# The Dynamics of Sectoral Labour Adjustment

Stephen Tapp*

Queen's University

January 2008

**Abstract**

This paper develops an equilibrium search and matching model to jointly study the aggregate, sectoral, and distributional impacts of labour adjustment. The model extends Pissarides (2000) to include multisector production and search and 'innovation' from investments that can potentially improve a match's productivity. These extensions deliver two mechanisms for inter-sectoral and intra-sectoral labour reallocation after productivity shocks. First, because workers search simultaneously in multiple sectors, changes in labour market conditions in one sector propagate to impact wages and hiring in the rest of the economy through a *reservation wage effect.* Second, a positive productivity shock causes firms to invest more resources in innovation. This *innovation effect* shifts production towards high-skill jobs and amplifies the impact of productivity shocks relative to the baseline model. I show that the model is useful for analyzing labour adjustments caused by a diverse set of factors including: technological change; persistent energy price and exchange rate shocks; and trade liberalization. Finally, because the transition dynamics between steady-states are tractable, the model can be readily-applied to the data to study particular labour adjustment episodes.

# 1 Introduction

Recent empirical evidence finds that sectoral job changes are common and have increased significantly in recent decades.[1] These findings raise several issues. For individuals, these job changes typically result in earnings losses that can be large and persistent, particularly for those with intervening unemployment spells.[2] Furthermore, the factors driving sectoral reallocation — such as technological change; persistent energy price and exchange rate movements; and trade liberalization — often have different effects on low-skill and high-skill workers. At the aggregate level, given the large differences in sectoral output per worker, reallocation can have important impacts on output and productivity as well as equilibrium wage spillovers among sectors.[3]

Existing approaches fail to jointly capture these important features of sectoral labour reallocation in a unified, tractable framework that explicitly considers transition dynamics. This paper attempts to fill this gap by developing a model to study the aggregate, sectoral, and distributional impacts of labour adjustment following unanticipated, sector-specific productivity shocks. I solve the model, derive the main analytical results and use simple quantitative examples to clearly illustrate the model's adjustment mechanisms. I then demonstrate that the model's results are consistent with facts from sectoral labour adjustments caused by a variety of factors. In addition, the model's transition dynamics are quite tractable, which facilitates applying the model to the data.

The model makes two key extensions to the baseline Pissarides (2000) labour search and matching model, where search frictions generate equilibrium unemployment. The first extension is multisector production and search, which delivers equilibrium wage spillovers across sectors. The second extension is an 'innovation' process that allows matches to acquire skills and become more productive. This extension parsimoniously models low-skill and high-skill workers in this environment to analyze how they might be impacted differently by shocks. Including the innovation process is also useful because it amplifies the model's response to

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[1]Kambourov and Manovskii (forthcoming) find that more than 10 percent of U.S. workers change sectors annually. Sectoral labour mobility more than doubled from 6 percent to 14 percent per year, over 1968–1997.

[2]For U.S. evidence, see *e.g.* McLaughlin and Bils (2001), Fallick (1996), Jacobson et al. (1993), Topel (1993). For Canadian evidence, see Morissette et al. (2007) and Galarneau and Stratychuk (2002).

[3]For aggregate impacts see Lee and Wolpin (2006); Tapp (2007) studies Canada following a global commodity price shock; for sectoral impacts, see Trefler (2004) for Canadian manufacturing industries responses following the Canada-U.S. Free Trade Agreement; Beaudry et al. (2007) find that changes in sectoral composition cause equilibrium wage spillovers.

productivity shocks and captures how reallocation within a sector can impact aggregate output and productivity.

While the model retains the well-studied features (and short-comings) of the basic one-sector model without innovation investment, the extensions provide some important new insights from two mechanisms that generate inter-sectoral and intra-sectoral reallocation of workers during labour adjustments. Inter-sectoral reallocation operates through a *reservation wage effect*, which describes how changes in workers' outside options cause sector-specific shocks to spillover to other sectors. In the model, workers search simultaneously in multiple sectors of the economy. Therefore, when a shock changes labour market conditions in one sector, this affects workers' value of search, causing them to update their reservation (and ultimately, their bargained) wages. This changes the cost of labour, which in turn, impacts profitability and, therefore, job creation in other sectors of the economy. The varied recruiting responses in different sectors result in inter-sectoral labour reallocation.

Workers not only move between sectors after productivity shocks, they can also move within a sector, as firms substitute between low and high-skill production. This intra-sectoral labour reallocation operates through an *innovation effect*. In the model, all new matches begin production as low-skill, but may become high-skill through a costly and uncertain productivity-enhancing investment. After a positive productivity shock in a sector, employing a high-skill worker becomes relatively more profitable, so firms expect a larger return from these investments. They respond by investing more resources into innovation with their low-skill workers. This accelerates skill acquisition, endogenously raises the share of high-skill production in the sector and amplifies the model's response to productivity shocks.

This paper argues that this general model is relevant for sectoral labour adjustments driven by various factors. The reason is that while the precise causes of sectoral reallocation are specific to the particular episode under study, the consequences are quite unified across episodes. Regardless of the exact shock, the salient effect is a change in the relative profitability of production across certain sectors. In sectors where production becomes more profitable, there is relatively more entry, and over time, employment increases. Typically, a positive shock makes high-skill production in these sectors more attractive, so more resources are devoted to productivity-improving, innovative efforts to chase these new profits. This results in relatively more high-skill production and rewards these high-skill workers with larger wage gains. In the model, sector-specific productivity shocks capture these effects quite well

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by changing the relative match surpluses across and within sectors to generate the labour reallocation observed in the data.

Several studies relate to the model developed here. In the search and matching literature, Albrecht et al. (2006a) and Acemoglu (2001) consider two-sector production. However, while my paper focuses on the impacts of productivity shocks for sectoral adjustment in the steady-state and transition, these paper focus on a different issue — the impacts of labour market policies (unemployment benefits, minimum wages, severance and payroll taxes) on the sectoral composition in the steady-state. In Acemoglu (2001), there are no productivity differences between workers within a sector, so distributional considerations are absent and the transition is not considered. Albrecht et al. (2006a) consider steady-state distributional effects, but not transition dynamics. Other approaches in the international trade literature model sectoral reallocation.[4] These models typically ignore unemployment and labour market frictions to make long-run statements about the benefits of reallocation. In this paper, in addition to addressing the long-run steady-state impacts, I focus on the short-run adjustment, which generates important costs for individual displaced workers and is a relevant concern for policymakers. Finally, none of these papers model productivity-enhancing investment and skill acquisition.

The outline for the remainder of the paper is as follows: Sections 2 and 3 present the model and transition dynamics. Section 4 quantitatively illustrates the model's mechanisms. Section 5 argues that the model is consistent with key characteristics of sectoral labour adjustments and Section 6 concludes. Proofs and derivations are in the Appendix.

# 2   Multisector Search Model with Innovation

This section presents a model of labour reallocation following unanticipated sector-specific productivity shocks. The model adopts the search and matching framework, which is a standard tool macroeconomists use to analyze labour market fluctuations.[5] In this environment,

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[4]Melitz (2003) is a prominent example featuring intra-sectoral reallocation that can be contrasted with the innovation effect here. In Melitz's model increasing trade exposure improves a sector's productivity through selection effects. My model features within-firm productivity improvements, which aggregate to change the sectoral composition of production. The 'Dutch disease' literature models inter-sectoral reallocation, but not due to changes in workers reservation wages. Corden (1984) summarizes earlier contributions.

[5]This approach was developed by Diamond (1982a,b); Mortensen (1982a,b); Pissarides (1979, 1985); and Mortensen and Pissarides (1994), among others. Pissarides (2000) provides a thorough overview of the basic model and various extensions. Rogerson, Shimer and Wright (2005) and Yashiv (2006) survey the recent labour market search literature.

search frictions cause firms and workers to use resources and take time to locate partners before new production can begin. This search process results in equilibrium unemployment. I add two key extensions relative to the baseline model of Pissarides (2000, Ch. 1). The first is multisector production and search, which effectively links labour market conditions across sectors and generates sectoral wage and hiring spillovers (inter-sectoral labour reallocation). The second extension is a process I call 'innovation' and skill acquisition. This formalizes the idea that acquiring skills in a job typically involves a costly investment process, where successful skill acquisition is uncertain and the match-specific component of the skills are not transferable to new jobs. Including innovation amplifies the model's response to productivity shocks through endogenous shifts in the skill-intensity of production (intra-sectoral labour reallocation). Unlike previous multisector versions of the model, I focus not only on the steady-state, but also on transition dynamics between steady-states. In addition, I allow for sector-specific separation rates and use sector-specific matching functions to capture the fact that job-finding and job-filling rates vary significantly by sector.

## 2.1 Environment and General Overview

The following is a general overview of the model's key ingredients and timing of events. The details are made explicit in subsequent sections. I focus first on the steady-state, so there is initially no aggregate uncertainty; later sections consider shocks and transition dynamics.

Time is discrete with an infinite horizon. There are multiple sectors of the economy indexed by $i \in \{1, 2, \ldots, I\}$ that produce a non-storable good. The model features two types of agents: workers and firms. Each type of agent is *ex ante* identical, infinitely-lived and risk-neutral, discounting future payoffs at rate $\delta$. There will be heterogeneity *ex post* in the sectors in which agents work and their match skill levels, based on the luck associated with job search and skill acquisition. Agents are either matched and productive, or searching for a partner to begin production.

Figure 1 describes the timing of events in a given period for unmatched agents. A recruiting stage begins the period when unemployed workers collect unemployment benefits and search for jobs, and firms post vacancies in decentralized labour markets. The matching stage follows when a subset of firms with vacancies and unemployed workers are brought together in pairwise matches. Once matched, the pair bargain over the worker's wage and the firm decides how much innovation to engage in. If there is agreement, the pair produce

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next period as a low-skill match.

Figure 2 describes the timing for producing agents. Production begins the period and wage payments follow. Firms in low-skill matches then attempt to innovate to improve their productivity. At the end of each period, some low-skill matches successfully acquire match-specific skills and become high-skill. Also at the end of the period, some low and high-skill matches terminate exogenously.

## 2.2   Workers

The labour force consists of a measure one continuum of potential workers. At any point in time, a given worker is in one of the following $(2 \times I + 1)$ states: *Unemployed* — receiving unemployment benefits, $z$, and searching for a job; or, *working* — receiving a wage in sector $i$ in a low-skill match of $w_i^L$ or in high-skill match of $w_i^H$. The expected present values in these states are denoted $U$, $W_i^L$ and $W_i^H$, respectively. Workers maximize the expected present value of their lifetime income subject to the random arrival of job offers when unemployed.

The unemployed search for jobs at no cost. As a result, their search is not directed to a particular sector, but rather simultaneous in all sectors.[6] There is no on-the-job search or quits.[7] Workers do not value leisure. Therefore, when unemployed they allocate all their time to search and when employed they inelastically supply one unit of labour each period. There are no savings in the model; workers simply consume their current income.[8]

## 2.3   Firms

There is a large measure of potential firms. Firms can be in one of the following $(3 \times I)$ states: posting a *vacancy* to recruit in sector $i$; or producing in sector $i$ in a low or high-skill *job* match. The expected present values in these states are denoted $V_i$, $J_i^L$ and $J_i^H$, respectively.

There is free entry and exit of vacancies and firms incur recruiting cost, $c$, each period their vacancy remains unfilled. In a low-skill matches, firms engage in innovation activities,

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[6]The Appendix discusses directed search.

[7]Evidence from the U.S. and Canada finds a significant number of workers who change sectors experience an intervening unemployment spell. For the U.S., see Kambourov and Manovskii (forthcoming) and for Canada, see Osberg (1991). The model abstracts from job-to-job transitions, so all movements are between employment and unemployment. The assumption ruling out quits is innocuous because in equilibrium workers' wages compensate them for their value of search, so working in any sector is strictly preferred to unemployment.

[8]Goods are not storable, so they have no value next period. Borrowing and lending contracts are ignored because agents are risk neutral and no one will lend at a rate higher than $r$, so workers will not prefer promised future consumption over current consumption because of discounting.

$x_i \in \mathbb{R} [0, 1]$, at cost $\chi(x_i)$ each period, where $\chi(0) = 0$ and $\chi'(x_i) > 0$. Innovation is a costly and uncertain process, where firms make a match-specific investment in an attempt to improve the match's productivity. This can be interpreted in several ways. First, it can represent the lower productivity of a new worker while learning match-specific skills. Second, it can represent an on-the-job training program. Empirical evidence suggests training costs can be substantial and are mainly paid by the firm.[9] Third, it can represent research and development (R&D) to improve the production technology. Innovation is beneficial because it makes skill acquisition more likely, reducing the expected time to become a high-skill match. If innovation is successful, the match becomes high-skill, produces more and requires no further innovation.

The model attempts to capture the fact that labour adjustments are typically costly for individual workers. Empirical work finds that following job loss, workers can suffer significant and persistent earning losses in their subsequent jobs, particularly those workers with longer tenure.[10] These findings suggest that some skills which are accumulated are useful only in the current match. In the model, skills are match-specific and therefore are lost when the match terminates.

Matches produce output using only labour with constant returns to scale, skill-specific technologies. Each period sector $i$ matches produce: $y_i^{SK} = A_i p_i^{SK} l_i^{SK}$, where $y$ is output; $i \in \{1, 2, ..., I\}$ subscripts the sector; $SK \in \{L, H\}$ superscripts low and high-skill matches; $A_i$ is a sector-specific parameter, which is constant and normalized to one in the steady state, but will later serve as the shock; $p$ is productivity, with $p_i^H > p_i^L$; and $l$ is labour. To simplify the exposition, I assume each firm employs one worker.

## 2.4   Matching Process and Transitions Between States

Unmatched firms post vacancies to attract unemployed workers in one of $I$ sectors. The unemployed search simultaneously in all sectors. Search is costly for two reasons: 1) firms explicitly use resources to attract workers; and 2) workers and firms implicitly forego the higher wage earnings and profits they would be receiving if they were matched. Search is also time-consuming because each period some agents are unsuccessful in finding a match.

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[9]For estimates of investment costs see, for example, Barron et al. (1989, 1999); Bartel (1995); and Dolfin (2006). Loewenstein and Spletzer (1998) analyze National Longitudinal Survey of Youth data and find employers pay the explicit cost of on-site investment over 90 percent of the time.

[10]For U.S. evidence, see *e.g.* Fallick (1996), Jacobson et al. (1993), Topel (1993). For Canadian evidence, see Galarneau and Stratychuk (2002) and Morissette et al. (2007).

Sector-specific matching functions capture this feature by determining the measure of pairwise matches per period in each sector. The matching functions have the Cobb-Douglas functional form:[11] $m_i(u, v_i) = \mu_i u^\alpha v_i^{1-\alpha}$, where $m_i$ is the measure of sector $i$ matches; $u$ is the measure of unemployed workers; $v_i$ is the measure of vacancies in sector $i$; $\mu_i$ is the recruiting effectiveness in sector $i$; and $\alpha$ is the elasticity of matches with respect to unemployment.

In previous labour search papers with multiple sectors, such as Acemoglu (2001) and Davis (2001), matching occurs through an aggregate matching function. The formulation here is more general. Sectors are allowed to vary in their recruiting effectiveness, $\mu_i$'s, because in some sectors assessing applicants is easier. This means that market tightness, and therefore job-finding and job-filling rates, can vary by sector. This formulation brings the model closer to the data which feature clear differences in search outcomes across sectors.[12]

Because the model is set in discrete, rather than continuous time, this more general matching process implies that workers could potentially receive multiple offers in a period. This is an interesting and complex issue, which is explored in detail in several recent papers.[13] To keep the model's labour adjustment mechanisms transparent and comparable to the baseline Pissarides (2000) model, matching is determined in the following manner to avoid multiple offers. At the begin of the matching stage the number of matches in each sector is determined. In each sector, these pairwise matches are randomly allocated. Once matched, the pair exit to the bargaining stage. Define $\theta_i \equiv \frac{v_i}{u}$ as market tightness in sector $i$ from the firm's perspective; $f_i(\theta_i) = \frac{m_i}{u}$ denotes an unemployed worker's job-finding probability in sector $i$;[14] and $q_i(\theta_i) = \frac{m_i}{v_i}$ denotes the job-filling probability for a sector $i$ vacancy, where $\sum_{i=1}^{I} f_i(\theta_i), q(\theta_i) \in [0, 1]$.

Each period some matches change states. All sector $i$ matches face exogenous probability $s_i$ of job destruction, where $s_i$ is the sector $i$ separation rate. Low-skill matches in sector $i$ become high-skill with probability $\lambda_i x_i$, where $\lambda_i$ is the exogenous skill arrival rate and $x_i$ is innovative investment.

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[11] Petrongolo and Pissarides (2001) survey the empirical literature on estimating matching functions. They conclude that existing evidence generally supports the Cobb-Douglas specification.

[12] U.S. data from the Job Openings and Labor Turnover Survey and recent research by Davis et al. (2007), for instance, find significant heterogeneity in vacancy-filling rates across sectors.

[13] See Julien et al. (2006) and Albrecht et al. (2006b) among others.

[14] The probability of matching in sector $i$ is the product of the probability of finding a job and the probability of that job being in sector $i$, $f_i = \frac{\sum m_i}{u} \times \frac{m_i}{\sum m_i} = \frac{m_i}{u}$.

## 2.5   Value Functions in the Steady-State

In the steady-state, the worker's Bellman equations are as follows. The expected present value of being unemployed, $U$, is:

$$U = z + \delta\left[\sum_{i=1}^{I} f_i(\theta_i)W_i^L + (1 - \sum_{i=1}^{I} f_i(\theta_i))U\right] \tag{1}$$

In the current period the worker receives unemployment benefits. With probability $f_i(\theta_i)$ the worker matches with a firm and receives an offer in sector $i$. In equilibrium she accepts all job offers,[15] and thus will begin next period working as a low-skill match — the present value of which is $W_i^L$. $\delta$ discounts next period's payoffs and the summation is over all sectors. With complementary probability the worker does not match and remains unemployed.

The expected present value of being a worker in a low-skill sector $i$ match, $W_i^L$, is:

$$W_i^L = w_i^L + \delta[s_iU + \lambda_i x_i W_i^H + (1 - s_i - \lambda_i x_i)W_i^L] \tag{2}$$

The current return is the low-skill wage in sector $i$. With probability $s_i$, the match separates and the worker becomes unemployed next period. With probability $\lambda_i x_i$, the match acquires skill and produces next period as high-skill. With complementary probability the worker keeps his current job.

The expected present value of working in a high-skill sector $i$ match, $W_i^H$, is:

$$W_i^H = w_i^H + \delta[s_iU + (1 - s_i)W_i^H] \tag{3}$$

The worker receives the high-skill wage in the current period. The job terminates with probability $s_i$, leaving the worker unemployed next period, otherwise the job continues.

The value functions for the firm are given by the following: The expected present value of posting a sector $i$ vacancy, $V_i$, is:

$$V_i = -c + \delta[q_i(\theta_i)J_i^L + (1 - q_i(\theta_i))V_i] \tag{4}$$

The firm incurs the recruiting cost in the current period. With probability $q_i(\theta_i)$, the job-filling rate, the firm matches with a worker and begins producing with a low-skill job next period, else the firm continues recruiting.

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[15]Section 2.9 derives the equilibrium wages, confirming this assertion.

The expected present value for a firm in a low-skill match in sector $i$, $J_i^L$, is:

$$J_i^L = \underbrace{A_i p_i^L - w_i^L - \chi(x_i)} + \delta[s_i V_i + \lambda_i x_i J_i^H + (1 - s_i - \lambda_i x_i) J_i^L] \tag{5}$$

The first term is the firm's current profit: the firm produces output $A_i p_i^L$, pays the worker wage $w_i^L$ and provides investment of $x_i$ at cost $\chi(x_i)$. The match separates with probability $s_i$, leaving the firm with a vacancy next period. With probability $\lambda_i x_i$, the match becomes high-skill next period. The expected present value for a firm in a high-skill match in sector $i$, $J_i^H$, is:

$$J_i^H = A_i p_i^H - w_i^H + \delta[s_i V_i + (1 - s_i) J_i^H] \tag{6}$$

The firm's current profit is its output less the wage since the firm no longer trains the worker. With probability $s_i$, the match terminates becoming a vacancy next period, otherwise high-skill production continues.

## 2.6 Wage Determination Through Bargaining

When unmatched firms and workers first meet, they begin producing next period in a low-skill match only if they agree on how to split the expected surplus from their partnership. This is done by generalized Nash Bargaining with full information where the threat points are the continuation values from no-agreement — which leaves the worker unemployed, with value $U$, and the firm with a vacancy, valued at $V_i$. Agreement allows production to begin in a low-skill match giving the worker $W_i^L$ and the firm $J_i^L$. Clearly, agreement requires a non-negative return for each agent, $W_i^L \geq U$ and $J_i^L \geq V_i$. The new match surplus, $S_i$, is what the pair gains from producing less what they give up, $S_i \equiv W_i^L - U + J_i^L - V_i$.

The wage paid each period to a worker in a low-skill match in sector $i$ is set efficiently to split the weighted product of worker's and firm's net gains from the match:

$$w_i^L = \arg\max[W_i^L(w_i^L) - U]^\beta [J_i^L(w_i^L) - V_i]^{1-\beta}$$

where $\beta$ is the worker's bargaining power and $\beta \in (0, 1)$ so both sides have incentive to produce. First-order conditions for this maximization imply:

$$W_i^L - U = \beta S_i; \quad J_i^L - V_i = (1 - \beta) S_i \tag{7}$$

Therefore, the low-skill wage in sector $i$, which I derive explicitly later, gives workers share $\beta$, and firms share $(1 - \beta)$, of the new match surplus.

If the match becomes high-skill, the pair once again splits the surplus via Nash Bargaining. The threat points are the values of continuing production as a low-skill match.[16] Define the sector $i$ skill premium, $SP_i$, as the incremental surplus generated when moving from a low to high-skill match, where $SP_i \equiv W_i^H - W_i^L + J_i^H - J_i^L$. Similarly, the high-skill wage in sector $i$ is:

$$w_i^H = \arg\max[W_i^H(w_i^H) - W_i^L]^\beta (J_i^H(w_i^H) - J_i^L)^{1-\beta}$$

The high-skill wage is set so the worker receives share $\beta$ of the skill premium and the firm receives the rest:

$$W_i^H - W_i^L = \beta SP_i; \quad J_i^H - J_i^L = (1 - \beta)SP_i \tag{8}$$

## 2.7 Equilibrium

**Definition:** Given a set of constant exogenous parameters, $\{A_i, p_i^L, p_i^H, s_i, \lambda_i, \mu_i, \alpha, c, r, z, \beta\}_{i=1}^I$, a *symmetric steady-state rational expectations equilibrium* is a set of value functions $\{U, W_i^L, W_i^H, V_i, J_i^L, J_i^H\}_{i=1}^I$; transition probabilities $\{f_i(\theta_i)\}_{i=1}^I$, $\{q_i(\theta_i)\}_{i=1}^I$; wages $\{w_i^L, w_i^H\}_{i=1}^I$; innovation policies $\{x_i\}_{i=1}^I$ and labour $\{e_i^L, e_i^H, u\}_{i=1}^I$, such that, in all sectors:

1. **Optimality**:

   (a) Taking job-filling probabilities and wages as given, firms maximize expected profit.

   (b) Taking job-finding probabilities and wages as given, workers maximize expected income.

2. **Free Entry and Exit of Vacancies:** In all sectors, zero profit conditions hold for the expected value of posting a vacancy (net of recruiting costs).

3. **Nash Bargaining:** Generalized Nash Bargaining splits the low and high-skill match surpluses.

4. **Rational Expectations:** Firms and workers correctly anticipate transition probabilities, wages and innovation investment.

5. **Stationary Labour Distribution:** There is a stationary distribution of workers over employment states.

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[16]Since both agents strictly prefer participating in a low-skill match to being unmatched in equilibrium, threats to 'endogenously' separate the match by either side are not credible.

A stationary distribution of labour requires that in sector $i$ the flow into unemployment equals the flow out. Also, the flow of workers into high-skill sector $i$ matches equals the flow out. The labour force must sum to one, the total measure of potential workers. These conditions are:

$$s_i(e_i^L + e_i^H) = f_i(\theta_i)u; \quad \lambda_i x_i e_i^L = s_i e_i^H; \quad \sum_{i=1}^{I}(e_i^L + e_i^H) + u = 1 \tag{9}$$

An equilibrium solves for $\{x_i^*, \theta_i^*, w_i^{L*}, w_i^{H*}, e_i^{L*}, e_i^{H*}, u^*\}_{i=1}^I$. A representative firm in each sector makes two crucial decisions which drive the results. When unmatched, firms decide whether to post a vacancy; and once in a low-skill match, firms decide how much innovation to undertake. While these actions are sequential, in equilibrium, firms correctly anticipate the innovation policies offered once a meeting occurs. Since, the firm's vacancy posting decision takes into account the innovation decision, I discuss the innovation decision first.

## 2.8 Intra-Sectoral Labour Reallocation: The Innovation Effect

Firms in low-skill matches in sector $i$ optimally choose their innovation policies taking as given wages, the skill arrival rate, and other firms' innovation decisions:

$$J_i^L = \max_{0 \leq x_i \leq 1} A_i p_i^L - w_i^L - \chi(x_i) + \delta[s_i V_i + \lambda_i x_i J_i^H + (1 - s_i - \lambda_i x_i)J_i^L]$$

The first order condition for an interior solution is (the Appendix considers corner solutions):

$$\chi'(x_i^*) = \delta\lambda_i(J_i^H - J_i^L)$$
$$= \delta\lambda_i(1 - \beta)SP_i$$

The LHS is the marginal cost and the RHS is the expected discounted marginal benefit of increasing innovation. The second equality uses the Nash Bargaining solution, equation (8). The benefit of innovating is the increase in the arrival rate $\lambda_i$, multiplied by the firm's share $(1 - \beta)$ of the skill premium — the increased production from becoming a high-skill match plus the foregone investment costs, because high-skill matches require no further investment.

**Proposition 2.1 (Optimal innovation policies)** *When the innovation investment cost function is linear, $\chi(x_i) = k_i x_i$, a threshold skill arrival rate, $\overline{\lambda}_i$, characterizes firms' innovation decisions. The optimal symmetric innovation policy in sector $i$ is:*[17]

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[17]I consider only the symmetric innovation equilibrium. Equilibria may exist where some firms in a sector offer lower starting wages, but innovate more, or higher wages and innovate less.

$$x_i^* = \begin{cases} 0 & \text{if } \lambda_i \leq \overline{\lambda_i} \\ \min\{\frac{(1-\beta)}{k_i\beta}A_i(p_i^H - p_i^L) - \frac{r+s_i}{\lambda_i\beta}, 1\} & \text{if } \lambda_i > \overline{\lambda_i} \end{cases} \qquad (10)$$

where: $\overline{\lambda_i} = \frac{k_i(r+s_i)}{(1-\beta)A_i(p_i^H - p_i^L)}$

Firms innovate only if the skill arrival rate is sufficiently high, $\lambda_i > \overline{\lambda_i}$. Innovation is increasing in the skill arrival rate and the difference between high and low-skill productivity. Innovation is also increasing in the sector-specific productivity shock, $A_i$. Therefore, when a sector's productivity rises, firms innovate more. These actions accelerate skill acquisition and endogenously increase the share of high-skill matches in the sector. As a result, the output response to the productivity shock is amplified relative to the baseline model.[18] I call this the *'innovation effect'*.

Conversely, higher interest rates and separation rates reduce innovation. In both cases firms discount future payoffs more — because borrowing funds is more costly or because jobs are shorter-lived — so the return to innovating falls. Similarly, as worker's bargaining power, $\beta$, increases, firms receive less of the skill premium and therefore innovate less.

Finally, note a few important factors that do not affect the innovation decision. In particular, innovation does not depend on market tightness and unemployment, so the availability of new workers is irrelevant for the decision to innovate with existing workers. The firm's innovation decision simply compares the benefit from moving an existing low-skill match to high-skill, against its cost. In other words, the firm's entry decision (pre-match) does not directly influence its innovation decision (post-match), because of the timing of events. This fact simplifies solving the model.

## 2.9 Inter-Sectoral Labour Reallocation: The Reservation Wage Effect

Now consider the firm's entry decision of whether to post a vacancy. In equilibrium, free entry drives the expected value of posting a vacancy to zero, $V_i = 0$, which implies:

$$\frac{c}{q_i(\theta_i^*)} = \frac{\pi_i^L}{(r + s_i + \lambda_i x_i^*)} + \lambda_i x_i^* \frac{\pi_i^H}{(r + s_i + \lambda_i x_i^*)(r + s_i)} \qquad (11)$$

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[18]In the steady-state, from equation (9), in sector $i$ the flow of workers into high-skill jobs equals the flow out: $\lambda_i x_i e_i^L = s_i e_i^H$. Rearranging: $x_i = \frac{s_i}{\lambda_i} \frac{e_i^H}{e_i^L}$. Since the first fraction is a constant, increasing investment raises the steady-state ratio of high-to-low skill matches in sector $i$.

where $\pi$ is current period profit. The LHS is the total expected recruiting cost: the per-period cost, $c$, times the expected number of periods to fill the vacancy, $\frac{1}{q_i(\theta_i^*)}$. The RHS is the expected discounted accounting profits earned in a match. Notice this anticipates the expected gain in value if the match becomes high-skill, which occurs with probability $\lambda_i x_i^*$, when innovation is optimal. In this way, the innovation decision influences the entry decision.

I derive equilibrium wages using the value functions, equations (1) - (6), the Nash bargaining solutions, equations (7) and (8), and the zero profit conditions $V_i = 0 \;\; \forall i$, giving:

$$w_i^{L*} = \overline{w} + \beta(A_i p_i^L - \chi(x_i^*) - \overline{w}) \tag{12}$$

$$w_i^{H*} = \overline{w} + \beta(A_i p_i^H - \overline{w}) \tag{13}$$

where $\overline{w} \equiv z + \delta \sum_{i=1}^I f_i(\theta_i^*)(W_i^L - U)$

Workers receive their reservation wage, $\overline{w}$, plus their bargaining power share $\beta$ of the low and high-skill per-period match values respectively. The reservation wage is the worker's outside option — the value of continuing to search while unemployed, or equivalently, what the worker foregoes by accepting the job (since there is no on-the-job search). The option value of search is the unemployment benefits the worker would collect, $z$, plus the expected gain in value from accepting a job in a given sector, $(W_i^L - U)$, weighted by the probabilities of receiving offers in these sectors, $f_i(\theta_i)$, summed over all sectors and discounted because production begins next period.

A key difference relative to the basic one-sector model, is that with multisector search, the outside option includes the possibility of working in other sectors. As a result, the worker's reservation wage updates when market conditions change in other sectors. Sectoral spillovers occur through this feature of the model, which effectively creates equilibrium linkages in labour market conditions across different sectors.

In addition to receiving their reservation wage, workers also get their share $\beta$ of the joint match value, thus verifying the earlier assertion for the value functions that workers accept all wage offers in equilibrium. The joint match value for low-skill matches is the output generated, $A_i p_i^L$, less investment costs, $\chi(x_i^*)$, less the worker's opportunity cost of search, $\overline{w}$. High-skill matches are more valuable, since more output is produced, $A_i p_i^H$, and there are no investment costs. As a result, high-skill wages exceed low-skill wages.[19] Low-skill wages are

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[19]$w_i^{H*} - w_i^{L*} = \beta[A_i(p_i^H - p_i^L) + \chi(x_i^*)] > 0.$

decreasing in investment costs. Wages are increasing in the value of search and output.

Equilibrium profits are:

$$\pi_i^{L*} = (1 - \beta)(A_i p_i^L - \chi(x_i^*) - \overline{w}) \tag{14}$$

$$\pi_i^{H*} = (1 - \beta)(A_i p_i^H - \overline{w}) \tag{15}$$

Firms receive their bargaining share, $(1 - \beta)$, of the per-period match surplus in low and high-skill matches respectively. Substituting equilibrium profits into the vacancy posting equation (11), illustrates that increasing the worker's value of search, $\overline{w}$, discourages entry:

$$\frac{c}{q_i(\theta_i^*)} = \frac{(1 - \beta)(A_i p_i^L - \chi(x_i^*) - \overline{w})}{(r + s_i + \lambda_i x_i^*)} + \lambda_i x_i^* \frac{(1 - \beta)(A_i p_i^H - \overline{w})}{(r + s_i + \lambda_i x_i^*)(r + s_i)} \tag{16}$$

This *'reservation wage effect'* leads to sectoral spillovers. For example, positive developments in one sector raise workers' reservation wage. As wages are bid up, labour becomes more expensive, new jobs become less profitable, and job creation falls in other sectors. This intuition is formalized in the following proposition:

**Proposition 2.2 (Sector-Specific Shocks and Equilibrium Market Tightness)** *A positive sector-specific productivity shock in sector $i$, $A_i$, causes equilibrium market tightness to rise in sector $i$, $\theta_i^*$, and fall in the other sectors, $\{\theta_j^*\}_{j \neq i}^I$. Conversely, a negative shock in sector $i$, reduces market tightness in that sector and increases market tightness in the other sectors.*

Finally, I give a break-even condition for a sector to engage in recruitment and production:

**Proposition 2.3 (Necessary Condition for Sector $i$ Production)** *Production requires a non-negative new match surplus, $S_i \geq 0$. This implies the value of low-skill output net of investment costs, plus the expected present value of the skill premium, must weakly exceed the worker's value of search, otherwise production in sector $i$ is not worthwhile:*

$$A_i p_i^L - \chi(x_i^*) + \delta \lambda_i x_i^* SP_i \geq \overline{w}$$

## 2.10   Solving the Model

The model is solved in stages. First, I find the optimal innovation policies, $\{x_i^*\}_{i=1}^I$, using equations (10). As described above, these solutions are independent of market tightness. Given these innovation policies, I solve for equilibrium market tightness, $\{\theta_i^*\}_{i=1}^I$, using equations (17) below. A key feature of the model is the interdependence of labour market conditions. For example, the decision to post a vacancy in sector $i$ depends on the expected ease

of finding a worker, which in turn, depends on the vacancy posting decisions made in other sectors. The model must therefore be solved simultaneously. Fortunately, the model can be distilled into the following system of $I$ simultaneous non-linear equations in $\{\theta_i\}_{i=1}^I$:

$$\frac{r+s_i}{q(\theta_i)} + \beta \sum_{i=1}^I \theta_i = \frac{(1-\beta)}{c}[A_i p_i^L - \chi(x_i^*) - z + \lambda_i x_i^* \cdot \frac{A_i(p_i^H - p_i^L) + \chi(x_i^*)}{r + s_i + \lambda_i x_i^*}] \qquad (17)$$

This expression provides a straight-forward generalization of the basic one-sector model without aggregate uncertainty and innovation investment (e.g., Shimer (2005) equation 6):

$$\frac{r+s}{q(\theta)} + \beta\theta = \frac{(1-\beta)}{c}(p-z)$$

Solving the system given by (17) yields equilibrium market tightness. Equilibrium wages, profits and employment shares are found using equations (9) and (12) — (15). Finally, substituting equilibrium expressions into Proposition 2.3 provides a threshold low-skill output level for production, $\overline{y_i}$, to verify a sector's viability after a productivity shock.

# 3 Transition Dynamics

The previous section establishes the model's steady-state properties. A fully-specified model of labour adjustment must detail how the economy adjusts when it is out of the steady-state. Therefore, this section characterizes the model's transition dynamics between steady-states.

To illustrate, assume the economy is in a steady-state and consider an unanticipated sector-specific productivity shock, denoted $\hat{A}_{i,t}$, that occurs in sector $i$, at the beginning of period $t$, where the hat superscript denotes an updated value. As in the baseline Pissarides (2000) model, labour contracts are costlessly renegotiated whenever shocks hit the economy. Therefore, prior to production in period $t$, existing matches renegotiate low and high-skill wages using the Nash bargaining solutions described above and firms update their innovation policies. In addition, prior to recruitment, unmatched firms optimally update their vacancy decisions. Because there is free entry and free disposal of vacancies, the value of a vacancy is zero for all sectors at all points in time. In Pissarides' terminology, wages, innovation investment and market tightness (vacancies) are 'jump variables' updating immediately in the period the shock hits, prior to production and search. Their new values are:

$$\hat{x}_{i,t}^* = \begin{cases} 0 & \text{if } \lambda_i \leq \hat{\overline{\lambda}}_i \\ \min\{\frac{(1-\beta)}{k_i\beta}\hat{A}_{i,t}(p_i^H - p_i^L) - \frac{r+s_i}{\lambda_i\beta}, 1\} & \text{if } \lambda_i > \hat{\overline{\lambda}}_i \end{cases}$$

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$$\frac{1+r}{q(\hat{\theta}_{i,t}^*)} = \frac{(1-\beta)}{c}[\hat{A}_{i,t}p_i^L - \chi(\hat{x}_{i,t}^*) - z + \lambda_i \hat{x}_{i,t}^* \cdot \frac{\hat{A}_{i,t}(p_i^H - p_i^L) + \chi(\hat{x}_{i,t}^*)}{r + s_i + \lambda_i \hat{x}_{i,t}^*}] + E_{\hat{t}}\{\frac{1-s_i}{q(\hat{\theta}_{i,t+1}^*)} - \beta \sum_{i=1}^{I} \hat{\theta}_{i,t+1}^*\}$$

$$\hat{w}_{i,t}^{L*} = \bar{\hat{w}}_t + \beta(\hat{A}_{i,t}p_i^L - \chi(\hat{x}_{i,t}^*) - \bar{\hat{w}}_t); \qquad \hat{w}_{i,t}^{H*} = \bar{\hat{w}}_t + \beta(\hat{A}_{i,t}p_i^H - \bar{\hat{w}}_t)$$

where $\bar{\hat{\lambda}}_i = \frac{k_i(r+s_i)}{(1-\beta)\hat{A}_{i,t}(p_i^H - p_i^L)}$ and $\bar{\hat{w}}_t = z + \delta E_{\hat{t}}\{\sum_{i=1}^{I} f_i(\hat{\theta}_{i,t}^*)(\hat{W}_{i,t}^L - \hat{U}_t)\}$

Notice these variables can jump to their new values because they do not depend directly on employment and unemployment levels. Given these new wages and equilibrium transition probabilities, the value functions also discretely update in period $t$. For example, in period $t$ prior to the shock, the present value of being unemployed is:

$$U_t = z + \delta E_t[\sum_{i=1}^{I} f_i(\theta_{i,t+1}^*)(W_{i,t+1}^L - U_{t+1}) + U_{t+1}]$$

After the shock in period $t$, the value of unemployment updates immediately to:

$$\hat{U}_t = z + \delta E_{\hat{t}}[\sum_{i=1}^{I} f_i(\hat{\theta}_{i,t+1}^*)(\hat{W}_{i,t+1}^L - \hat{U}_{t+1}) + \hat{U}_{t+1}]$$

Similarly, the other value functions update to:

$$\hat{W}_{i,t}^L = \hat{w}_{i,t}^{L*} + \delta E_{\hat{t}}[s_i(\hat{U}_{t+1} - \hat{W}_{i,t+1}^L) + \lambda_i \hat{x}_{i,t+1}^*(\hat{W}_{i,t+1}^H - \hat{W}_{i,t+1}^L) + \hat{W}_{i,t+1}^L]$$

$$\hat{W}_{i,t}^H = \hat{w}_{i,t}^{H*} + \delta E_{\hat{t}}[s_i(\hat{U}_{t+1} - \hat{W}_{i,t+1}^H) + \hat{W}_{i,t+1}^H]$$

$$\hat{V}_{i,t} = -c + \delta E_{\hat{t}}[q_i(\hat{\theta}_{i,t+1}^*)(\hat{J}_{i,t+1}^L - \hat{V}_{i,t+1}) + \hat{V}_{i,t+1}]$$

$$\hat{J}_{i,t}^L = \hat{A}_{i,t}p_i^L - \hat{w}_{i,t}^{L*} - \chi(\hat{x}_{i,t}^*) + \delta E_{\hat{t}}[s_i(\hat{V}_{i,t+1} - \hat{J}_{i,t+1}^L) + \lambda_i \hat{x}_{i,t+1}^*(\hat{J}_{i,t+1}^H - \hat{J}_{i,t+1}^L) + \hat{J}_{i,t+1}^L]$$

$$\hat{J}_{i,t}^H = \hat{A}_{i,t}p_i^H - \hat{w}_{i,t}^{H*} + \delta E_{\hat{t}}[s_i(\hat{V}_{i,t+1} - \hat{J}_{i,t+1}^H) + \hat{J}_{i,t+1}^H]$$

Free entry and exit of vacancies imply $\hat{V}_{i,t} = \hat{V}_{i,t+1} = 0$. Nash Bargaining implies $\hat{J}_{i,t}^L = (1-\beta)\hat{S}_{i,t}$ and $(\hat{W}_{i,t}^L - \hat{U}_{i,t}) = \beta \hat{S}_{i,t}$, so one can succinctly write the updated joint value of a low-skill match in sector $i$ as $\hat{S}_{i,t} = \frac{c}{(1-\beta)\delta q_i(\hat{\theta}_{i,t}^*)}$ or equivalently:

$$\hat{S}_{i,t} = \hat{A}_{i,t}p_i^L - \chi(\hat{x}_{i,t}^*) - z + \delta E_{\hat{t}}\{\lambda_i \hat{x}_{i,t+1}^* \hat{SP}_{i,t+1} + [1 - s_i - f_i(\hat{\theta}_{i,t+1}^*)]\hat{S}_{i,t+1} - \sum_{j \neq i} f_j(\hat{\theta}_{j,t+1}^*)\hat{S}_{j,t+1}\}$$

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Other variables, such as employment and unemployment, evolve more slowly to their new steady-state values according to the following difference equations:

$$\hat{e}_{i,t+1}^{L} = f_i(\hat{\theta}_{i,t}^*)u_t + (1 - s_i - \lambda_i\hat{x}_i^*)e_{i,t}^{L}$$

$$\hat{e}_{i,t+1}^{H} = \lambda_i\hat{x}_i^* e_{i,t}^{L} + (1 - s_i)e_{i,t}^{H}$$

$$\hat{u}_{t+1} = \sum_{i=1}^{I} s_i(e_{i,t}^{L} + e_{i,t}^{H}) + [1 - \sum_{i=1}^{I} f_i(\hat{\theta}_{i,t}^*)]u_t$$

Finally, output moves along with changes in employment during the transition:

$$\hat{Y}_t = \sum_{i=1}^{I} \sum_{SK=L}^{H} \hat{A}_{i,t} p_i^{SK} e_{i,t}^{SK}$$

A stable transition requires that each sector's market tightness updates immediately to its new steady-state value, $\hat{\theta}_{i,t}^*$. However, since market tightness is $\theta_{i,t} \equiv \frac{v_{i,t}}{u_t}$, vacancies overshoot their steady-state level and move in the same direction as unemployment so that market tightness remains constant at its new steady-state value during the transition. See Pissarides (1985) or (2000, Ch. 1.7).

# 4    General versus Sector-Specific Productivity Shocks

The model results suggest that when a sector's productivity increases their firms innovate, investing resources to create more skilled jobs, resulting in intra-sectoral labour reallocation (Proposition 2.1). Furthermore, when relative productivity changes across sectors, this causes equilibrium wage and recruiting spillovers, resulting in inter-sectoral labour reallocation (Proposition 2.2). This section presents simple quantitative examples to illustrate these model mechanisms through the innovation and reservation wage effects.

## 4.1    Quantitative Approach

I compare the model economy's response to an equal-sized productivity shock in two scenarios. The first scenario is a general shock that affects all sectors equally. As a result, there is intra-sectoral labour reallocation but not inter-sectoral reallocation in the model's new steady-state. The second scenario is a sector-specific shock, which *directly* affects only one sector. This results in intra-sectoral reallocation in the sector where the shock occurs as well as inter-sectoral reallocation between sectors.

To keep the results transparent and emphasize the model's adjustment mechanisms, I parameterize a benchmark economy consisting of two perfectly symmetric sectors. Each sector uses the same production technologies and each has half of the economy's employed workers, of which half are in low-skill and half are in high-skill matches. Table 1 reports the parameter values for the benchmark model. In these examples, the only parameters that change are the sector-specific productivity terms, $A_1$ and $A_2$.

To quantify a reasonable size for the productivity shocks, Table 2 reports summary statistics using Canadian data for sectoral and aggregate output per worker, expressed in log deviations from their HP-filtered trends.[20] The table shows that productivity is considerably more volatile at the sectoral level than the aggregate level. In the resource and manufacturing sectors, productivity is often 3-4 percent or more away from its trend growth. Furthermore, these deviations from trend are quite persistent with autocorrelations of 0.86 and higher. In the numerical example, I use 3 percent for the sector-specific shock. The equivalent-sized general productivity shock in the two-sector economy is 1.5 percent, since the 3 percent shock directly affects half of the economy. I assume the shock is unanticipated and permanent.

## 4.2   Quantitative Results

Table 3 compares the results in the new steady-states following the general productivity shock to the equal-sized, sector-specific productivity shock. While the overall differences for social welfare are small, there are important distinctions for the sectoral and skill composition of production, aggregate productivity and the wage distributions.

First, consider the model economy's response to the general productivity shock. This case isolates the innovation effect and demonstrates that firms' endogenous innovation responses amplify the impacts of productivity shocks. The productivity shock was an increase of 1.5 percent, however, aggregate output rises by 2.4 percent because the economy invests more resources in innovation to substitute toward high-skill production (whose share of overall production increases from 50 percent to 51.2 percent after the shock).

The economy's response to the sector-specific productivity shock is quite different due to the asymmetric nature of the shock. The sector-specific shock raises aggregate output and output per worker more than the general shock (2.7 percent rather than 2.4 percent). The reason is that the economy concentrates production in high-skill jobs in the more productive

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[20]Output per worker proxies productivity here because labour is the only factor of production in the model.

sector, through inter-sectoral and intra-sectoral labour reallocation. While the shock directly affects Sector 1, there are negative equilibrium wage and hiring spillovers on Sector 2 through the reservation wage effect. This result is consistent with recent empirical findings by Beaudry et al. (2007) who show that changes in the sectoral composition in U.S. cities have equilibrium spillovers on the level of wages, after controlling for observable characteristics.

The mechanism here works as follows: Firms post vacancies and increase investment in Sector 1 to take advantage of the now-more-productive workers. The increase in Sector 1 vacancies changes the composition of job postings, making matches with Sector 1 firms more likely. These firms are now more productive and invest more resources in becoming high-skill to take advantage of the improved productivity. Therefore, workers in Sector 1 generally receive higher starting wages and also expect to earn high-skill wages sooner because, on average, they will acquire skills faster in this sector. The value of search for the unemployed rises because of the improved probability of getting these better paying jobs, pushing up the reservation wage.

The increase in the reservation wage has second-round equilibrium effects. Wages are re-bargained in Sector 2 to reflect workers' improved outside option. With more expensive labour in Sector 2 and no change in the productivity of their workers, these jobs become less profitable so recruiting falls in this sector. Thus in the new steady-state, the asymmetric recruiting responses — vacancies rise in Sector 1 and fall in Sector 2 — lead to inter-sectoral reallocation, shifting labour into the more productive sector. These productivity-enhancing labour movements between sectors are re-enforced by the shift within the more productive sector to high-skill matches due to a larger innovation effect after the sector-specific shock.

Finally, the sector-specific shock has larger distributional consequences for wages. Relative to the general shock scenario, high-skill workers in Sector 1 are the major winners and high-skill workers in Sector 2 are the major losers (as wages rise by 1 percent and fall by 0.9 percent respectively).

Theses effects are steady-state comparisons. Figure 3 shows the transition dynamics to illustrate the sectoral employment responses. After the sector-specific shock, the composition of vacancies shifts immediately and a larger proportion of new hires work in Sector 1 each period. Over time, employment rises in Sector 1 and falls in Sector 2.

# 5 Facts from Sectoral Labour Adjustment Episodes

The model describes a general process of sectoral labour adjustment driven by changes in relative productivities and profitability between and within sectors. This section demonstrates a key benefit of this general framework — that the model's predictions are broadly consistent with the sectoral labour adjustments experienced in several countries that occurred for disparate reasons. I summarize three important elements of these adjustments regarding inter-sectoral and intra-sectoral labour reallocation and relative wage effects between low and high-skill workers.

## 5.1 Summarizing The Facts

This section presents some new evidence and draws on existing findings for clearly-defined events related to persistent relative price shocks (energy prices and exchange rates) and trade liberalization as well as broader technological change. The key characteristic these events share, and which the model captures quite well, is that they can change a sector's production possibilities and the relative profitability between and within sectors. For example, in sectors where production possibilities expand and become more profitable, there is increased entry of new firms, increased employment in the sector and firms undertake costly productivity-enhancing investments to capture the new profit opportunities.

To be more concrete, consider the following examples. Both a reduction in trade barriers or a rapid exchange rate depreciation effectively improve market access for exporters. They respond to these new profit opportunities by entering and undertaking investments to improve their productivity. Similarly, a large increase in energy prices makes resource sector jobs more profitable, spurring new investments and employment in the sector. At the same time, energy input costs rise in the manufacturing sector reducing profitability and leading to labour movements to other sectors. Another example is improvements in computing technologies. Such improvements disproportionately benefits information-intensive sectors, and because they increase the relative productivity differences between low and high-skill workers, firms invest in these new technologies and increase the employment share of high-skill workers.

**Fact 1:** *Inter-Sectoral Labour Adjustment*

Consider the case of energy price shocks. Figure 4, reproduced from Blanchard and

Gali (2007), identifies four oil price shocks: 1973, 1979, 1999, and 2002.[21] These shocks are a particularly convenient way to investigate inter-sectoral labour reallocation, because they are relatively discrete episodes with some persistence. In addition, these shocks can reasonably be treated as unanticipated and exogenous from the point of view of the economies I study. I analyze internationally-comparable employment data for the G7 countries (Canada, France, Germany, Italy, Japan, U.K. and U.S.), from the OECD's Structural Analysis Database.

Figure 5 shows that there is an asymmetric negative impact on manufacturing employment following oil price shocks. I use the dates identified by Blanchard and Gali (2007) and normalize employment to 100 at each shock, so the relative changes are comparable. The reported results are averaged over the three shocks, since the same trends occur in each episode (1973, 1979, and 1999).[22] In the four years following the oil shocks, there was a substantial drop in manufacturing employment, which fell by an average of 7.6 percent. Figure 6 disaggregates the employment dynamics for each country's manufacturing sector before and after the oil price shocks. This drop occurred in all countries except Italy, where employment rose a mere 0.6 percent.[23]

Conversely, Figure 7 shows that in the four years after the shocks, non-manufacturing employment continued to grow in all economies, at or only slightly below trend. Not surprisingly, while there is a general increase in employment in the non-manufacturing sectors, using more detailed data, reveals that the largest employment gains occur in the resource sector. Figure 8 shows the average response in the U.S. economy after the four oil price shocks. Figure 9 shows the particularly dramatic response in Canada during the most recent oil price shock, which Tapp (2007) studies in detail.

These empirical findings are consistent with the model's response in Section 4 for the sector-specific productivity shocks (comparing Figures 3 and 5) because the increase in the price of oil raises the profitability of non-manufacturing relative to manufacturing production. Given that manufacturing production is more energy-intensive, its production costs are more adversely affected.[24]

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[21]They define a shock as an increase in the real oil price of more than 50 percent which persists longer than four quarters (where the real price is the West Texas Intermediate price deflated by the U.S. GDP deflator).

[22]Due to the lag in reporting internationally-comparable employment data, the results for the 2002 shock are not yet available.

[23]There are likely two effects at play here. First, manufacturers are the most energy-intensive producers so their input costs increase more than in other sectors. Second, there is generally an endogenous monetary policy response which raises interest rates to fight the inflationary impacts of the oil price shocks. Manufacturers and are more sensitive to interest rates as their sales are often financed by borrowing.

[24]This is consistent with findings by Davis and Haltiwanger (2001) who analyze plant-level data within the

**Facts 2 and 3:** *Intra-Sectoral Labour Adjustment; and Relative Wage Gains for High-Skill Workers*

Not only are there movements of workers between sectors, but often there is a shift from low to high-skill workers within sectors that become relatively more profitable.

Keane and Prasad (1996) find such a shift following rising oil prices, as the relative employment and wages of high-skill workers rose. These results are found using three proxies for 'high-skill': job tenure; labour force experience; and those with a college degree. They use individual-level panel data from the National Longitudinal Survey of Youth covering 1966–1981 and control for individual fixed effects and sample selection bias. Tapp (2007) also finds that wage gains were concentrated in the upper end of the distribution in the Canadian resource sector following the most recent global commodity price shock (Figure 10).

Verhoogen (2007) studies another important relative price change: the exchange rate. In 1994, a rapid depreciation of the Mexican peso expanded opportunities for exporters. Firms responded by increasing the quality of goods produced to export abroad. This, in turn, resulted in a relative increase in employment and wages of high-skilled workers in the Mexican manufacturing sector.

Other research in the international trade context provides similar results of so-called, skill-upgrading. Using detailed plant-level data, Trefler (2004) finds a relative increase in the employment of high-skill relative to low-skill workers in Canadian manufacturing industries following the Canada-U.S. Free Trade Agreement. For this episode, the relative employment shift to high-skill workers is associated with investments that increased productivity within plants, particularly for those that entered export markets after trade liberalization (Lileeva and Trefler, 2007; and Lileeva, 2007). Several other recent papers for a variety of countries suggest that trade liberalization increases firms' incentives to invest in productivity-enhancing investments (e.g. on-the-job training, R&D, technological adoption) and raises productivity within plants.[25]

Finally, similar responses occurred with technological changes from computerization and general R&D. There were considerable intra-sector employment shifts towards high-skill labour. These effects were largest in computer-intensive industries, particularly after 1970 (Autor, Katz and Kreuger, 1998). And in the U.K., Haskel and Hayden (1999) find most of

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manufacturing sector. They find larger employment reductions in more energy-intensive plants following oil price increases.

[25]See Costantini and Melitz (2007); Aw et al. (2007); and Bustos (2005).

the aggregate skill upgrading was due to employing more skilled workers within continuing establishments and was related to computer usage. Similar results hold in the manufacturing sector and are correlated with computer and R&D investment (Berman, Bound, and Griliches, 1994). Finally, Machin and Reenen (1998) link the within-industry increases in the proportion of skilled-workers in several OECD countries to broader technological change through R&D intensity.

The model's predictions are consistent with these facts. As Section 4 shows, increased productivity leads to increased innovation investment and larger wage gains for high-skill workers in the affected sectors. It is straight-forward to show these results analytically.

For the intra-sectoral reallocation result: In the steady-state, from equation (9), in sector $i$ the flow of workers into high-skill jobs equals the flow out: $\lambda_i x_i^* e_i^{L*} = s_i e_i^{H*}$. Rearranging: $x_i^* = \frac{s_i}{\lambda_i} \frac{e_i^{H*}}{e_i^{L*}}$. By Proposition 2.1, innovation investment (the LHS) increases with a sector's productivity. Therefore, since the first fraction is constant, productivity increases investment which, in turn, raises the steady-state ratio of high-to-low skill workers in sector $i$.

In addition, the relative wages of high-skill to low-skill workers rise as productivity increases. The wage differential can be expressed as: $w_i^{H*} - w_i^{L*} = \beta[A_i(p_i^H - p_i^L) + \chi(x_i^*)] > 0$. This expression is directly increasing in a sector's productivity $A_i$, which, in turn, increases innovation costs, $\chi(x_i)$, and causes further wage dispersion.

# 6  Conclusions

This paper presented a general model of sectoral labour reallocation. I demonstrated that the model's implications are consistent with the results from several labour adjustment episodes. The analysis, therefore, suggests that the widely-used search and matching framework is well-suited to tackle, not only the aggregate and distributional issues to which it is generally applied, but also to study issues at the sectoral level such as labour reallocation.

The model's transition dynamics are quite tractable, which facilitates taking the model to the data to study particular labour adjustment episodes. In a related paper, I apply this model to the data to quantify the aggregate costs of labour adjustment in the Canadian economy following a global commodity price shock and analyze how labour market policies affect social welfare, allocations and the speed of adjustment (Tapp, 2007).

There are several potentially interesting extensions to the model, such as comparing the role of general, sector-specific and match-specific skills and adding physical capital.

# Appendix

## A   Tables

Table 1: **Parameter Values for the Benchmark Model**

| Variable | Parameter | Value | Rationale |
|---|---|---|---|
| Real Interest Rate | $r$ | 0.33% | 4 percent annual |
| Discount Factor | $\delta$ | 0.997 | $\delta = \frac{1}{1+r}$ |
| Separation Rate, Sector 1 | $s_1$ | 3.4% | Shimer (2005) |
| Separation Rate, Sector 2 | $s_2$ | 3.4% | Shimer (2005) |
| Low-Skill Productivity, Sector 1 | $p_1^L$ | 1.0 | |
| Low-Skill Productivity, Sector 2 | $p_2^L$ | 1.0 | |
| High-Skill Productivity, Sector 1 | $p_1^H$ | 2.0 | |
| High-Skill Productivity, Sector 2 | $p_2^H$ | 2.0 | |
| Recruiting Cost | $c$ | 0.1 | Tapp (2007) |
| Investment Cost Scale, Sector 1 | $k_1$ | 1.0 | |
| Investment Cost Scale, Sector 2 | $k_2$ | 1.0 | |
| Productivity Shock, Sector 1 | $A_1$ | 1.0 | Steady-State |
| Productivity Shock, Sector 2 | $A_2$ | 1.0 | Steady-State |
| Matching Fx. Scale, Sector 1 | $\mu_1$ | 0.13 | Sector 1 Employment Share $= \frac{1}{2}$ |
| Matching Fx. Scale, Sector 2 | $\mu_2$ | 0.13 | Sector 2 Employment Share $= \frac{1}{2}$ |
| Skill Arrival Rate, Sector 1 | $\lambda_1$ | 0.11 | High-Skill Employment Share $= \frac{1}{2}$ |
| Skill Arrival Rate, Sector 2 | $\lambda_2$ | 0.11 | High-Skill Employment Share $= \frac{1}{2}$ |
| Unemployment Income | $z$ | 0.6 | 40% Replacement Rate $= 0.4(\frac{p_1^L + p_2^L}{2})$ |
| Matching Function Elasticity | $\alpha$ | 0.6 | Petrongolo and Pissarides (2001) |
| Workers' Bargaining Power | $\beta$ | 0.5 | Equal split of surplus |
| Model Period = 1 Month | | | |

Table 2: **Sectoral and Aggregate Output per Worker, Summary Statistics, Canada, 1987Q1–2001Q4**

|  | Resources | Manufacturing | Aggregate Economy |
|---|---|---|---|
| Standard Deviation | 0.043 | 0.030 | 0.012 |
| Quarterly Autocorrelation | 0.86 | 0.89 | 0.95 |
| Correlation with Resources | 1 | 0.52 | 0.30 |
| Correlation with Manufacturing |  | 1 | 0.53 |
| Correlation with Aggregate Economy |  |  | 1 |

Note: All variables are reported in logs as deviations from an HP trend with smoothing parameter of $10^5$. The results obtained using a smoothing parameter of 1600 are similar. All data are from Cansim. Aggregate, Resource and Manufacturing output series are seasonally adjusted at annual rates expressed in 1997 constant dollars: v2036138; v2036146; and v2036171. Aggregate, Resource and Manufacturing Employment series are: v13682073; v13682076; and v13682079.

Table 3: **Steady-State Impacts of General versus Sector-Specific Shocks**

| Aggregate Impacts | Benchmark | General Shock | Sector-Specific Shock | Sector-Specific Effect |
|---|---|---|---|---|
| Social Welfare (a+b-c-d) | 100 | 102.3 | 102.5 | 0.2 |
| Social Net Production (a-c-d) | 100 | 102.4 | 102.6 | 0.1 |
| a) Output | 100 | 102.4 | 102.7 | 0.2 |
| b) Unemployment Benefits | 100 | 98.4 | 100.6 | 2.2 |
| c) Innovation Investment Costs | 100 | 102.5 | 102.7 | 0.2 |
| d) Recruiting Costs | 100 | 102.8 | 105.5 | 2.7 |
| Employment | 100 | 100.1 | 100.0 | -0.2 |
| % High-Skill | 50.0 | 51.2 | 51.4 | 0.2 |
| | | | | |
| Output per Worker | 100 | 102.3 | 102.7 | 0.4 |
| | | | | |
| Reservation Wage | 100 | 102.2 | 102.5 | 0.2 |
| Unemployment | 100 | 98.4 | 100.6 | 2.2 |
| Unemp. Duration (months) | 2.4 | 2.3 | 2.4 | 0.1 |
| | | | | |
| **Sectoral Impacts** | | | | |
| Output - Sector 1 | 100 | 102.4 | 124.1 | 21.7 |
| Sector 2 | 100 | 102.4 | 81.2 | -21.2 |
| | | | | |
| Employment - Sector 1 | 100 | 100.1 | 118.7 | 18.6 |
| Sector 2 | 100 | 100.1 | 81.2 | -18.9 |
| % High Skill - Sector 1 | 50.0 | 51.2 | 52.3 | 1.1 |
| Sector 2 | 50.0 | 51.2 | 50.0 | -1.2 |
| | | | | |
| Profits - Sector 1 | 100 | 102.7 | 128.2 | 25.5 |
| Sector 2 | 100 | 102.7 | 72.5 | -30.2 |
| | | | | |
| Market Tightness - Sector 1 | 100 | 104.5 | 151.3 | 46.8 |
| Sector 2 | 100 | 104.5 | 58.5 | -46.0 |
| | | | | |
| **Distributional Impacts** | | | | |
| Low-Skill Wages - Sector 1 | 100 | 101.4 | 101.6 | 0.1 |
| Sector 2 | 100 | 101.4 | 101.6 | 0.1 |
| High-Skill Wages - Sector 1 | 100 | 101.8 | 102.8 | 1.0 |
| Sector 2 | 100 | 101.8 | 100.9 | -0.9 |

Note: Steady-state comparison following unanticipated, permanent productivity shocks which are general versus sector-specific. The relevant variables in the benchmark steady-state are normalized to 100. In the Benchmark model $A_1 = A_2 = 1$; General Shock $A_1 = A_2 = 1.015$; Sector-Specific Shock for Sector 1: $A_1 = 1.03, A_2 = 1$. The sector-specific effect of the shock is the specific shock minus the general shock.

# B   Figures

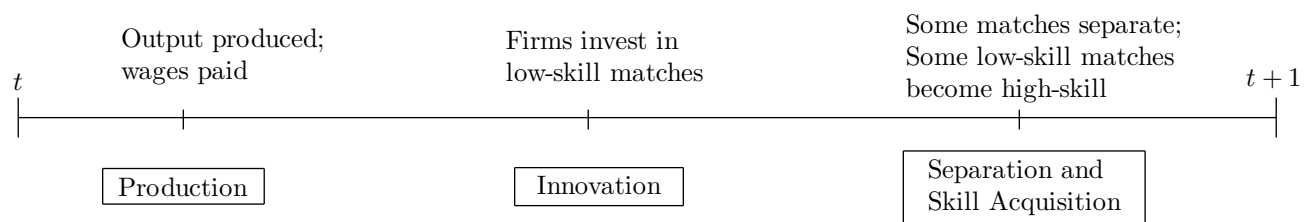Figure 1: **Model Timing: Unmatched Workers and Firms**



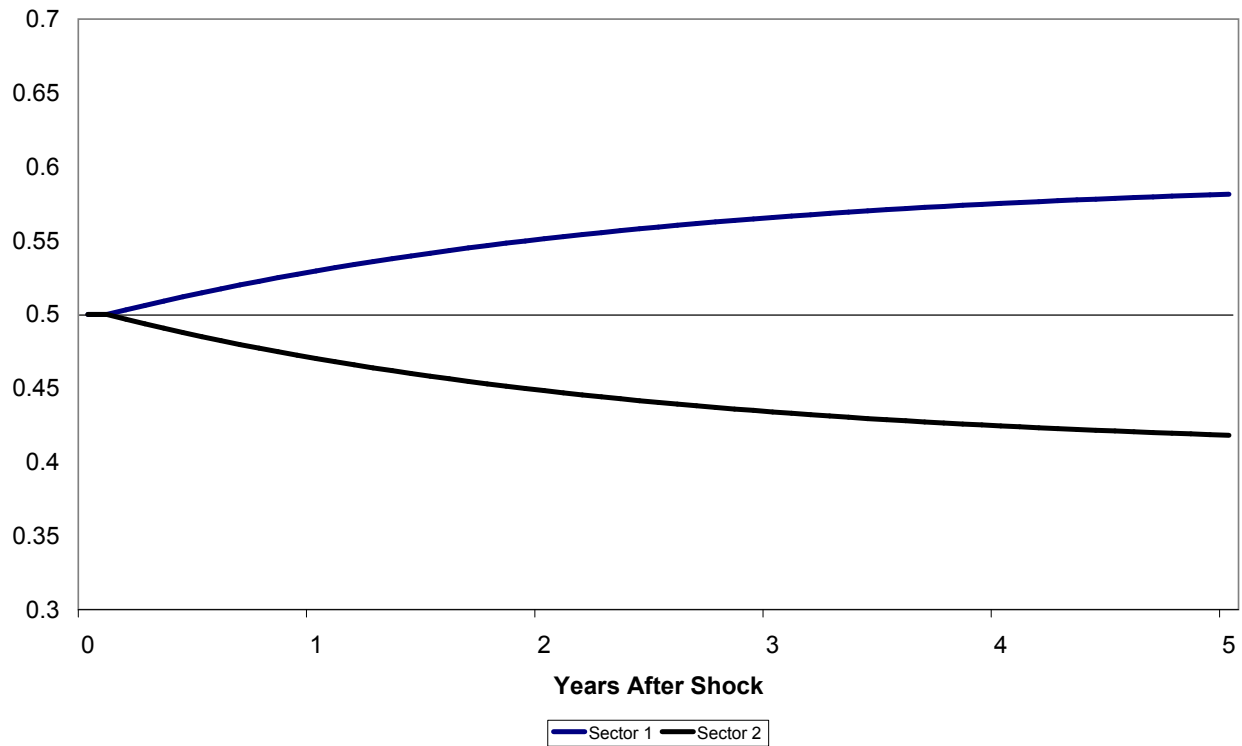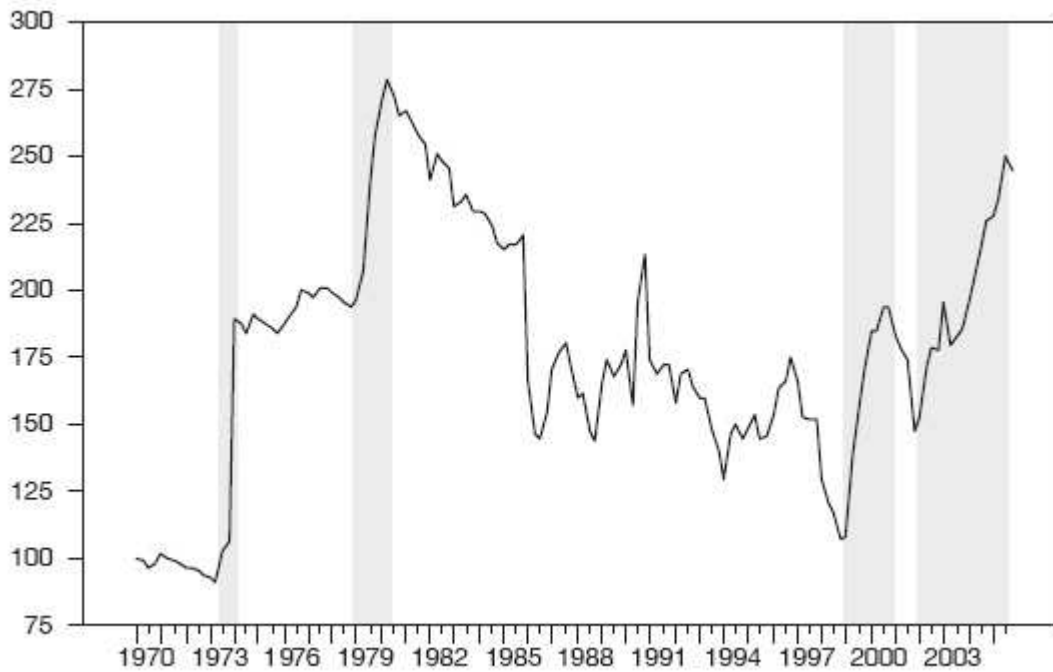Figure 2: **Model Timing: Producing Workers and Firms**

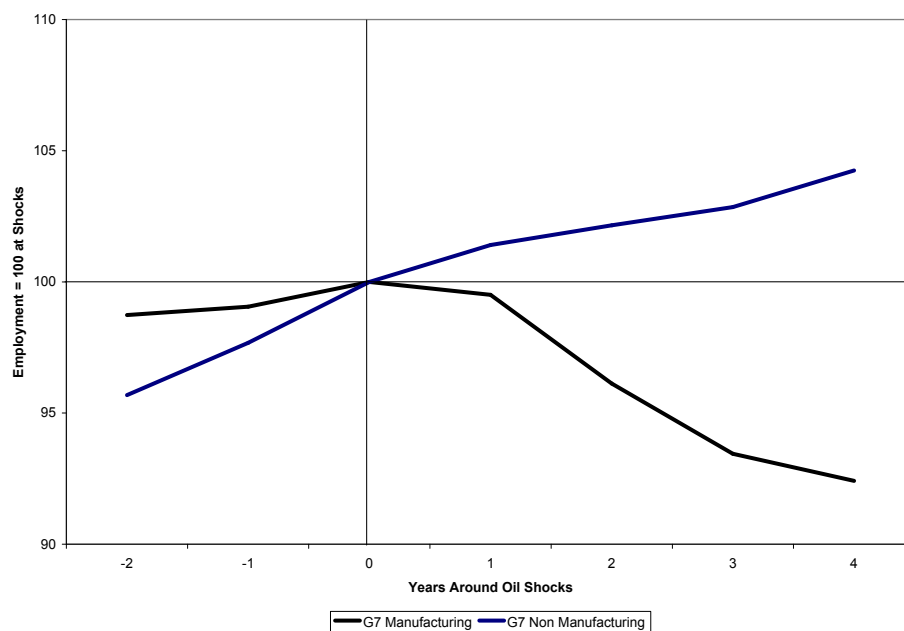Figure 3: **Model's Employment Response After Sector-Specific Productivity Shock**

Benchmark model's dynamic response to permanent productivity shock to sector 1, $A_1 = 1.03; A_2 = 1$.

Figure 4: **Log Real Oil Price (1970=100)**



Source: Reproduced from Blanchard and Gali (2007), (Figure 3 of their paper). Shading indicates oil price shocks as defined by 50 percent increase in the real price of oil, sustained for at least four quarters.

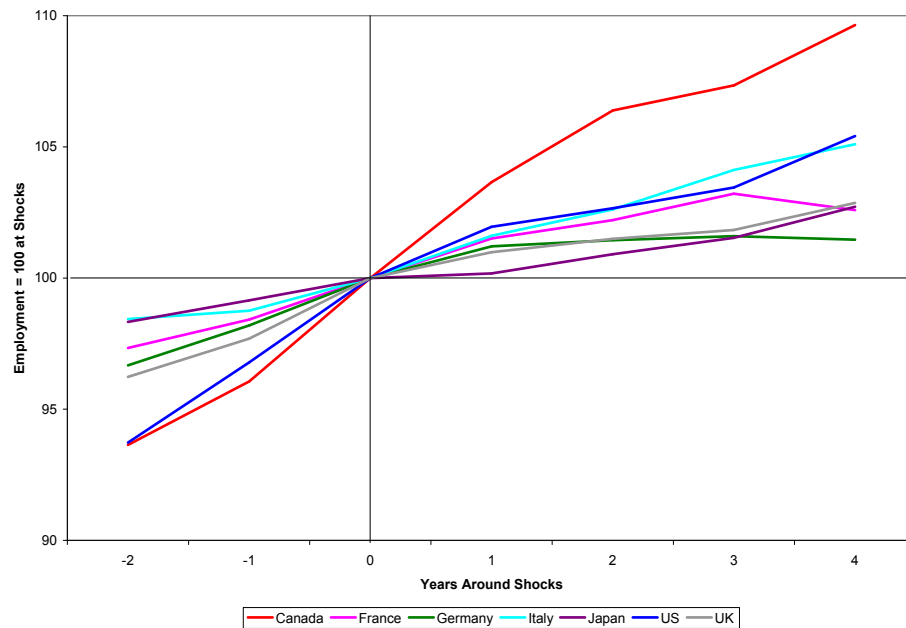Figure 5: **Average Employment Response After Oil Shocks, G7 Economies**



Data Source: OECD Structural Analysis (STAN) Database.

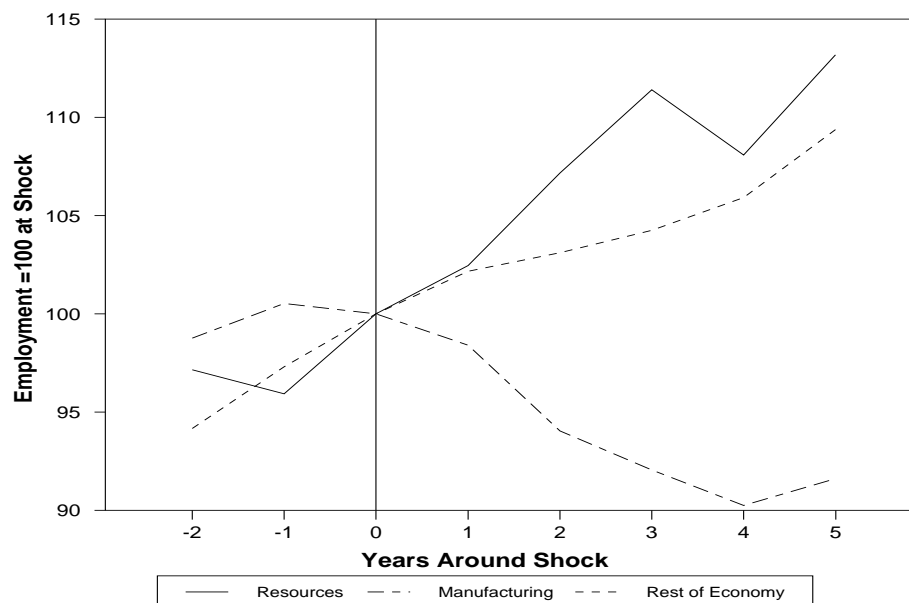Figure 6: **Average Manufacturing Employment Response After Oil Shocks, G7 Economies**
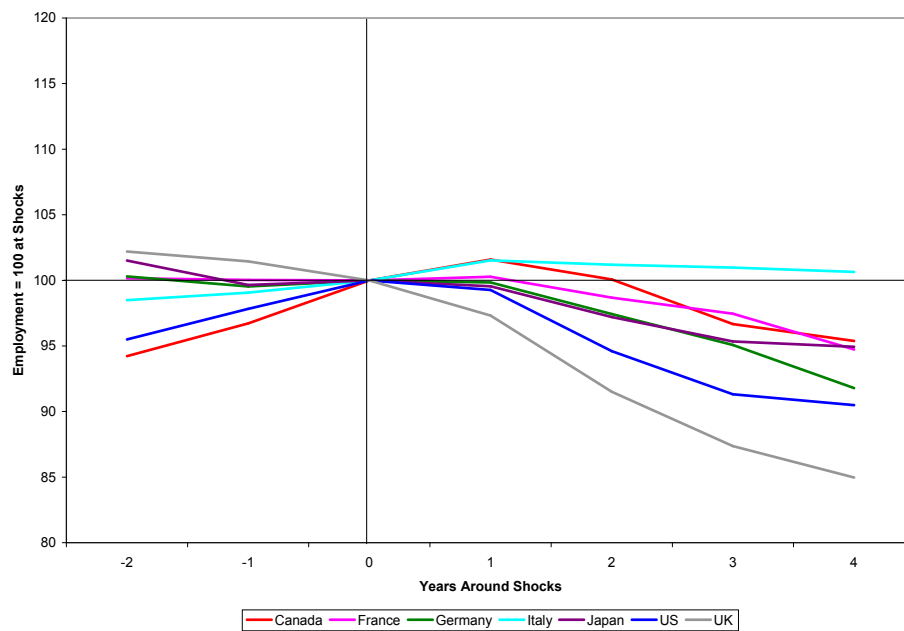


Data Source: OECD Structural Analysis (STAN) Database.

Figure 7: **Average Non-Manufacturing Employment Response After Oil Shocks, G7 Economies**



Data Source: OECD Structural Analysis (STAN) Database.

Figure 8: **Average Employment Response After Oil Shocks, U.S.**



Data Source: Current Employment Statistics (CES) Survey, Resources = Natural Resources and Mining, CES1000000001; Manufacturing CES3000000001; Rest of Economy = Total Nonfarm Employment, CES0000000001, less Resources and Manufacturing Employment.

Figure 9: **Relative Employment Responses After An Oil Price Shock, Canada**
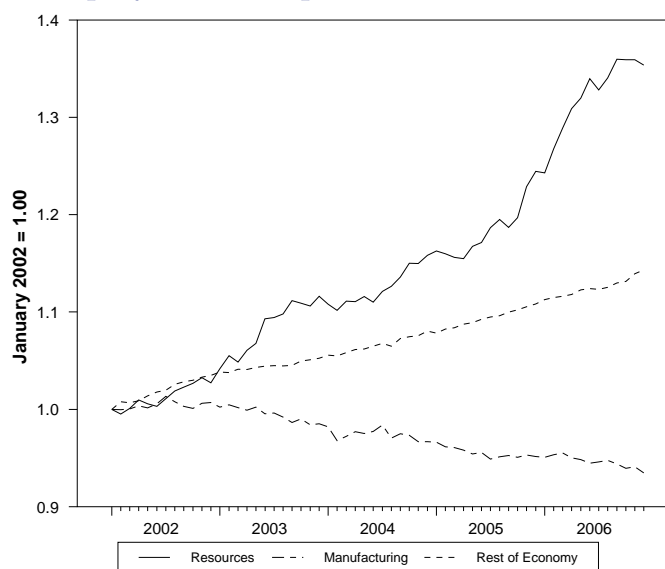


Source: Cansim, Survey of Employment, Payrolls and Hours (SEPH), seasonally adjusted employment. Manufacturing v1596771. Mining and oil and gas extraction v1596768. Rest of Economy = Industrial aggregate excluding unclassified, v1596764, less manufacturing and mining, oil and gas employment.

Figure 10: **Real Hourly Wages in the Resource Sector, Canada**
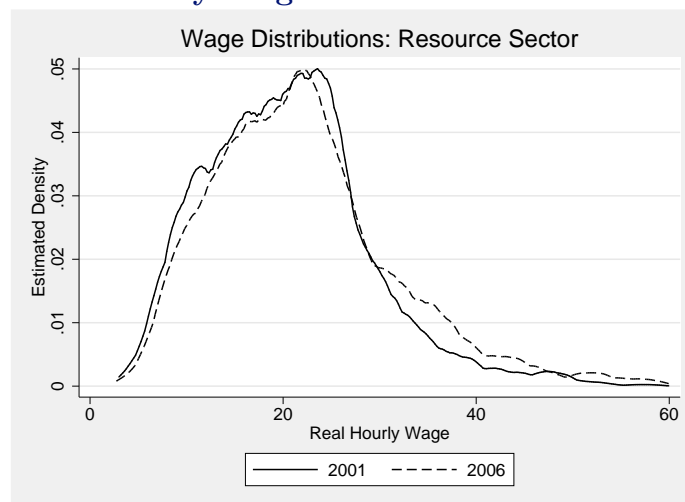


Source: *Labour Force Survey* Public Use Microdata Files. Kernel density estimates of 2001 and 2006 surveys, Hourly Earnings variable for workers in the Oil & Gas; Forestry; Fishing; and Mining sectors. The solid line shows 2001, the year prior to the shock; the dashed line shows 2006, the latest available data. I deflate nominal earning using the Consumer Price Index. The estimation applies the Epanenchiknov smoothing kernel with optimal weights from Silverman (1986).

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# C  Model Derivations and Proofs of Propositions

***Proof of Proposition 2.1:***

$$J_i^L = \max_{x_i} A_i p_i^L - w_i^L - \chi(x_i) + \delta[s_i V_i + \lambda_i x_i J_i^H + (1 - s_i - \lambda_i x_i) J_i^L(x_i)] \ \ s.t. \ 0 \le x_i; \ x_i \le 1$$

The associated optimization problem is:
$$\mathcal{L} = A_{i,t} p_{i,t}^L - w_{i,t}^L - \chi(x_{i,t}) + \delta E_t[s_i V_{i,t+1} + \lambda_i x_{i,t+1} J_{i,t+1}^H + (1 - s_i - \lambda_i x_{i,t+1}) J_{i,t+1}^L(x_{i,t+1})]$$
$$-\gamma_1(-x_{i,t}) - \gamma_2(x_{i,t} - 1)$$

I focus on stationary innovation policies, where $x_{i,t} = x_{i,t+1}$. The Kuhn-Tucker conditions are:

$\frac{\partial \mathcal{L}}{\partial x_{i,t}} = -\chi'(x_{i,t}) + \delta E_t[\lambda_i(J_{i,t+1}^H - J_{i,t+1}^L) + (1 - s_i - \lambda_i x_{i,t+1})\frac{\partial J_i^L}{\partial x_i^*}] + \gamma_1 - \gamma_2$

$\frac{\partial \mathcal{L}}{\gamma_1} = x_i$

$\frac{\partial \mathcal{L}}{\gamma_2} = 1 - x_i$

There are three cases to consider: the two corner solutions $x_i^* = 0, x_i^* = 1$ and interior solutions $x_i^* \in (0,1)$.

*Case 1:* $x_i^* = 0$. If the first constraint holds, $x_i^* = 0$, so $\gamma_1 > 0$ by the complementary slackness condition. The second constraint is satisfied, so $\gamma_2 = 0$. Collecting terms on the first order condition for investment gives: $\frac{\partial \mathcal{L}}{\partial x_i^*}[1 - \delta(1 - s_i)] = -\chi'(x_{i,t}) + \delta\lambda_i E_t[(J_{i,t+1}^H - J_{i,t+1}^L)] + \gamma_1$. Given the boundary solution, this expression is non-positive so: $\chi'(x_{i,t}) \ge \delta\lambda_i E_t[(J_{i,t+1}^H - J_{i,t+1}^L)] + \gamma_1$. Since $\gamma_1$ is positive, this implies the marginal investment cost exceeds the expected marginal benefit at $x_i^* = 0$.

*Case 2:* $x_i^* = 1$. If the second constraint holds, $x_i^* = 1$, so $\gamma_2 > 0$ by the complementary slackness condition. The first constraint is satisfied, so $\gamma_1 = 0$. Collecting terms on the first order condition for investment gives: $\frac{\partial \mathcal{L}}{\partial x_i^*}[1 - \delta(1 - s_i - \lambda_i)] = -\chi'(x_{i,t}) + \delta\lambda_i E_t[(J_{i,t+1}^H - J_{i,t+1}^L)] - \gamma_2$. Given the boundary solution, this expression is non-negative so: $\delta\lambda_i E_t(J_{i,t+1}^H - J_{i,t+1}^L) \ge \chi'(x_{i,t}) + \gamma_2$. Since $\gamma_2$ is positive, the expected marginal benefit of investment exceeds the marginal cost at $x_i^* = 1$.

*Case 3:* $x_i^* \in (0,1)$. Both constraints are satisfied so $\gamma_1 = \gamma_2 = 0$. By the envelope theorem, $\frac{\partial J_i^L}{\partial x_{i,t}^*} = 0$, so the first order condition for investment simplifies to:

$$\frac{\partial \mathcal{L}}{\partial x_{i,t}^*} = -\chi'(x_{i,t}) + \delta E_t[\lambda_i(J_{i,t+1}^H - J_{i,t+1}^L)] = 0$$

For interior solutions, the marginal benefit of investment equals the marginal cost. Substituting into the first order necessary condition for investment using $J_{i,t+1}^H - J_{i,t+1}^L = (1 - \beta)SP_{i,t+1} = (1 - \beta)\frac{A_{i,t+1}(p_i^H - p_i^L) + k_i x_i}{\delta(r + s_i + \lambda_i x_i)}$ from the Nash Bargaining solution, equation (8), and using the properties of investment cost function, $\chi(x_i) = k_i x_i$, gives: $k_i = \lambda_i(1 - \beta)\frac{A_{i,t+1}(p_i^H - p_i^L) + k_i x_{i,t+1}}{(r + s_i + \lambda_i x_{i,t+1})}$, which after lagging one-period, simplifies to:

$$x_{i,t}^* = \frac{(1 - \beta)}{k_i \beta} A_{i,t}(p_i^H - p_i^L) - \frac{r + s_i}{\lambda_i \beta}$$

Finally, when $\lambda_i \le \bar{\lambda}_i$, the skill arrival rate is sufficiently low so no investment is offered.

**Proof of Proposition 2.2:** The sector that received the positive shock is now more productive, so its surplus from a new match increases. This in turn, means jobs in this sector are more profitable, so vacancy posting and market tightness increase in this sector.

Now, assume unemployed workers' reservation wage falls. With cheaper labour, jobs in all other sectors also become more profitable. Therefore, vacancy posting increases, raising market tightness in these other sectors, $\{\theta_j^*\}_{j\neq i}^I$. The reservation wage can be expressed as $\overline{w} = z + \frac{\beta c}{1-\beta}\sum_i \theta_i$. Therefore, because $z, c, \beta$ are fixed, the reservation wage would increase. However, this contradicts the original assumption that the reservation wage falls.

Thus, it must be the case that following a positive productivity shock in sector $i$, workers' reservation wage increases. Jobs in the other sectors are therefore less profitable at the higher wage, so from the zero profit conditions, the RHS of equation (16) falls. For the zero profit condition to hold in the new equilibrium, firms expected recruiting costs must also fall — the LHS of equation (16). Given the cost of a vacancy, $c$, is fixed, the job filling rates in these other sectors must increase, $\{q(\theta_j)\}_{j\neq i}^I$, which requires that market tightness fall in the other sectors, $\{\theta_j^*\}_{j\neq i}^I$.

The same argument applies after a negative shock in sector $i$, but in the opposite direction.

## Proof of Proposition 2.3:

The social present value of a low-skill match is $S_i = W_i^L - U + J_i^L - V_i$. The skill premium of a high-skill relative to a low-skill match is $SP_i = W_i^H - W_i^L + J_i^H - J_i^L$. Using the worker's and firm's value functions, equations (1) – (6), and the free entry/zero profit condition, $V_i = 0$, gives an expression for low-skill match surplus:

$$S_i = A_i p_i^L - \chi(x_i) - z - \delta \sum_i^I f_i(\theta_i)(W_i^L - U) + \delta \lambda_i x_i SP_i + \delta(1 - s_i)S_i \qquad (18)$$

Substituting in for the worker's reservation wage, $\bar{w} = z + \delta \sum_i^I f_i(\theta_i)(W_i^L - U)$, and rearranging using $\delta = \frac{1}{1+r}$, gives:

$$\delta(r + s_i)S_i = A_i p_i^L - \chi(x_i) - \bar{w} + \delta \lambda_i x_i SP_i$$

Production requires the match surplus be non-negative, $S_i \geq 0$. This implies the value of low-skill output, net of investment costs, plus the expected present value of the skill premium covers the worker's reservation wage:

$$A_i p_i^L - \chi(x_i) + \delta \lambda_i x_i SP_i \geq \bar{w}$$

## Corollary of Propositions 2.1 and 2.3:

Case 1) A given sector will not produce if:

$$y_i^L - \chi(x_i^*) + \lambda_i x_i^* \frac{y_i^H - y_i^L + \chi(x_i^*)}{(r + s_i + \lambda_i x_i^*)} < \bar{w}$$

Case 2) A given sector produces only low-skill output if:

$$i)\quad y_i^L \geq \bar{w} \qquad \& \qquad ii)\quad \lambda_i \leq \overline{\lambda_i}$$

Case 3) A given sector produces both low-skill and high-skill output if:

$$i) \quad y_i^L - \chi(x_i^*) + \lambda_i x_i^* \frac{y_i^H - y_i^L + \chi(x_i^*)}{(r + s_i + \lambda_i x_i^*)} \geq \bar{w} \qquad \& \qquad ii) \quad \lambda_i > \overline{\lambda_i}$$

where for Case 3) $\chi(x_i^*) = k_i x_i^* = \frac{(1-\beta)}{\beta}(y_i^H - y_i^L) - \frac{k_i(r+s_i)}{\lambda_i \beta}$

### *Derivation of Equilibrium Wages:*

***Low-Skill Wage in Sector*** $i$***:*** From (18) as described above, the low-skill match surplus can be expressed as:

$$S_i = A_i p_i^L - \chi(x_i) - z - \delta \sum_i^I f_i(\theta_i)(W_i^L - U) + \delta \lambda_i x_i S P_i + \delta(1 - s_i) S_i \qquad (19)$$

Using equation (5) and $V_i = 0$ gives:

$$J_i^L = A_i p_i^L - w_i^L - \chi(x_i) + \delta \lambda_i x_i (J_i^H - J_i^L) + \delta(1 - s_i) J_i^L$$

Substituting in $J_i^L = (1 - \beta) S_i$, and $J_i^H - J_i^L = (1 - \beta) S P_i$ from the Nash Bargaining solutions, equations (7) and (8), gives another expression in the low-skill surplus:

$$(1 - \beta) S_i = A_i p_i^L - w_i^L - \chi(x_i) + \delta \lambda_i x_i (1 - \beta) S P_i + \delta(1 - s_i)(1 - \beta) S_i \qquad (20)$$

Multiplying (19) by $(1 - \beta)$ gives:

$$(1 - \beta) S_i = (1 - \beta)[A_i p_i^L - \chi(x_i) - z - \delta \sum_i^I f_i(\theta_i)(W_i^L - U) + \delta \lambda_i x_i S P_i + \delta(1 - s_i) S_i] \quad (21)$$

Equating the RHS of (20) and (21) and simplifying gives the equilibrium low-skill wage in sector $i$, equation (12) in the paper:

$$w_i^{L*} = \overline{w} + \beta(\underbrace{A_i p_i^L - \chi(x_i^*) - \overline{w}})$$

where $\overline{w} = z + \delta \sum_i^I f_i(\theta_i^*)(W_i^L - U)$

### *High-Skill Wage in Sector* $i$***:***
Subtracting the worker's value functions, equations (3) from (2) gives:

$$W_i^H - W_i^L = w_i^H - w_i^L + \delta[(1 - s_i - \lambda_i x_i)(W_i^H - W_i^L)]$$

Using the fact that $\delta = \frac{1}{1+r}$ and simplifying gives:

$$\delta(r + s_i + \lambda_i x_i)(W_i^H - W_i^L) = w_i^H - w_i^L$$

Substituting in $W_i^H - W_i^L = \beta S P_i$ from the Nash Bargaining solution, (8) gives:

$$\delta(r + s_i + \lambda_i x_i)\beta S P_i = w_i^H - w_i^L \qquad (22)$$

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Then explicitly solve for the skill premium using the worker's and firm's value functions, equations (2) and (3) and (5) and (6) and $\delta = \frac{1}{1+r}$:

$$SP_i = \frac{A_i(p_i^H - p_i^L) + \chi(x_i)}{\delta(r + s_i + \lambda_i x_i)} \tag{23}$$

Substituting into (22) for the skill premium and the low-skill wage and simplifying gives the high-skill wage in sector $i$, equation (13) in the paper:

$$w_i^{H*} = \overline{w} + \beta(A_i p_i^H - \overline{w})$$

where $\overline{w} = z + \delta \sum_i^I f_i(\theta_i^*)(W_i^L - U)$

### Equilibrium System of Equations in Market Tightness:

Using the zero profit condition, $V_i = 0$ in the firm's value of a vacancy equation, (4), gives $J_i^L = \frac{c}{\delta q_i(\theta_i)}$. From the firm's Nash Bargaining, (7), $(1-\beta)S_i = J_i^L$. So,

$$(1-\beta)S_i = \frac{c}{\delta q_i(\theta_i)}$$

Substitute in for $(1-\beta)S_i$ using (21):

$$(1-\beta)[A_i p_i^L - \chi(x_i) - z - \delta \sum_i^I f_i(\theta_i)(W_i^L - U) + \delta \lambda_i x_i SP_i + \delta(1-s_i)S_i] = \frac{c}{\delta q_i(\theta_i)}$$

Use the worker's Nash Bargaining solution, (7), $\beta S_i = W_i^L - U$ and use $S_i = \frac{c}{\delta(1-\beta)q_i(\theta_i)}$,

$$(1-\beta)[A_i p_i^L - \chi(x_i) - z - \delta \sum_i^I f_i(\theta_i)\frac{\beta c}{\delta(1-\beta)q_i(\theta_i)} + \delta \lambda_i x_i SP_i + \delta(1-s_i)\frac{c}{\delta(1-\beta)q_i(\theta_i)}] = \frac{c}{\delta q_i(\theta_i)}$$

Use the fact that $f_i(\theta_i) = \theta_i q_i(\theta_i)$ and $\frac{1}{\delta} = 1 + r$ to get:

$$(1-\beta)[A_i p_i^L - \chi(x_i) - z - \sum_i^I \frac{\beta c \theta_i}{(1-\beta)} + \delta \lambda_i x_i SP_i] = \frac{c(r+s_i)}{q_i(\theta_i)}$$

Divide both sides by $c$, substitute in for the skill premium, $SP_i$, from (23) and rearrange to get the equilibrium system of equations in $\{\theta_i\}_1^I$ given in equation (17) of the paper:

$$\frac{r+s_i}{q(\theta_i)} + \beta \sum_i \theta_i = \frac{(1-\beta)}{c}[A_i p_i^L - \chi(x_i^*) - z + \lambda_i x_i^* \cdot \frac{A_i(p_i^H - p_i^L) + \chi(x_i^*)}{r + s_i + \lambda_i x_i^*}]$$

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# D   Model Extension: Directed Search

One potential objection to the model formulation is that since jobs in more productive sectors pay higher wages and workers have full information, they may direct their search to these high-wage sectors rather than apply for jobs in all sectors. The model can easily be amended to give workers the option of directing their search to a particular sector.

To simplify the analysis and highlight the main results, I ignore innovation investment and assume workers can costlessly switch their job search between sectors at the beginning of each period, but can only apply to one sector per period. While the value functions for firms are essentially unchanged, workers' value functions become:

$$U_i = z + \delta[f_i(\theta_i)W_i + (1 - f_i(\theta_i))U_i] \tag{24}$$

$$W_i = w_i + \delta[s_i U_i + (1 - s_i)W_i] \tag{25}$$

There are equilibria where some sectors do not produce because no workers apply for jobs. For practical applications, the relevant equilibrium features production in all sectors under study. In this case, workers must be indifferent between search in any sector, i.e. $U_i = \underline{U} \quad \forall i$. Since the expected present value of searching in each sector must be the same, considering sectors $i$ and $j$:

$$\underline{U} = z + \delta[f_i(\theta_i)W_i + (1 - f_i(\theta_i))\underline{U}] = z + \delta[f_j(\theta_j)W_j + (1 - f_j(\theta_j))\underline{U}]$$

Nash Bargaining implies $W_i - \underline{U} = \beta S_i$, therefore the above expression simplifies to:

$$\Rightarrow f_i(\theta_i)S_i = f_j(\theta_j)S_j$$

More productive sectors have a higher match surplus, $S_i$, so they pay workers higher wages. In equilibrium, however, workers take longer to find a job in these sectors, because $f_i(\theta_i)$ is lower.[26]

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[26]Wages are given as before by: $w_i^* = \overline{w} + \beta(A_i p_i - \overline{w})$ except now the worker's reservation wage is: $\overline{w} = z + \delta f_i(\theta_i^*)(W_i - \underline{U})$. From above, workers reservation wages are the same in each sector, so more productive sectors pay higher wages, $p_i > p_j \Rightarrow w_i > w_j$.

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