# Calculus/Optimization and Convexity I: Practice Problems

September 7, 2011

1. Consider the function
$$f(x) = \frac{3}{x^4 - x^2 + 1}$$
   (a) Compare $f'(x)$ and find all local maximum and minimum points for $f$. Has $f$ any global extreme points?
   (b) Draw the graph of $f$

2. A function $f$ is given by:

$$f(x) = (1 + 2/x)\sqrt{x + 6}$$

(a) Find the domain of f, the zeros of $f$, and the intervals where $f(x)$ is positive.

(b) Find possible local extreme points and values.

3. (a) If $a_1 < \cdots < a_n$, find the minimum value of $f(x) = \sum_{i=1}^{n}(x - a_i)^2$

   (b) Find the minimum value of $f(x) = \sum_{i=1}^{n} |x - a_i|$. Calculus will not help in this problem.

   (c) Let $a > 0$. Show that the maximum value of

$$f(x) = \frac{1}{1 + |x|} + \frac{1}{1 + |x - a|}$$

   is $\frac{2+a}{1+a}$. (The derivative can be found on each of the intervals $(-\infty, 0), (0, a)$ and $(a, \infty)$ separately.)

4. Let the total cost of producing $Q$ units of a commodity be

$$C(Q) = aQ^2 + bQ + c, Q > 0$$

where $a, b$, and $c$ are positive constants. Prove that the average cost function $A(Q) = C(Q)/Q$ has a minimum at $Q^* = \sqrt{c/a}$.

5. A competitive firm receives a price $p$ for each unit of its output, pays a price $w$ for each unit of its only variable input, and incurs fixed costs of $F$. Its output from using $x$ units of variable input is $f(x) = \sqrt{x}$.

   (a) Write the firm's revenue, cost and profit functions.

   (b) Write the first-order conditions for profit maximization, and give it an economic interpretation.

   (c) Check that profit really is maximized at a point satisfying the first-order condition.

   (d) Explain how your answers would change if $f(x) = x^2$.

6. Suppose that a function $f$ is concave.

   (a) What restrictions on $a$ and $b$ will guarantee that $g(x) = af(x) + b$ is also concave?

   (b) Prove that if $f$ and $g$ are both concave, then

$$h(x) = min\{f(x), g(x)\}$$

   is concave. Illustrate.