EC 450
Advanced Macroeconomics
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Winter 2008

Assignment 4 (REQUIRED)

Submisson Deadline and Location: March 27 in Class

Total Marks: 50

Part A Short Questions [10 marks]

Explain why the following statement is True, False, or Uncertain according to economic principles. Use diagrams and / or numerical examples where appropriate. Unsupported answers will receive no marks. It is the explanation that is important. Each question is worth 10 marks.

A1. According to the growth theory, an increase in the investment rate in physical capital in a closed economy does not have any impact on the long-run growth rate of output per worker. [Diagrams required]

Part B Problem Solving Questions [40 marks]

Read each part of the question very carefully. Show all the steps of your calculations to get full marks.

B1. [40 Marks]

Consider an economy with the following Cobb-Douglas aggregate production function.

$$Y_t = K_t^{\alpha}\left(A_t L_t\right)^{1-\alpha}, \qquad 0 < \alpha < 1,$$

where Y_t is aggregate output, K_t is the stock of aggregate capital, L_t is total labor and A_t is the effectiveness of labor at period *t*. Assume that L_t grows exogenously at constant rates *n*. Capital depreciates at a constant rate δ.

The evolution of aggregate capital in the economy is given by

$$K_{t+1} - K_t = sY_t - \delta K_t, \qquad 0 < s < 1, \qquad 0 < \delta < 1,$$

where s is a constant and exogenous saving rate.

Assume that the labour productivity variable, A_t, depends on aggregate output because of learning-by-doing productive externalities which arise from workers being involved in production. The following equation shows this external learning by doing effect.

$$A_t = Y_t^{\phi}, \qquad\qquad 0 < \phi < \frac{1}{1-\alpha}$$

1. Show that the aggregate production function in this model is:

$$Y_t = K_t^{\alpha/[1-\phi(1-\alpha)]} L_t^{(1-\alpha)/[1-\phi(1-\alpha)]}.$$

Why have we assumed $\phi < \frac{1}{1-\alpha}$? Show that the aggregate production function has increasing returns to (K_t, L_t) whenever $\phi > 0$. Show also that if $\phi = 1$ it has constant returns to K_t alone. [5 marks]

2. Show that in this model:

$$\frac{A_{t+1}}{A_t} = \left(\frac{K_{t+1}}{K_t}\right)^{\alpha\phi/[1-\phi(1-\alpha)]} \left(\frac{L_{t+1}}{L_t}\right)^{[\phi(1-\alpha)]/[1-\phi(1-\alpha)]}.$$

[3 marks]

Assume now that $\phi < 1$. Let $\tilde{y}_t$ and $\tilde{k}_t$ be defined as usual: $\tilde{k}_t \equiv k_t / A_t \equiv K_t / (A_t L_t)$ and $\tilde{y}_t \equiv y_t / A_t \equiv Y_t / (A_t L_t)$.

3. Show that the transition equation for $\tilde{k}_t$ is:

$$\tilde{k}_{t+1} = \left(\frac{1}{1+n}\right)^{\frac{1}{1-\varphi(1-\alpha)}} \tilde{k}_t \left[s\tilde{k}_t^{\alpha-1} + (1-\delta)\right]^{\frac{1-\phi}{1-\phi(1-\alpha)}}$$

$$= \left(\frac{1}{1+n}\right)^{\frac{1}{1-\varphi(1-\alpha)}} \left[s\tilde{k}_t^{\frac{\alpha}{1-\phi}} + (1-\delta)\tilde{k}_t^{\frac{1-\phi(1-\alpha)}{1-\phi}}\right]^{\frac{1-\phi}{1-\phi(1-\alpha)}}$$

Find the steady state values for $\tilde{k}_t$ and $\tilde{y}_t$, and show that these are meaningful whenever $(1+n)^{1/(1-\phi)} > (1-\delta)$ (which we assume). Show also that the transition equation implies convergence to steady state. [16 marks]

4. Find the expression for the growth rate, g_{se}, of output per worker in steady state. Comment with respect to what creates growth in output per worker in this model (when $\phi < 1$). [6 marks]

Now assume $\phi = 1$ and $n = 0$, so L_t is equal to some L for all t.

5. Show that the model can be condensed to the two equations:

$$Y_t = AK_t, \qquad A \equiv L^{\frac{1-\alpha}{\alpha}},$$
$$K_{t+1} = sY_t + (1-\delta)K_t.$$

Find the growth rate, g_e, of output per worker. Comment with respect to what creates growth in output per worker in this model (when $\phi = 1$). [10 marks]