EC 450
Advanced Macroeconomics
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Assignment 3 (OPTIONAL)

Total Marks: 50

Part A Short Questions [20 marks]

For the question A1 only:

Explain why the following statement is True, False, or Uncertain according to economic principles. Use diagrams and / or numerical examples where appropriate. Unsupported answers will receive no marks. It is the explanation that is important. Each question is worth 10 marks.

A1. In the Solow model with human capital, a rise in the investment rate in human capital increases the long-run level of human capital per effective worker but the leaves the long-run level of human capital per effective worker unchanged. [Diagrams required]

A2. Consider an economy which has the following aggregate production function and human capital function:

$$Y_t = K_t^{\alpha}\left(A_t h(u_t) L_t\right)^{1-\alpha}, \qquad 0 < \alpha < 1,$$

and

$$h(u_t) = \exp(\psi u_t), \qquad \psi > 0,$$

where Y is aggregate output, K is the stock of aggregate capital, L is total labor, A is the effectiveness of labor, $h(u_t)$ is the human capital function and u is the years of education.

Derive an appropriate growth accounting formula that splits up the average annual growth rate of GDP per worker into components coming from growth in A_t, growth in capital per worker, and growth in education.

Part B Problem Solving Questions [30 marks]

Read each part of the question very carefully. Show all the steps of your calculations to get full marks.

B1. [30 Marks]

Consider the Solow model with human capital. Assume that the economy has a Cobb-Douglas aggregate production function with labor-augmenting technological progress:

$$Y_t = K_t^{\lambda} H_t^{\varphi} \left(A_t N_t\right)^{1-\lambda-\varphi}, \qquad 0<\lambda<1, \qquad 0<\varphi<1, \qquad \lambda+\varphi<1,$$

where Y is aggregate output, K is the stock of aggregate physical capital, H the stock of aggregate capital, N is total labor and A is the effectiveness of labor. Assume that L and A grow exogenously at constant rates n and g, respectively. Capital depreciates at a constant rate δ.

In this economy, hiring one more (marginal) unit of labor means hiring one more unit endowed with the average amount, h_t, of human capital per worker. Hence, a firm cannot increase the input of 'raw' labor, N_t, without increasing proportionally the input of human capital, $H_t = h_t L_t$.

Denote with lower case letters the variables in unit of effective worker. That means,

$$\tilde{y} \equiv \frac{Y}{AL}, \ \tilde{k} \equiv \frac{K}{AL} \text{ and } \tilde{h} \equiv \frac{H}{AL}.$$

The evolution of aggregate physical capital in the economy is given by

$$K_{t+1} - K_t = s_K Y_t - \delta K_t$$

where s_K is a constant and exogenous physical capital investment rate.

Similarly, the evolution of aggregate human capital in the economy is given by

$$H_{t+1} - H_t = s_H Y_t - \delta H_t$$

where s_H is a constant and exogenous human capital investment rate.

(a) Derive the laws of motion, or the transition equations, for physical capital per effective worker and human capital per effective worker. [4 marks]

(b) Solve for the steady state equilibrium values of physical capital per effective worker, human capital per effective worker and output per effective worker. [6 marks]

(c) Draw a phase diagram and illustrate the steady-state equilibrium values of physical capital per effective worker and human capital per effective worker. Explain how the economy converges to the steady-state equilibrium starting from a given set of initial positive levels of physical capital per effective worker and human capital per effective worker. [6 marks]

(d) Find the growth rates of physical capital per effective worker, human capital per effective worker, output per effective worker, output per worker, physical capital per worker, human capital per worker, aggregate output, aggregate capital and aggregate human capital on the balanced growth path. [9 marks]

(e) Find the steady-state equilibrium value of output per worker. Find also the elasticity of the steady-state equilibrium output per worker with respect to the physical capital investment rate. Compare this elasticity with its counterpart in the general Solow model. Which elasticity is larger? Explain why? [5 marks]