only if $y \geq y_R$. Thus, the value of a firm with $e = 0$ in the CM is

$$W_0^f(x, m) = x + \phi m + \max \left\{ 0, -k + \beta \lambda_f \int_{y_R} W_y^f(0, 0) dF(y) + \beta [1 - \lambda_f + \lambda_f F(y_R)] W_0^f(0, 0) \right\}. \quad (10)$$

where λ_f is the job arrival rate for the firm. By constant returns, $\lambda_f = \mathcal{M}(u, v)/v = \mathcal{M}(1/\sigma, 1)$. Under the free entry condition, the last term in (10) equals 0, so

$W_0^f(x, m) = x + \phi m$, and

$$k = \beta \lambda_f \int_{y_R} W_y^f(0, 0) dF(y) + \beta [1 - \lambda_f + \lambda_f F(y_R)] W_0^f(0, 0). \quad (11)$$

In steady state $V_y^f = \hat{V}_y^f$, which by (8) and (9) can be rewritten as

$$V_y^f = \frac{y - w(y) + \alpha_f (\phi d_f - q_f)}{1 - \beta(1 - \delta)}. \quad (12)$$

It is useful to define $\pi = \alpha_f (\phi d_f - q_f)$ as the expected profits from trade in the DM. Using (12), the steady state version of (11) is

$$k = \frac{\beta^2 \lambda_f}{1 - \beta(1 - \delta)} \int_{y_R} [y - w(y) + \pi] dF(y). \quad (13)$$

Given the above value functions, each period the average profits across all firms that are in

production is $(1 - u) \int_{y_R} [y - w(y) + \pi] dF(y) / [1 - F(y_R)] - vk$. Assume that each worker, independent of his or her type, holds the same portfolio of shares. Then the real dividend Ψ , received by each worker in the CM is simply equal to the average profits.⁸

3 Equilibrium

To characterize the equilibrium, it remains to specify the pricing mechanisms in each of the three markets. Following the standard specifications in the related literature, I consider price taking in the CM and Nash wage bargaining in the LM. The terms of trade in the DM are determined by pairwise Nash bargaining, although one can easily modify the model to allow other pricing options⁹. The reason for this choice is that we can use seller's bargaining power as a proxy for the degree of competition in the goods market, and examine how different market structures affect the implied inflation-unemployment relation.

3.1 Optimal Decisions in the DM

Suppose that when a worker and firm meet in the DM, they bargain over the terms of trade (q, d) , subject to the worker's cash constraint $d \leq m$, where m is the amount of money held by any type of worker. Let η denote the seller's bargaining power. According to expressions (6), (7) and (12), the surplus for a worker from trading q units of goods is $v(q) - \phi d$, and the surplus for the firm is $(1 - s)\phi d - q$, as the government collects $s\phi d$ dollars of sales tax from each transaction¹⁰. The generalized Nash bargaining problem is to

$$\max_{q,d} [v(q) - \phi d]^{1-\eta} [(1 - s)\phi d - q]^\eta \quad \text{s.t.} \quad d \leq m.$$

The standard results in Lagos and Wright (2005) and related models apply here. First, $d = m$ for any $q \leq q^*$, where q^* is the first best allocation satisfying $v'(q^*) = 1$. Secondly, from the first order condition the solution for q satisfies

$$\phi m = g(q) \equiv \frac{\eta v(q) + (1 - \eta)v'(q)q}{(1 - s)(1 - \eta)v'(q) + \eta}. \quad (14)$$

Given the bargaining solutions, one can easily solve for workers' optimal choices of \hat{m} using (3), (6) and (7). Inserting $d = m$ into (6) and (7) and differentiating with respect to m , the

⁸Note that, due to the quasi-linear utility, the distribution of firms' dividends does not affect the main analysis in this model.

⁹See Berentsen, Menzio and Wright (2008) for alternative specifications under price taking and price posting with directed search. In general the qualitative results under bargaining and price taking are consistent, as long as firm's profits are positive. The modification to directed search, however, requires some additional changes in this environment, as it considers endogenous trading probabilities in the DM.

¹⁰An alternative way to model the sale tax is to let workers (buyers) pay $(1 + s)\phi d$ when they make the purchase. Given the cash constraint, in equilibrium workers pay ϕm and firms receive $\phi m / (1 + s)$, which leads to similar results as in the above setup.

first order condition in (3) becomes

$$\frac{\phi}{\hat{\phi}} = \beta \left[\frac{\alpha_0 v'(\hat{q}_0)}{g'(\hat{q}_0)} + (1 - \alpha_0) \right] \quad \text{and} \quad \frac{\phi}{\hat{\phi}} = \beta \left[\frac{\alpha_1 v'(\hat{q}_1)}{g'(\hat{q}_1)} + (1 - \alpha_1) \right]. \quad (15)$$

The second expression in (15) implies that, independent of their wages $w(y)$, all employed workers that trade in the DM purchase and consume the same amount of goods. In a stationary monetary equilibrium, with q constant over time and $\phi/\hat{\phi} = \gamma$, (15) pins down two equilibrium conditions,

$$\frac{v'(q_0)}{g'(q_0)} = \frac{i}{\alpha_0} + 1 \quad \text{and} \quad \frac{v'(q_1)}{g'(q_1)} = \frac{i}{\alpha_1} + 1, \quad (16)$$

where we have replaced $(\gamma/\beta - 1)$ by the nominal interest rate, i , using the Fisher equation $1 + i = \gamma/\beta$.

In general, the comparison between q_0 and q_1 depends on the properties of the LHS of (16). Under the conditions that guarantee unique solutions for positive q_0 and q_1 , $v'(q)/g'(q)$ is strictly decreasing¹¹. In this case, using m_0 and m_1 as the optimal money holdings for the employed and the unemployed, respectively, we have

Proposition 1 *For all $i > 0$, $\alpha_0 > \alpha_1$ implies that $m_0 > m_1$ and $q_0 > q_1$. Both q_0 and q_1 are decreasing in i , but the difference between them is increasing. In the limit as $i \rightarrow 0$, q_0 and q_1 converge to the same level, q where $q \leq q^*$ with $q = q^*$ iff $\eta = 0$ and $s = 0$.*

Proof: See appendix.

Given the results of q_0 and q_1 in (16), one can obtain m_0 and m_1 using (14). In this economy the difference in trading probabilities induces employed and unemployed people to carry different money balances into the DM, thereby causing them to consume different amounts of q . Inflation, as a tax on cash-intensive goods, reduces both types of workers' DM consumption, but the difference between q_0 and q_1 diminishes as interest rate goes down. At the Friedman rule, which requires $i = 0$, there is no cost of holding nominal balances, hence the two types of workers take the same amount of money.

On the firm's side, the expected sale in the DM depends on q_0 , q_1 and trading probabilities α_0 and α_1 . Suppose that all the firms that produce in the LM have an equal probability of meeting a worker in the DM. Given that the total measure of potential buyers is $\alpha_0 u$ unemployed workers plus $\alpha_1(1 - u)$ employed workers, where u denotes the unemployment rate, the expected sale of each firm is $\alpha_f q_f = [\alpha_0 u q_0 + \alpha_1(1 - u) q_1] / (1 - u)$. Similarly, the expected cash receipts are $\alpha_f d_f = (1 - s) [\alpha_0 u m_0 + \alpha_1(1 - u) m_1] / (1 - u)$, where $d = m$ has

¹¹The conditions that guarantee a unique solution in Lagos and Wright (2005) are: $v'(q)$ is log-concave; or $\eta \approx 0$. Wright(2008) recently proves that there is generically a unique steady state q even if the LHS of (16) is not monotone. Moreover, he shows q is increasing with the trading probabilities.

been inserted. Using the bargaining solution in (14), one can rewrite the firm's expected profits

$$\pi = \frac{\alpha_0 u}{1-u} [(1-s)g(q_0) - q_0] + \alpha_1 [(1-s)g(q_1) - q_1]. \quad (17)$$

Finally, taking as given the equilibrium quantities exchanged in the DM the government meets its budget constraint at each period:

$$G + bu = s [\alpha_0 u g(q_0) + \alpha_1 (1-u)g(q_1)] + (\gamma - 1)\phi M.$$

3.2 Optimal Decisions in the LM

In the LM, when a firm with a vacancy meet with an unemployed worker, they first learn their potential productivity y , and then bargain over wage $w(y)$. Assume that $w(y)$ is determined by the generalized Nash bargaining solution. The surplus for a worker from a successful negotiation is equal to $S_y^w = W_y^w(m) - W_0^w(m) = W_y^w(0) - W_0^w(0)$, by linearity of W_e^w . Using (2), (6) and (7), one can derive the following expression for S_y^w :

$$\begin{aligned} S_y^w = & \beta \{w(y) - b - l + i [g(\hat{q}_0) - g(\hat{q}_1)] + \alpha_1 [v(\hat{q}_1) - g(\hat{q}_1)] - \alpha_0 [v(\hat{q}_0) - g(\hat{q}_0)]\} \\ & + \beta(1 - \delta)\hat{S}_y^w + \beta\lambda_w \int_{y_R} \hat{S}_y^w dF(y), \end{aligned} \quad (18)$$

where we have inserted workers' optimal choices in (14) and (16).

The surplus for a firm from a successful match is $S_y^f = W_y^f(x, m) - W_0^f(x, m)$, Applying the optimal decisions in (8) to (10), $S_y^f = \beta [y - w(y) + \pi + (1 - \delta)\hat{S}_y^f]$. Let θ denote the worker's bargaining power. The equilibrium wage solves the following Nash product

$$\max_w [S_y^w(w)]^\theta [S_y^f(w)]^{1-\theta}$$

Taking as given future surpluses \hat{S}_y^i , the solution to the maximization problem satisfies

$$\theta S_y^f = (1 - \theta)S_y^w. \quad (19)$$

We now proceed to solve for the equilibrium wage $w(y)$. First consider the reservation value for an unemployed worker. By definition, y_R is the reservation productivity such that workers and firms agree to match if and only if $y \geq y_R$. At y_R , the total surplus from a job match satisfies $S_{y_R}^w + S_{y_R}^f = 0$. By virtue of (19), this is equivalent to saying that $S_{y_R}^w = 0$. Replacing y by y_R in (18), one can obtain an expression for $S_{y_R}^w$. Substituting it into the general expression of S_y^w in (18) and imposing the steady state condition, we get

$$S_y^w = \frac{\beta [w(y) - w(y_R)]}{1 - \beta(1 - \delta)}. \quad (20)$$

Moreover, by the result in (12), the surplus for the firm in steady state is

$$S_y^f = \frac{\beta [y - w(y) + \pi]}{1 - \beta(1 - \delta)}. \quad (21)$$

Plugging (20) and (21) into (19), after some algebra, the equilibrium bargaining wage is given by

$$w(y) = \theta y + (1 - \theta)y_R + \pi. \quad (22)$$

Inserting (22) into (13), firm's free entry condition becomes

$$k = \frac{\beta^2(1 - \theta)\mathcal{M}(1/\sigma, 1)}{1 - \beta(1 - \delta)} \int_{y_R} (y - y_R)dF(y), \quad (23)$$

where we have replaced the arrival rate λ_f using the matching function. Note that (23) defines one equation for the equilibrium y_R and σ .

To obtain a second equation, replacing y by y_R in (18), the bargaining wage at the reservation level is then characterized by

$$w(y_R) = b + l + i [g(q_1) - g(q_0)] + \alpha_0 [v(q_0) - g(q_0)] - \alpha_1 [v(q_1) - g(q_1)] + \frac{\sigma\theta k}{\beta(1 - \theta)}, \quad (24)$$

where the last term is obtained using (23). For the convenience of later use, define $w_{y_R} \equiv w(y_R)$ as the reservation wage for workers. Using (22), the reservation productivity equals $y_R = w_{y_R} - \pi$. In the steady state, the equilibrium unemployment rate satisfies

$$u = \frac{\delta}{\delta + \lambda_w[1 - F(y_R)]} = \frac{\delta}{\delta + \mathcal{M}(1, \sigma)[1 - F(y_R)]}. \quad (25)$$

Together with the equation for π in (17), we then arrive at a second equation for (y_R, σ) :

$$\begin{aligned} y_R = & b + l + i [g(q_1) - g(q_0)] + \alpha_0 [v(q_0) - g(q_0)] - \alpha_1 [v(q_1) - g(q_1)] + \frac{\sigma\theta k}{\beta(1 - \theta)} \\ & - \frac{\alpha_0\delta}{\mathcal{M}(1, \sigma)[1 - F(y_R)]} [(1 - s)g(q_0) - q_0] - \alpha_1 [(1 - s)g(q_1) - q_1]. \end{aligned} \quad (26)$$

A nice property of this model is that the equilibrium in the DM can be solved separately from the LM optimal choices. As demonstrated above, (16) determines unique solutions for q_0 and q_1 . To ensure the existence of an equilibrium in the LM, the solution of y_R needs to be positive. This can be satisfied by choosing appropriate parameter values for b , l and k . Under such conditions, one can solve for (y_R, σ) using (23) and (26). The free entry condition in (23) describes a downward sloping curve *FE* over (y_R, σ) space. Higher y_R makes job matches less profitable for firms, so they reduce vacancy postings. The expression in (26), on the other hand, presents an upward sloping reservation productivity (*RP*) curve between y_R and σ . Larger σ implies that it is easier for workers and firms to find a match, so they are

more willing to turn down a potential match at low productivity. As shown in Figure 2, the FE curve and the RP curve together define unique solutions for y_R and σ as long as $y_R > 0$. Proposition 2 summarizes the existence and properties of the equilibrium.

Proposition 2 *Under standard conditions that entails $y_R > 0$ (or $u < 1$), there exists a unique stationary monetary equilibrium with (q_0, q_1, y_R, σ) characterized by equations (16), (23) and (26).*

Proof: See appendix.

Before turning to examine the relationship between inflation and unemployment, it is important to notice that in this economy, the overall levels of consumption and welfare for unemployed workers are lower than for employed people, although the unemployed face a higher trading probability in the DM. The intuition is as follows. First of all, the total income available is higher for people who are working and this is guaranteed by the Nash bargaining solutions. By consuming more in the DM, unemployed people are able to consume only a much smaller amount in the CM, so their total level of consumption is lower than the employed. Moreover, with a positive nominal interest rate, unemployed workers are subject to a larger cost of inflation as they carry more money to consume cash-intensive goods. This reduces their utility further. The fact that unemployed workers endure a higher burden of the inflation tax has important implications for the inflation-unemployment relation, which is explored in the next section.

4 Inflation and Unemployment

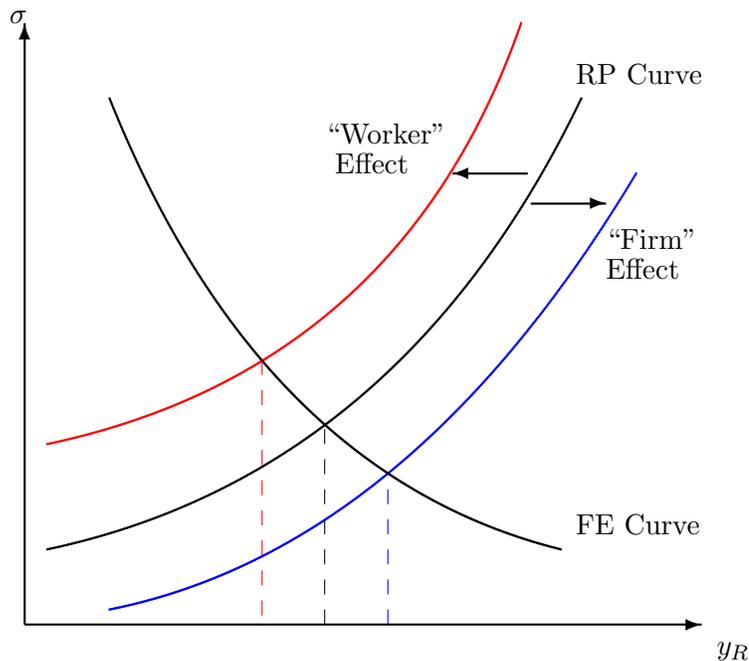
We now turn to examine how changes in monetary policy, in particular inflation or interest rate, affect the unemployment rate in the steady state. In this economy monetary factors impact labor market decisions through their effects on DM transactions. According to Proposition 1, higher rate of inflation or interest rate reduces consumption in the DM. Since y_R and σ are functions of q_0 and q_1 , we first examine how (23) and (26) respond to a change in i .

First notice that i does not enter directly into equation (23), so a change in inflation has no effect on the position of the FE curve. Keeping σ constant, totally differentiating (26) yields the sign of $\partial y_R / \partial i$ as

$$[g(q_1) - g(q_0)] - \left\{ \frac{\alpha_0 u}{1 - u} [(1 - s)g'(q_0) - 1] \frac{\partial q_0}{\partial i} + \alpha_1 [(1 - s)g'(q_1) - 1] \frac{\partial q_1}{\partial i} \right\}. \quad (27)$$

The first term is the same as $\partial w_{y_R} / \partial i$, which comes from the differential effect of inflation. For convenience, we name it the “worker” effect. With $\alpha_0 > \alpha_1$, $q_1 < q_0$ and $\partial w_{y_R} / \partial i < 0$. That is, higher consumption of cash goods makes the unemployed more exposed to the inflation tax. Consequently, an increase in inflation or nominal interest rate reduces the value of outside options for workers, and lowers the reservation productivity at which firms and workers agree

Figure 2: Effects of increase in i on σ and y_R



to match. This shifts the RP curve to the left. The second term on the RHS of (27) reflects the effect of inflation on firm's profits from trading in the DM. We call it the "firm" effect. Since $\partial q/\partial i < 0$ and $(1-s)g'(q) - 1 > 0$ for both types of workers, the "firm" effect shifts the RP curve to the right. Higher rate of inflation raises the tax on cash-intensive goods, and lowers the quantities exchanged in the DM market, as well as firm's profits. A lower return from job creation discourages firm's willingness to match with a worker, thereby raising y_R .

The two effects of inflation are generated through an uneven distribution of the burden of inflation tax on the one hand, and changes in firm's profits on the other. In the special case with $\alpha_0 = \alpha_1$, unemployed and employed people consume the same amount of cash goods and face the same cost of the inflation tax. In this case, the negative effect vanishes and inflation always increases unemployment by reducing firm's trading surplus. This is the same result as in Berentsen, Menzio and Wright (2008), although productivity is heterogenous in the present economy. On the other hand, if firms make zero profits in the cash-intensive goods market (this corresponds to the case of $\eta = 0$), inflation has no effect on firm's trading surplus. With the negative effect on the value of worker's outside option alone, the inflation-unemployment relation always slopes downward in the steady state.

In general, the two opposite effects imply that an increase in i may shift the RP curve either to the left or to the right, as illustrated in Figure 2. When the "firm" effect dominates the "worker" effect, the RP curve shifts to the right, which increases y_R and decreases σ , since (23) is downward sloping. By virtue of (25), this leads to a rise in the steady state unemployment rate. On the other hand, if the RP curve shifts to the left, y_R goes down and σ goes up. The

equilibrium unemployment rate falls. Summarizing these results, we establish the following proposition

Proposition 3 *Define $g(q_1) - g(q_0)$ as the “worker” effect and $\frac{\alpha_0 u}{1-u} [(1-s)g'(q_0) - 1] \frac{\partial q_0}{\partial i} + \alpha_1 [(1-s)g'(q_1) - 1] \frac{\partial q_1}{\partial i}$ as the “firm” effect. In the steady state, $\partial u / \partial i > 0$ iff the “firm” effect dominates the “worker” effect; otherwise, $\partial u / \partial i < 0$.*

A formal proof of this proposition is omitted as the results are intuitive. The two conflicting effects of inflation raise the question of what factors ultimately determine the slope of the Phillips curve. Since the impact of inflation on labor decisions is carried out through activities in the DM, the first group of factors we consider is the parameters that determine the goods market structure and characteristics.

First note that the magnitude of the “worker” effect, $\partial w_{yR} / \partial i$, is directly determined by the difference between q_1 and q_0 . Large difference in the real balances of employed and unemployed workers amplifies the “worker” effect, which tends to generate an inflation-unemployment trade-off if other factors remain the same. Secondly, the relative size of the two effects is closely linked to the relative surplus of workers and firms obtained in the DM trade. With Nash bargaining, firm’s bargaining power, η , determines how the surplus is split: bigger η yields more surplus to firms than to workers. Intuitively, if we view η as the proxy for the degree of competition in the goods market, then the difference between the “firm” effect and the “worker” effect is larger in a less competitive market, which leads to a positively sloped Phillips curve. Last but not least, the sales tax also changes the surplus in the trading sector. Higher s reduces the quantity exchanged in the DM, as well as firm’s profits from each sale. As both terms in (27) go down in this case, it is not clear which effect dominates the other one.

The inflation-unemployment relationship involves optimal decisions in the labor market, so intuitively labor market institutions may also play certain roles in determining the sign of $\partial u / \partial i$. Notice that q_0 and q_1 are chosen independently from parameters in the LM, so the impact of labor institutions works through the equilibrium unemployment rate, which enters in the “firm” effect only. Following the standard result in labor search theory, a higher UI benefit, b , or the worker’s wage bargaining power, θ , raises the steady state unemployment rate. As sellers in this economy are firms that have a job match with workers, the increase in u lowers the number of sellers and raises the number of buyers. This outcome generates higher profits for each firm who trades in the DM, thereby raising the “firm” effect. Nevertheless, as the effect works only through a general equilibrium channel, one would expect it may not be very significant.

One exception is the case in which the UI benefit is not perfectly indexed to inflation. For example, suppose now when a worker becomes unemployed, the UI benefit received is indexed to the average nominal earnings in the previous period. With inflation rate γ , the real value

of the UI benefit in the steady state now becomes b/γ^{12} . Accordingly, the expression in (27) changes to

$$-\frac{b}{\beta(1+i)^2} + [g(q_1) - g(q_0)] - \left\{ \frac{\alpha_0 u}{1-u} [(1-s)g'(q_0) - 1] \frac{\partial q_0}{\partial i} + \alpha_1 [(1-s)g'(q_1) - 1] \frac{\partial q_1}{\partial i} \right\}. \quad (28)$$

The first term in (28) means that the UI benefit now directly influence $\partial y_R/\partial i$. With imperfect indexation, a more generous UI scheme makes unemployed workers subject to a larger risk of inflation. As a result, a higher b leads the “worker” effect to dominate the “firm” effect and produces a trade-off between inflation and unemployment. Note that this result is independent of the assumption $\alpha_0 > \alpha_1$. In an economy where employed and unemployed workers have the same money holdings ($m_0 = m_1$), the imperfect indexation in UI benefit still generates a channel through which inflation hurts the unemployed more heavily than the employed.

5 Quantitative Examples

Due to the complex functional form of $g(q)$, it is difficult to study the comparative statics of the economy analytically. This section presents some numerical examples to illustrate how different factors in goods and labor markets affect the inflation and unemployment relation. To determine the parameter values, the length of each period is considered to be one quarter. Utility in the DM has the form of $v(q) = Aq^{1-\nu}/(1-\nu)$. The Matching function in the LM is assumed to be $\mathcal{M}(u, v) = Zu^{1-\rho}v^\rho$.

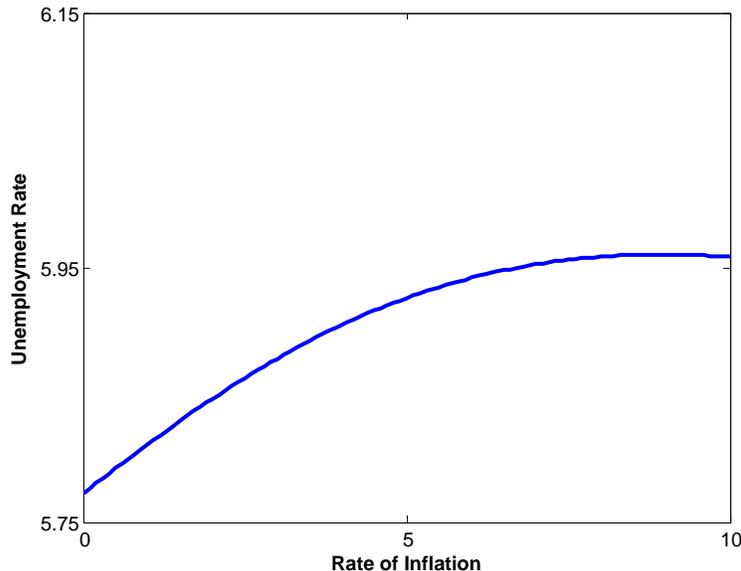
Table 1: Parameter Values in the Benchmark Economy

Description	Parameter
Discount factor	$\beta = 0.992$
DM utility function	$A = 1.013$
	$\nu = 0.2$
LM matching function	$Z = 0.364$
	$\rho = 0.28$
Worker’s wage bargaining power	$\theta = 0.72$
Job separation rate	$\delta = 0.05$
Value of nonmarket activity	$l = 0.417$
Vacancy posting cost	$k = 1.25 * 10^{-3}$

For simplicity, the productivity y is assumed to be homogeneous and normalized to one. This means the negative effect of inflation on unemployment works only through the bargained wage, but not the reservation productivity. Nevertheless, the comparison of the two conflicting effects in (27) are the same as in the general version. In a benchmark economy, the UI benefit

¹²In an alternative case, one can assume the government adjusts the nominal UI benefit to inflation with one period lag. Note that, different from nominal wage rigidities, this assumption does not hinge on any restriction on agent’s optimal choices. In any context, the government policy as an exogenous variable may fail to adjust perfectly.

Figure 3: Benchmark Economy



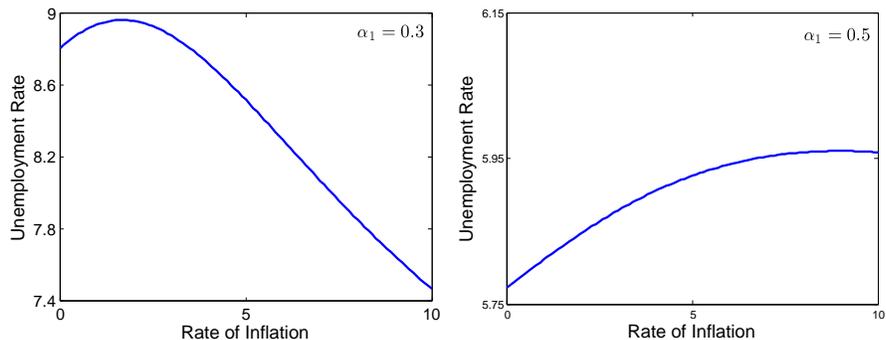
$b = 0.5$, which is chosen to meet a replacement ratio of 0.5. We start with firm’s bargaining power in the DM $\eta = 0.35$, but later compare the equilibrium under different values. Worker’s wage bargaining power θ is set to equal the elasticity of the matching function, which satisfies the Hosios condition. To focus on the effects of other parameters, initially the sales tax $s = 0$. Finally set the trading probabilities to $\alpha_0 = 0.9$ and $\alpha_1 = 0.5$, which implies a 10% difference between the real balances held by unemployed and employed workers. Notice that this is consistent with the estimation result of Italian Household data in Table A.1. The values of the rest parameters are reported in Table 1, in accordance with the calibration exercises in Berentsen, Menzio and Wright (2008) except a few variations¹³. Figure 3 displays the benchmark economy. Following a rise in inflation from 0 to 10%, the unemployment rate first increases by approximately 0.19 percentage point up to 9% inflation, and then starts to decline¹⁴.

The first exercise is to reduce α_1 from 0.5 to 0.3, to demonstrate the effect of heterogeneous money holdings. The change in the trading probability leads to a larger difference in unemployed and employed workers’ real balances (a rise from 10% to 25%). As a result, the Phillips curve becomes negatively sloped as shown in part (a) of Figure 4. The intuition is as follows. A larger gap between unemployed and employed workers’ cash holdings makes the value of outside options more sensitive to the inflation tax, and therefore causes the “worker” effect to dominate the “firm” effect. Secondly, to see the effect of changes in the government sales tax,

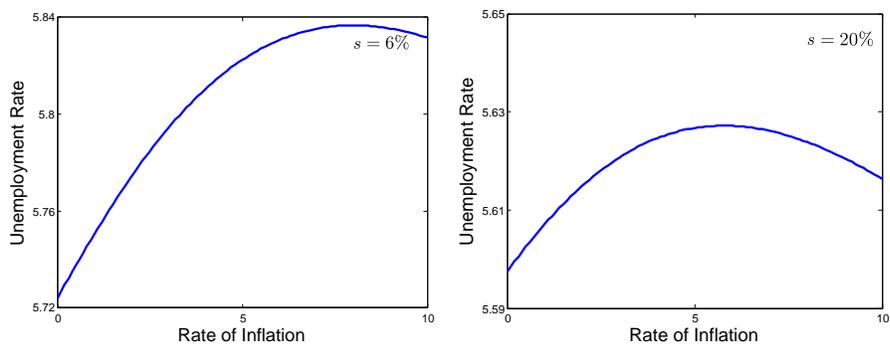
¹³Adjustments have been made for k , l and ν . This is to guarantee the existence of equilibrium for a large set of parameters.

¹⁴Notice that the change in the unemployment rate is quite small in the benchmark economy, but one can obtain a large variation simply by adjusting the value of non-market activity l . See Berentsen Menzio and Wright (2008) for details. Here the value of l is chosen to ensure the existence of equilibrium in a large range of parameter values, for the purpose of later sensitivity analysis.

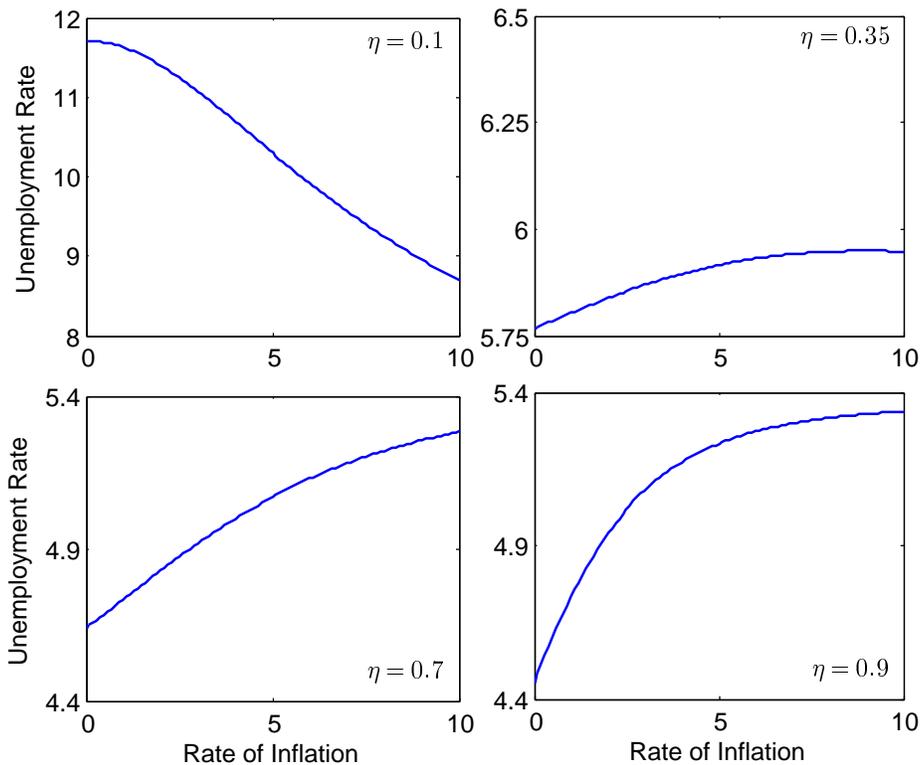
Figure 4: Goods Market Institutions



(a) Effects of Heterogeneity in Money Holdings



(b) Effects of Sales Tax



(c) Effects of Firm's Bargaining Power

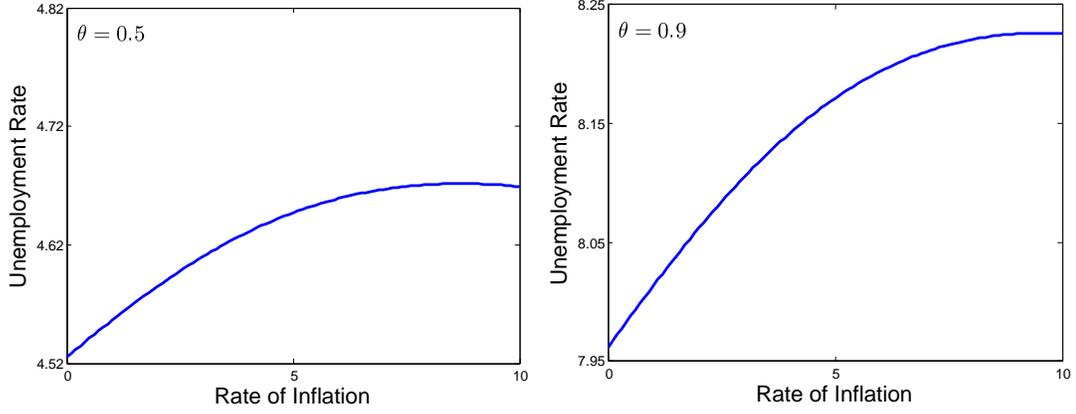
part (b) of Figure 4 compares the equilibria with and without sales tax. Although positive s reduces the trading surplus for both workers and firms, the numerical result suggests a larger impact on the firm’s side. One reason is that the magnitude of the “worker” effect depends on the difference between q_0 and q_1 . Although there is a drop in the quantities exchanged in the DM, the gap of q_0 and q_1 is not enlarged too much. On the firm’s side, however, the positive sales tax directly reduces the profits for each trade. As a consequence, the impacts on the “firm” effect seem to be large enough to outweigh the drop in the “worker” effect. This leads to an equilibrium that favors the negatively sloped Philips curve.

To illustrate the effects of the firm’s bargaining power, we compute the equilibrium under four different values of η . As can be seen in part (c) of Figure 4, the Phillips curve is downward sloping when $\eta = 0.1$. Following a rise in inflation from 0 to 10 percent, the unemployment rate is reduced from 11.6 to 8.7 percent. In the benchmark economy with $\eta = 0.35$, unemployment becomes increasing in inflation. A further rise in η generates a large but still positive change in u . Overall with a reasonable range of parameter values, the inflation-unemployment relation tends to be negative in a more competitive market (lower η).

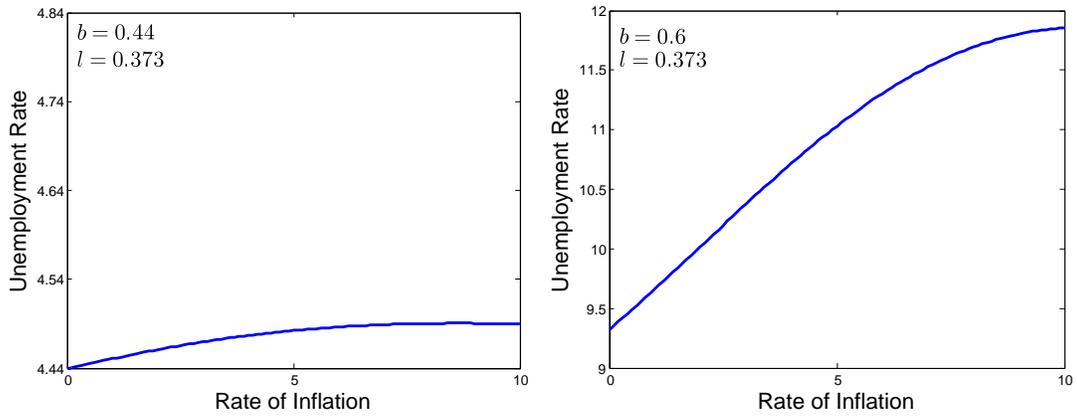
The sensitivity analysis for θ and b is presented in parts (a) and (b) of Figure 5. Following a rise in worker’s wage bargaining power or UI benefit, the unemployment rate goes up at every level of inflation, but the direction of the Philips curve remains unchanged if everything else is held constant. In both cases, larger values of θ or b only make the slope of the Philips curve steeper. The modest effect of labor market institutions is consistent with our conjecture in the previous section. In this economy, monetary factors impact labor market outcomes through the inflation tax mechanism in the cash-intensive goods market. Changes in θ or b are transmitted to the goods market only through a second order effect on the equilibrium unemployment rate. Therefore, with reasonable values of θ or b , the overall influence is generally not very significant and hardly generates different directions of the Philips curve. This result, however, is not necessarily true when other types of labor market frictions are considered. As shown in (28), with imperfect indexation the “worker” effect in $\partial y_R/\partial i$ is a function of b . A generous UI benefit strongly favors the negative effect of inflation on unemployment and makes the Philips curve downward sloping. To illustrate that this effect is independent of the assumption $\alpha_0 > \alpha_1$, we equalize worker’s trading probabilities at this moment and adjust firm’s bargaining power to $\eta = 0.5$. In this case, part (c) of Figure 5 shows that the slope changes from positive to negative when b is increased from 0.44 to 0.6.

The above numerical exercises have provided some general pictures about how different goods and labor market institutions affect the long-run Philips curve. In general, the model predicts that the inflation-unemployment relation can be quite complicated, even without nominal rigidities or other types of imperfection. Moreover, the numerical results also show that this relationship may vary across different monetary regimes. In most of the numerical examples, the curve exhibits a hump shape. This is because the heterogeneity effect is decreasing when inflation or interest rate goes down, which makes the positive effect more likely to domi-

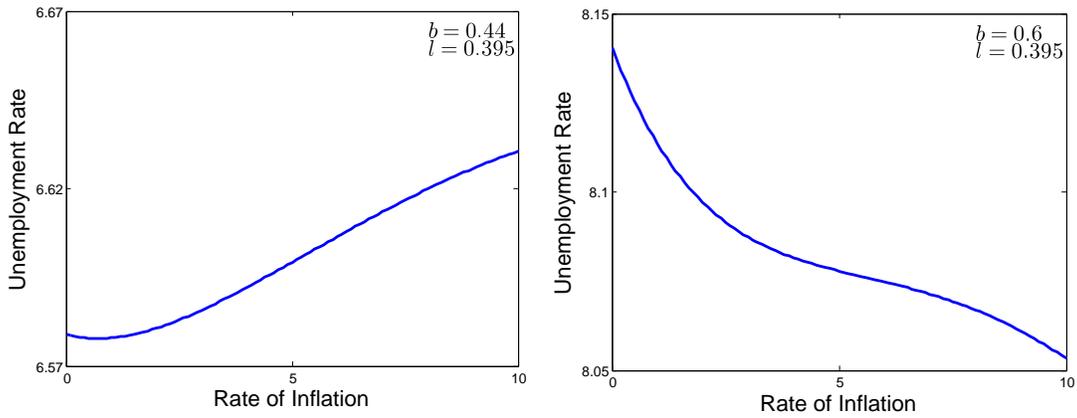
Figure 5: Labor Market Institutions



(a) Effects of Wage Bargaining Power



(b) Effects of UI Benefit with Perfect Indexation



(c) Effects of UI Benefit under Imperfect Indexation with $\alpha_0 = \alpha_1 = 0.9$ and $\eta = 0.5$

nate. In particular, as nominal interest rate approaches zero, there is no cost of holding money and the asymmetric effect diminishes.

Given the results in the quantitative analysis, one important question is whether the model can qualitatively account for some cross-country differences in the slopes of the Phillips curve. As reviewed earlier, empirical literature seems to suggest that the Phillips curve tends to slope positively in the U.S., while negatively in Europe. Overall the goods and labor markets in the two areas differ in many aspects. Here we focus on two parameters: the sales tax in the goods market and the UI benefit in the labor market. In an OECD working paper, Carey and Tchilinguirian (2000) estimate the average effective tax rates for different countries and areas. They show that, between 1980 and 1997, the consumption tax in U.S. is around 6%, while the average of European Union is close to 18.6%. By incorporating this large difference in the sales tax, the present model suggests that European countries on average tend to have a downward sloping Phillips curve, if other factors remain the same. This is qualitatively consistent with the empirical evidence. Secondly, for the labor market institutions, there is a consensus that Europe tends to have a more generous UI benefit scheme than the U.S.. In my model, when the nominal UI is not perfectly indexed to inflation, higher benefit tends to generate a negatively sloped inflation-unemployment relationship.

Finally the model has some implications for the optimal monetary policy in the long run. In general it is difficult to draw one conclusion from this model, since the optimal rate of inflation depends on labor market efficiency, as well as the particular inflation-unemployment relation. For a simple scenario, suppose now the Hosios condition is satisfied in the labor market, with worker's wage bargaining power θ equals the elasticity of the matching function ρ . This may be achieved through government fiscal policy, or simply efficient market outcomes. In this case, the optimal monetary policy is to set $i = 0$, no matter inflation and unemployment are positively or negatively related. Intuitively, a positive interest rate imposes a tax on the consumption of cash-intensive goods, which reduces the welfare for both employed and unemployed workers. Even under the case where inflation encourages employment and output in the long-run, the first order welfare cost always dominates the second order employment effect, and the Friedman rule (with $i = 0$) is optimal.

Suppose now the Hosios condition is violated and the labor market is inefficient. With certain restrictions on the fiscal policy (UI benefit or income tax), the equilibrium unemployment rate may be above or below the efficient level. Under these circumstances, the optimal inflation is to deviate from the Friedman rule. Nevertheless, the specific level of inflation rate depends on labor market outcomes and the underlying inflation-unemployment relation. To infer a detailed optimal policy rule, we need additional welfare analysis and a close examination of the market characteristics. This is left for future work.

6 Conclusion

This paper has developed a general equilibrium model to study the long-run relationship between inflation and unemployment. The model takes seriously the observation that unemployed people are affected more heavily by the inflation tax than employed ones, and studies its implication for the inflation-unemployment relation. Overall the model predicts that changes in monetary conditions have two opposing effects on the labor market. Following a rise in inflation or nominal interest rate, the asymmetric effect of inflation reduces the value of outside options for workers, which lowers both the bargained wage and reservation productivity, thereby decreasing unemployment. At the same time, inflation as a tax on the cash-intensive activities, also reduces firm's return of job creation and raises unemployment. In equilibrium the implied inflation-unemployment relationship can be either positive or negative.

The quantitative examples show that goods market factors, such as the degree of heterogeneity, the level of sales tax and the overall competitiveness, play major roles in determining which of the two effects dominates. In contrast, labor market institutions, like UI benefit or wage bargaining power, only affects the inflation-unemployment relationship modestly. The exception is when UI benefit is not perfectly indexed to inflation, in which case unemployed workers suffer an additional cost of inflation due to the real value erosion of the nominal benefit. Overall by comparing the sales tax and UI benefit, the model generates an inflation-unemployment relation qualitatively consistent with empirical evidence.

The paper also provide some implications for the optimal policy. With no restrictions on policy instruments, the optimum is to achieve labor market efficiency using fiscal policy, and set the monetary policy at the Friedman rule. This is welfare improving even when inflation encourages employment. In the presence of distortions in the labor market, deviating from the Friedman rule may be optimal. In general, the model provides a framework that is exploitable by monetary policy to improve economic efficiency. The implementation, however, requires detailed analysis of the goods market structure and labor market institutions.

Several extensions can be made for future research. First, the numerical examples in Section 5 only demonstrates that the model implications are qualitatively consistent with empirical evidence. One extension is to calibrate the model and quantitatively examine the cross-country difference in the relationship between inflation and unemployment. Secondly, the degree of goods market competitiveness in the present economy is modelled exogenously through firm's bargaining power. As it is important for determining the long-run Philips curve, endogenizing the goods market structure may provide further insights for the effects of monetary factors on the labor market. This is of particular interest if we incorporate the correlation between inflation and market power into the model. Lastly, the present paper focuses on the steady state analysis. To understand how labor and goods markets frictions affect the conduct of monetary policy, one can introduce short-run stochastic shocks into the model and follow the line of Berentsen and Waller (2006) to study the optimal stabilization policy.

Appendix

A. Evidence of the Asymmetric Effects of Inflation

Table A.1: Results of Linear Regression for cash holding and spending

	Cash Holdings (as a percentage of total annual income)	Monthly Consumption in Cash (as a percentage of total expenditure)
Unemployment	1.6004 (0.2942)***	10.7343 (2.7569)***
Male	0.3232 (0.1601)**	6.0200 (1.5006)***
Married	-0.3054 (0.1585)**	-3.8708 (1.4851)***
University degree	-0.3292 (0.1924)*	-15.1348 (1.8030)***
Age3140	0.0471 (0.2818)	-4.3032 (2.6407)*
Age4150	-0.0241 (0.2755)	-5.3244 (2.5816)**
Age5165	0.2730 (0.2795)	-3.3632 (2.6190)
Age65	0.2203 (0.6783)	-3.1902 (6.3564)
Constant	1.5658 (0.2688)***	81.5602 (2.5191)***
Summary statistics:		
Mean of dependent variables:		
<i>Employed</i>	1.6226%	76.8952%
<i>Unemployed</i>	3.2789%	89.1514%
# of Observations	3730	3730
Standard errors in parentheses.		
* significant at 10%; ** significant at 5%; *** significant at 1%		
Data: Italian Survey of Household Income and Wealth 2004 (excluding Not in Labor Force)		

Table A.2: Average minutes by activity, region and employment status

	Employed					Unemployed				
	US	Canada	Western Europe	Eastern Europe	Nordic	US	Canada	Western Europe	Eastern Europe	Nordic
Sleep	494	475	490	484	485	550	515	528	544	518
Personal care	46	43	49	48	44	43	42	53	48	44
Eating	66	60	96	89	84	55	71	107	108	89
Work	323	353	310	332	281	11	38	16	8	40
Job search	1	0	0	n.a.	n.a.	32	27	11	11	4
Education	10	7	6	6	9	21	6	21	16	39
Home Production and care of others	129	130	134	158	152	220	167	211	260	202
<i>of which: childcare</i>	27	23	20	24	23	42	34	25	34	27
Shopping and services	28	30	24	18	25	36	50	40	28	29
Voluntary: religious and civic activities	13	9	8	8	6	20	7	10	7	7
Sport	17	24	23	16	27	19	43	37	29	37
Leisure and socializing	222	222	211	203	237	352	393	323	305	336
<i>Of which: TV</i>	124	101	98	118	102	203	171	151	165	155
Travel	84	88	85	73	85	70	81	79	71	75
Other	7	0	3	5	6	11	0	4	5	20

(Western Europe: Austria, Belgium, France, Germany, Italy, Spain, UK; Eastern Europe: Bulgaria, Slovenia, Poland; Nordic: Finland, Sweden)

Notes: Survey weights were used to compute country averages. Region averages are weighted by the size of the labor force of each country. Universe: Labor force, age 20-65.

Sources: HETUS, MTUS (Canada, Austria, Germany 1991-92, France), ATUS.

The table is reproduced from Table 3 in Krueger and Mueller (2008).

B. Proofs

B.1 Proof of Proposition 1

Proof. With $v'(q)/g'(q)$ strictly decreasing in q , $q_0 > q_1$ follows directly from (16). To prove $m_0 > m_1$, differentiating (14) with respect to q

$$g'(q) = \frac{(1-s)(1-\eta)v'(q)^2 + \eta v'(q) + \eta(1-\eta)v''(q)[q - (1-s)v(q)]}{[(1-s)(1-\eta)v'(q) + \eta]^2}.$$

To see the sign of $g'(q)$, we need to determine the term $[q - (1-s)v(q)]$. By Nash bargaining, the surplus for the firm is positive as long as $\eta > 0$, that is, $(1-s)g(q) - q > 0$. Using (14), we obtain $(1-s)v(q) - q > 0$. With $v'(q) > 0$ and $v''(q) < 0$, this gives $g'(q) > 0$. Hence, $m_0 > m_1$ when $q_0 > q_1$.

The results of $\partial q_0/\partial i < 0$ and $\partial q_1/\partial i < 0$ again follow directly from (16). To see the effect of change in i on $q_0 - q_1$, we can rewrite (16) in a general function form

$$h(q) = \frac{i}{\alpha} + 1,$$

where $h(q) \equiv \frac{v'(q)}{g'(q)}$ and α represents any arbitrary number of trading probability. Differentiating q with respect to α yields $dq/d\alpha = -i/\left[\alpha^2 h'(q)\right]$. With $v'(q)/g'(q)$ strictly decreasing in q , $dq/d\alpha$ is positive and increasing in i . That is, the size of the difference in q resulting from different α is increasing in i .

Finally, the results in the limiting case, $i \rightarrow 0$, follow the same arguments as in the standard model of Lagos and Wright (2005), except here the efficient allocation q^* requires both $\eta = 0$ and $s = 0$. ■

B.2 Proof of Proposition 2

Proof. The uniqueness of q_0 and q_1 has been established in Proposition 1. To examine the properties of equilibrium y_R and σ , we first totally differentiate (23). The sign of $\partial\sigma/\partial y_R$ is the same as

$$-\frac{[\mathcal{M}(1, \sigma)]^2 [1 - F(y_R)]}{\mathcal{M}_1},$$

where $\mathcal{M}_1 = \partial\mathcal{M}(u, v)/\partial u$. Given the standard assumptions on $\mathcal{M}(u, v)$, $\partial\sigma/\partial y_R$ is always positive. Similarly, totally differentiating (26) yields

$$\frac{\partial\sigma}{\partial y_R} = \frac{1 + \frac{\Delta f(y_R)}{\mathcal{M}[1-F(y_R)]^2}}{\frac{\theta k}{\beta(1-\theta)} + \frac{\Delta\mathcal{M}_2}{\mathcal{M}^2[1-F(y_R)]}} > 0,$$

where $\mathcal{M}_2 = \partial\mathcal{M}(u, v)/\partial v$, and $\Delta = \alpha_0\delta[(1-s)g(q_0) - q_0]$. Therefore, under the condition that guarantees $y_R > 0$ (which can be satisfied when $b + l$ is sufficiently high), (23) and (26)

together determine unique solutions for y_R and σ . ■

C. Data

The data is from Italian Survey of Household Income and Wealth 2004, excluding households who are not in labor force. The total number of observation is 3730. The *cash holdings as a percentage of total annual income* is calculated as the average money holdings in the house divided by net annual disposable income. The *monthly consumption in cash as a percentage of total expenditure* is calculated as the ratio of monthly cash spending and monthly average spending on all consumption.

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