EC 450
Advanced Macroeconomics
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Winter 2008

Suggested Solutions to Assignment 6 (OPTIONAL)

Part A Short Questions

A1.

Suppose real GDP of an economy is given by following Cobb-Douglas production function:

$$Y_t = B_t K_t^{\alpha} L_t^{1-\alpha}, \qquad 0 < \alpha < 1,$$

where K_t is the aggregate capital stock, L_t is the aggregate number of hours worked, and B_t is the 'total factor productivity' measuring the combined productivity of capital and labour. By definition, total working hours are given as:

$$L_t = (1 - u_t) N_t H_t,$$

where u_t is the unemployment rate, N_t is the labor force, and H_t is the average number of working hours per person employed.

(a) **Define the output gap for this economy.**

(b) **Explain the production function approach of measuring and decomposing the output gap.**

See Pages 421-422 of the textbook.

A2.

Explain the method of detrending macro variables using Hodrick-Prescott (HP) filter. Explain the sensitivity of this method to the value of λ. What are the 'reasonable' values of λ for detrending monthly, quarterly, and annual data?

See Pages 403-405 of the textbook.

The 'reasonable' value of λ for detrending monthly data is 14400.

The 'reasonable' value of λ for detrending quaterly data is 1600.

The 'reasonable' value of λ for detrending annual data is 100.

A3.

(a) Explain the conflicting evidence on the relationship between the average propensity to consume and disposable income found in microeconomic cross-section data and in macroeconomic time series data.

See Pages 466-468 of the textbook.

(b) Explain how the theory of consumption presented in Chapter 16 of the textbook helps to resolve the apparent inconsistency between the two types of evidence.

See Pages 478-479 of the textbook.

Part B Problem Solving Questions

Read each part of the question very carefully. Show all the steps of your calculations to get full marks.

B1.

Exercise 2 of Chapter 16 of the textbook: Part 1, 2, 3 and 4.

ting, the consumer's intertemporal budget constraint reads

$$C_1 + \frac{C_2}{1+r} = V_1 + Y_1^L - T_1 + \frac{Y_2^L - T_2}{1+r}, \tag{A.1}$$

where we use the notation of the text. The government's intertemporal budget constraint says that the present value of net taxes must be sufficient to cover the present value of public consumption plus the initial public debt. In our two-period framework, using the notation of the text, we thus have

$$D_1 + G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r}. \tag{A.2}$$

Inserting (A.1) into (A.2), we get

$$C_1 + \frac{C_2}{1+r} = V_1 - D_1 + Y_1^L - G_1 + \frac{Y_2^L - G_2}{1+r}. \tag{A.3}$$

Eq. (A.3) shows that if it does not affect pre-tax labour income (an important proviso), a rise in public consmption requires a corresponding cut in private consumption, measured in present value terms. This is intuitive: if a rise in public consumption does not affect the economy's overall resource constraint, it has to crowd out a corresponding amount of private consumption.

Exercise 16.2. The consumption function with endogenous labour supply

1. Inserting eq. (34) into (eq. (33) and dividing by $1+r$, we get the consumer's intertemporal budget constraint in the conventional form

$$C_1 + \frac{C_2}{1+r} = V_1 + w\,(1-\tau)\,h, \tag{A.4}$$

which says that (the present value of) lifetime consumption must correspond to the sum of lifetime disposable labour income and the consumer's initial non-human wealth. But since the consumption of leisure $(1-h)$ is an argument in the utility function, and since the after-tax wage rate $w(1-\tau)$ may be seen as the price of leisure, it is convenient to rewrite (A.4) as

$$C_1 + \frac{C_2}{1+r} + w(1-\tau)(1-h) = V_1 + w(1-\tau). \tag{A.5}$$

The third term on the left-hand side of (A.5) is the value of the consumption of leisure, and $w(1-\tau)$ is *potential* labour income, i.e., the maximum income the consumer could earn if he spent all of his time endowment working in the labour market. Thus eq. (A.5) says that the value of total lifetime consumption, including the consmption of leisure, is constrained by the sum of the consumer's initial non-human wealth and his potential disposable lifetime labour income.

2. Defining $F \equiv 1-h$ and using (A.5) to eliminate C_2, we may write the consumer's lifetime utility (35) as

$$U = \ln C_1 + \beta \ln F + \frac{\ln\left\{(1+r)\left[V_1 + w(1-\tau) - C_1 - w(1-\tau)F\right]\right\}}{1+\phi}. \tag{A.6}$$

Maximization of (A.6) w.r.t. C_1 og F yields the first-order conditions

$$\frac{\partial U}{\partial C_1} = 0 \Longleftrightarrow$$

$$\frac{1}{C_1} - \frac{1}{(1+\phi)\left[V_1 + w(1-\tau) - C_1 - w(1-\tau)F\right]} = 0 \tag{A.7}$$

$$\frac{\partial U}{\partial F} = 0 \Longleftrightarrow$$

$$\frac{\beta}{F} - \frac{w(1-\tau)}{(1+\phi)\left[V_1 + w(1-\tau) - C_1 - w(1-\tau)F\right]} = 0 \tag{A.8}$$

Substitution of (A.7) into (A.8) gives

$$\frac{\beta}{F} - \frac{w(1-\tau)}{C_1} = 0 \Longleftrightarrow$$

$$F = \frac{\beta C_1}{w(1-\tau)} \tag{A.9}$$

which we may insert into (A.7) to obtain the optimal consumption level in period 1:

$$\frac{1}{C_1} - \frac{1}{(1+\phi)\left[V_1 + w(1-\tau) - (1+\beta)C_1\right]} = 0 \Longleftrightarrow$$

$$C_1\left[1 + (1+\phi)(1+\beta)\right] = (1+\phi)\left[V_1 + w(1-\tau)\right] \Longleftrightarrow$$

$$C_1 = \widetilde{\theta} \cdot \left[V_1 + w(1-\tau)\right], \tag{A.10}$$

$$0 < \widetilde{\theta} \equiv \frac{1+\phi}{1+(1+\phi)(1+\beta)} < 1 \tag{A.11}$$

According to (A.10) the optimal consumption level is proportional to the sum of initial non-human wealth and *potential* litetime labour income. By contrast, the consumption function (17) derived in the chapter text implies that consumption depends on *actual* lifetime labour income. The difference is due to the fact that, in the present exercise, actual labour income is no longer exogenous from the viewpoint of the consumer, but rather an endogenous outcome of his choice of working hours.

We see from (A.11) that the marginal propensity to consume wealth is still between zero and one, as in the consumption function given by (17) in the chapter text. However, in the present exercise the propensity to consume does not depend on the interest rate, since the logarithmic utility function (which has an intertemporal substitution elasticity equal to one, like a Cobb-Douglas utility function) implies that the substitution and income effects of a change in the interest rate exactly offset each other. Finally, it follows from (A.11) that the propensity to spend potential income is decreasing in the parameter β, since a stronger preference for leisure will reduce labour supply, thereby reducing material consumption possibilities through a reduction of actual income.

3. Substituting (A.10) and (A.11) into (A.9), one finds that

$$F \equiv 1 - h = \left(\frac{\beta (1+\phi)}{1 + (1+\phi)(1+\beta)} \right) \left[\frac{V_1}{w(1-\tau)} + 1 \right] \Longleftrightarrow$$

$$h = 1 - \left(\frac{\beta (1+\phi)}{1 + (1+\phi)(1+\beta)} \right) \left[\frac{V_1}{w(1-\tau)} + 1 \right] \tag{A.12}$$

The first equality in (A.12) shows that the consumption of leisure will always be positive. However, according to the second equality in (A.12) one cannot exclude the possibility that h can become negative provided the stock of initial non-human wealth is sufficiently large relative to the after-tax wage rate. Of course, literally speaking labour supply cannot become negative, but an interpretation of (A.12) might be that very wealthy persons would choose to hire domestic servants to avoid having to do household work at home, thereby increasing the effective amount of time available for pure leisure.

It follows from (A.12) that a higher labour income tax rate τ will reduce labour supply provided the consumer has positive net wealth, since

$$\frac{\partial h}{\partial \tau} = -\left(\frac{\beta (1+\phi)}{1 + (1+\phi)(1+\beta)} \right) \left(\frac{w V_1}{[w(1-\tau)]^2} \right) < 0 \qquad \text{for} \qquad V_1 > 0. \tag{A.13}$$

If V_1 were zero, the income and substitution effects of a rise in the tax rate would exactly offset each other, given the logarithmic utility function assumed here. But when $V_1 > 0$ and the tax rate goes up, the relative fall in the price of leisure, $w(1-\tau)$, is larger than the relative fall in the consumer's total potential consumption, $V_1 + w(1-\tau)$. The presence of untaxed wealth reduces the importance of the income effect on the demand for leisure generated by a change in the tax rate. Hence the substitution effect on labour supply comes to dominate.

4. The consumption of leisure now becomes $F = 1 - \overline{h}$ which is exogenously given to the consumer. It is then more convenient to work with the budget constraint (A.4) which implies that

$$C_2 = (1+r)\left[V_1 + w(1-\tau)\overline{h} - C_1\right] \tag{A.14}$$

Substitution of (A.14) into (A.13) yields

$$U = \ln C_1 + \beta \ln\left(1 - \overline{h}\right) + \frac{\ln\left\{(1+r)\left[V_1 + w(1-\tau)\overline{h} - C_1\right]\right\}}{1+\phi}$$

so the first-order condition for utility maximization becomes

$$\frac{dU}{dC_1} = 0 \Longleftrightarrow \frac{1}{C_1} - \frac{1}{(1+\phi)\left[V_1 + w(1-\tau)\overline{h} - C_1\right]} = 0 \Longleftrightarrow$$

$$C_1 = \overline{\theta} \cdot \left[V_1 + w(1-\tau)\overline{h}\right], \qquad 0 < \overline{\theta} \equiv \frac{1+\phi}{2+\phi} < 1 \tag{A.15}$$

We see that consumption no longer depends on potential but rather on *actual* labour income, in accordance with the chapter text where working hours and actual labour income was also exogenous to the consumer. The propensity to consume in (A.15) is between zero and one and corresponds to the consumption propensity which emerges from eq. (17) in the chapter text when the intertemporal substitution elasticity σ is equal to one.

Exercise 16.3: Fiscal policy and consumption with credit-constrained consumers

1. Aggregate private consumption in period 1 is given by