EC 450 Department of Economics
Advanced Macroeconomics Wilfrid Laurier University
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Suggested Solutions to Assignment 7 (OPTIONAL)

Part B Problem Solving Questions

Read each part of the question very carefully. Show all the steps of your calculations to get full marks.

B1.

Consider a closed economy where aggregate demand for goods and services consists of the sum of real private consumption, *C*, real private investment, *I*, and real government demand for goods and services, *G*. Based on the theory of private investment we can summarize private investment behaviour in an investment function of the form $I = I(Y, r, \varepsilon)$, where *Y* is total output of the economy, *r* is the real interest rate and ε is a parameter capturing the 'state of confidence', reflecting the expected growth of income and demand. Investment increases with current output and with growth expectations, ε, whereas it decreases with the real interest rate. Based on the theory of private consumption we can summarize private consumption behaviour in an consumption function of the form $C = C(Y - T, r, \varepsilon)$, where *T* denotes the total tax payments so that $Y - T$ is current disposable income. Consumption increases with current disposable income and with growth expectations, ε. The marginal propensity to consume current disposable income is assumed to be less than 1. The real interest rate has an ambiguous effect on consumption, due to offsetting income and substitution effects on consumption.
To avoid complications arising from the dynamics of government debt accumulation, we assume that the government balances its budget so that $T = G$.

(1) Derive the goods market equilibrium condition in the log-linearized form.

See Pages 498-501 of the textbook.

Assume that the central bank in this economy can observe the expected inflation rate π_{+1}^{e} and its inflation target is π^{*}. Since private saving and investment decisions depend on the *ex ante* real interest rate $r = i - \pi_{+1}^{e}$, we assume that the central bank sets the short-term nominal interest rate in accordance with the following slightly modified version of the Taylor rule:

$$i = \bar{r} + \pi_{+1}^{e} + h\left(\pi - \pi^{*}\right) + b\left(y - \bar{y}\right), \qquad h > 0, \qquad b > 0.$$

(2) Derive the aggregate demand curve in the log-linearized form.

See Pages 514-515 of the textbook.

(3) Identify the determinants of the position and the slope of the *AD* curve in the (y, π) plane. Explain with a diagram how the slope of the *AD* curve varies with a change in the values of the parameters *h* and *b* in the Taylor rule.

See Pages 515-516 of the textbook.

B2.

Exercise 1 of Chapter 17 of the textbook: Part 1 and Part 2

B3.

Exercise 2 of Chapter 17 of the textbook: Part 1, 2 and 3.

B4.

Exercise 3 of Chapter 17 of the textbook: Part 1 and 2.

Chapter 17
Solutions to Exercises

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Exercise 17.1. Topics in the theory of aggregate demand

1. The *ex post* real interest rate is the actual real interest rate implied by the actual rate of inflation, measured after the relevant time period has passed. This measure of the real interest rate, which may be approximated by $r^a = i - \pi_{+1}$, is relevant for judging how inflation affects the distribution of income between borrowers and lenders. The *ex ante* real interest rate, proxied by $r = i - \pi^e_{+1}$, is the real interest rate expected to prevail over the next period, given the rate of inflation expected for that period. This is the measure of the real interest rate which is relevant as an indicator of the incentives to save and invest. If expectations of inflation are static so that $\pi^e_{+1} = \pi$, the ex ante real interest rate is equal to the popular measure of the real interest rate, $r = i - \pi$. Furthermore, if inflation is stable over time so that $\pi_{+1} = \pi$, the ex post real interest rate is likewise equal to the popular measure.

2. The AD curve is derived on the assumption that monetary policy follows a Taylor rule with $h > 0$. If the inflation rate goes up, the central bank reacts by raising the real interest rate which reduces aggregate demand. This explains why the AD curve slopes downwards. The numerical slope of the curve is given by $1/\alpha = (1 + \alpha_2 b)/\alpha_2 h$. This shows that, the stronger the interest rate response to higher inflation (the higher the value of h), the larger is the fall in aggregate demand, so the flatter is the AD curve. A higher value

of b implies a larger rise in the real interest rate in response to a rise in economic activity. This means that, as output expands in reaction to a fall in inflation, a larger part of the initial fall in the real interest rate is reversed, implying a smaller increase in demand and hence a steeper AD curve. The AD curve shifts downwards in case of a drop in the private sector's growth expectations (the 'confidence' variable ε), a tightening of fiscal policy, or a drop in the central bank's target inflation rate.

Exercise 17.2. Interest rate setting under a constant money growth rule

1. In long run equilibrium the nominal interest rate must be constant over time. When i is constant and output is on its long run growth trend, we find by taking logs in (36) that the growth rate of the demand for real money balances is approximately equal to

$$\ln L - \ln L_{-1} = \eta \left(\ln \overline{Y} - \ln \overline{Y}_{-1} \right) \approx \eta x, \tag{A.1}$$

where the last equality is obtained by using (35).

Since $M = (1+\mu) M_{-1}$ and $P \equiv (1+\pi) P_{-1}$, the growth rate of the real money supply is approximately equal to

$$\begin{aligned} \ln(M/P) - \ln(M_{-1}/P_{-1}) &= \ln(1+\mu) + \ln M_{-1} - \ln P - \ln M_{-1} + \ln P_{-1} \\ &\approx \mu - \pi, \qquad \pi \equiv \ln P - \ln P_{-1} \end{aligned} \tag{A.2}$$

In equilibrium, the growth rate of the real money supply must equal the rate of growth in real money demand, so by equating the expressions in (A.1) and (A.2) we obtain the long run equilibrium inflation rate:

$$\mu - \pi = \eta x \Longleftrightarrow \pi = \mu - \eta x \tag{A.3}$$

Thus the long run inflation rate equals the growth rate in the nominal money supply minus the product of the growth rate of real income and the income elasticity of money demand.

2. Taking logs on both sides of (37) and approximating, we get

$$\mu - \pi = \eta\left(y - \overline{y}_{-1}\right) + \beta\left(\overline{r} + \mu - \eta x - i\right) \tag{A.4}$$

According to (35) we have $\overline{y}_{-1} \approx \overline{y} - x$. Inserting this into (A.4), and rearranging, we obtain

$$\begin{aligned}
\mu - \pi = \eta\left(y - \overline{y} + x\right) + \beta\left(\overline{r} + \mu - \eta x - i\right) \Longleftrightarrow \\
i = \overline{r} + \mu - \eta x + \left(\frac{1}{\beta}\right)(\pi - \mu) + \left(\frac{\eta}{\beta}\right)(y - \overline{y} + x) \Longleftrightarrow \\
i = \overline{r} + \pi + \left(\frac{\eta}{\beta}\right)(y - \overline{y}) + \left(\frac{1-\beta}{\beta}\right)[\pi - (\mu - \eta x)]
\end{aligned} \tag{38}$$

The magnitude $\mu - \eta x$ may be seen as the central bank's inflation target. Equation (20) in the main text is the special case of (A.5) where the growth rate of trend output is zero ($x = 0$).

3. If β is close to zero, it follows from (38) that fluctuations in the output gap and the rate of inflation must lead to large fluctuations in the nominal and real interest rate to maintain a constant growth rate of the nominal money stock. This may be problematic, since sharp fluctuations in the real interest rate will tend to cause a significant redistribution of income between borrowers and lenders.

Exercise 17.3. Nominal GDP targeting

1. Inserting (40) and (41) into (39) and solving for the nominal interest rate, we obtain the monetary policy rule under nominal GDP targeting:

$$\overbrace{\overline{y} - \alpha_2\left(i - \pi - \overline{r}\right)}^{=y} = y_{-1} + \mu - \pi \Longleftrightarrow$$

$$\begin{aligned}
\overline{y}_{-1} + x - \alpha_2\left(i - \pi - \overline{r}\right) = y_{-1} + \mu - \pi \Longleftrightarrow \\
i = \overline{r} + \pi + \left(\frac{1}{\alpha_2}\right)[\pi - (\mu - x)] - \left(\frac{1}{\alpha_2}\right)\left(y_{-1} - \overline{y}_{-1}\right)
\end{aligned} \tag{A.5}$$

We see that the real interest rate responds positively to a rise in current inflation. At the same time it reacts *negatively* to the output gap observed in the *previous* period. The reason is that, ceteris paribus, a high value of y_{-1} reduces the likelihood of a high real growth rate in the current period. This leaves room for a more expansionary interest rate policy, given the desire to maintain a constant rate of growth of money income. The larger the magnitude of the parameter α_2, the stronger is the response of aggregate demand to a change in the real interest rate, so the weaker is the interest rate response to changes in output and inflation needed to stabilize the growth rate of nominal income, as shown in (A.5).

2. Under all three monetary policy rules the interest rate reacts positively to a rise in the current inflation rate. Under the Taylor rule and Friedman's constant money growth rule, the interest rate also responds positively to the current output gap, whereas under nominal GDP targeting, the interest rate reacts negatively to the lagged output gap, as shown in (A.5). One advantage of directly targeting nominal GDP rather than following a constant money growth rule is that the latter rule does not ensure constant growth of nominal income if the money demand function is unstable, say, due to financial innovations.

Exercise 17.4. Topics in monetary policy

1. Under Friedman's assumption of a stable money demand function with a low interest elasticity, a constant growth rate of nominal money supply will ensure stable growth in nominal money income (see p. 504). This will also ensure a high degree of stability in the growth of real income, given Friedman's belief that market forces tend to pull real output fairly quickly towards its 'natural' level. The constant money growth rule as well as the Taylor rule require that the central bank should raise the interest rate in response to a rise in the inflation gap and in the output gap. Empirically it may therefore be difficult to determine whether a central bank follows one rule or the other. However, if the demand