EC 450
Advanced Macroeconomics
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Assignment 2 (OPTIONAL)

Total Marks: 50

Part A — Short Questions — [20 marks]

For the question A1 only:

Explain why the following statement is True, False, or Uncertain according to economic principles. Use diagrams and / or numerical examples where appropriate. Unsupported answers will receive no marks. It is the explanation that is important. Each question is worth 10 marks.

A1. In the general Solow model, an increase in the savings rate raises the long-run growth rate of aggregate output per worker. [Diagrams required]

False

In the general Solow model with exogenous technological progress, an increase in the level of the savings rate will have no effect on the long-run growth rate of output per worker, y_t. It will cause a shift in the steady-state growth path of y_t from one level to a new and higher level, with the long-run growth rate being the same before and after as the rate of exogenous technological progress, namely g.

See Figure 5.3, Figure 5.4, Figure 5.5, Figure 5.6, and pages 135 to 138 of the textbook for a graphical explanation.

A2. Consider a general Solow economy on its balanced growth path. Assume that the production function is Cobb-Douglas:

$$Y_t = K_t^{\lambda}(A_t N_t)^{1-\lambda}, \qquad 0 < \lambda < 1,$$

where Y is aggregate output, K is the stock of aggregate capital, N is total labor and A is the effectiveness of labor. Assume that N and A grow exogenously at constant rates n and g, respectively. Suppose the growth accounting techniques are applied to this economy.

(a) What fraction of growth in output per worker does growth accounting attribute to growth in capital per worker? What fraction does it attribute to technological progress? [5 marks]

Define $y_t \equiv \frac{Y_t}{N_t}$ and $k_t \equiv \frac{K_t}{N_t}$. Thus, y_t denotes denote output per worker and k_t denote capital per worker.

Production function: $Y_t = K_t^{\lambda}(A_t N_t)^{1-\lambda}$ (1)

Dividing both sides of the production function, equation (1), by N_t we get:

$$\frac{Y_t}{N_t} = \frac{K_t^{\lambda}(A_t N_t)^{1-\lambda}}{N_t}$$

$$\Rightarrow \frac{Y_t}{N_t} = \frac{A_t^{1-\lambda}K_t^{\lambda}}{N_t^{1-1+\lambda}}$$

$$\Rightarrow \frac{Y_t}{N_t} = \frac{A_t^{1-\lambda}K_t^{\lambda}}{N_t^{\lambda}}$$

$$\Rightarrow \frac{Y_t}{N_t} = A_t^{1-\lambda}\left(\frac{K_t}{N_t}\right)^{\lambda}$$

$$\Rightarrow y_t = A_t^{1-\lambda}k_t^{\lambda} \qquad (2) \qquad \text{[Using the definition of } y_t \text{ and } k_t\text{]}$$

For another and later year $T > t$, we have:

$$y_T = A_T^{1-\lambda}k_T^{\lambda} \qquad (3)$$

If we take logs on both sides of (2) and (3), we get:

$$\ln y_t = (1-\lambda)\ln A_t + \lambda \ln k_t \qquad (4)$$

$$\ln y_T = (1-\lambda)\ln A_T + \lambda \ln k_T \qquad (5)$$

Subtracting (4) from (5), and divide on both sides by *T-t*, we get:

$$\frac{\ln y_T - \ln y_t}{T-t} = (1-\lambda)\frac{\ln A_T - \ln A_t}{T-t} + \lambda \frac{\ln k_T - \ln kt}{T-t} \qquad (6)$$

Let g^y denote the growth rate in output per worker, g^A denote the growth rate of A_t and g^k denote the growth rate in capital per worker. So, we can rewrite (6) as,

$$g^y = (1-\lambda)g^A + \lambda g^k \qquad (7)$$

The growth-accounting equation (7) splits up the growth in GDP per worker into contributions from growth in capital per worker and growth in the effectiveness of labor (technological progress). Now imagine applying this growth-accounting equation to a Solow economy that is on its balanced growth path. On the balanced growth path, the growth rates of output per worker and capital per worker are both equal to *g*, the growth rate of *A*. Thus equation (1) implies that growth accounting would attribute a fraction λ of growth in output per worker to growth in capital per worker. It would attribute the rest – fraction $(1-\lambda)$ - to technological progress. So, with our usual estimate of $\lambda = 1/3$, growth accounting would attribute 67 percent of the growth in output per worker to technological progress and about 33 percent of the growth in output per worker to growth in capital per worker.

(b) How can you reconcile your results in (a) with the fact that the general Solow model implies that the growth rate of output per worker on the balanced growth path is determined solely by the rate of technological progress? [5 marks]

In an accounting sense, the result in part **(a)** would be true, but in deeper sense it would not: the reason that the capital-labor ratio grows at rate *g* on the balanced growth path is because the effectiveness of labor is growing at rate *g*. That is, the growth in the effectiveness of labor – the growth in A – raises output per worker through two channels. It raises output per worker not only by directly raising output but also by (for a given saving rate) increasing the resources devoted to capital accumulation and thereby raising the capital-labor ratio. Growth accounting attributes the rise in output per worker through the second channel to growth in the capital-labor ratio, not to its underlying source. Thus, although growth accounting is often instructive, it is not appropriate to interpret it as shedding light on the underlying determinants of growth.

Part B   Problem Solving Questions   [30 marks]

Read each part of the question very carefully. Show all the steps of your calculations to get full marks.

B1. [30 Marks]

Consider a general Solow economy with labor-augmenting technological progress.

$Y_t = K_t^{\lambda}(A_t N_t)^{1-\lambda}$, $0 < \lambda < 1$,

where *Y* is aggregate output, *K* is the stock of aggregate capital, *N* is total labor and *A* is the effectiveness of labor. Assume that *N* and *A* grow exogenously at constant rates *n* and *g*, respectively. Capital depreciates at a constant rate δ. Denote with lower case letters the variables in unit of effective labor. That means,

$$\tilde{y} \equiv \frac{Y}{AN} \quad \textbf{and} \quad \tilde{k} \equiv \frac{K}{AN}.$$

The evolution of aggregate capital in the economy is given by

$$K_{t+1} - K_t = sY_t - \delta K_t$$

where s is a constant and exogenous saving rate.

(a) Derive the law of motion, or the transition equation, for capital per effective worker. What is the economic interpretation of this equation? Plot the transition equation in a diagram. Explain how the level of capital per worker will converge to the steady state value from a given positive initial value. [7 marks]

Define $\tilde{y}_t \equiv \frac{Y_t}{A_t N_t}$ and $\tilde{k}_t \equiv \frac{K_t}{A_t N_t}$. Thus, $\tilde{y}_t$ denotes denote output per effective worker and $\tilde{k}_t$ denote capital per effective worker.

Production function: $Y_t = K_t^{\lambda}(A_t N_t)^{1-\lambda}$ (1)

An exogenous growth in labor: $N_{t+1} = (1+n)N_t$ (2)

An exogenous growth in the effectiveness of labor : $A_{t+1} = (1+g)A_t$ (3)

The evolution equation of aggregate capital: $K_{t+1} - K_t = sY_t - \delta K_t$ (4)

Dividing both sides of the production function, equation (1), by $A_t N_t$ we get:

$$\frac{Y_t}{A_t N_t} = \frac{K_t^{\lambda}(A_t N_t)^{1-\lambda}}{A_t N_t}$$

$$\Rightarrow \frac{Y_t}{A_t N_t} = \frac{K_t^{\lambda}}{(A_t N_t)^{1-1+\lambda}}$$

$$\Rightarrow \frac{Y_t}{A_t N_t} = \frac{K_t^{\lambda}}{(A_t N_t)^{\lambda}}$$

$$\Rightarrow \frac{Y_t}{A_t N_t} = \left(\frac{K_t}{A_t N_t}\right)^{\lambda}$$

$$\Rightarrow \tilde{y}_t = \tilde{k}_t^{\lambda} \qquad (5) \qquad \text{[Using the definitions of } \tilde{y}_t \text{ and } \tilde{k}_t\text{]}$$

Rewrite equation (4) as:

$$K_{t+1} = sY_t + (1-\delta)K_t . \qquad (4.1)$$

Dividing both sides of equation (4.1) by $A_{t+1}N_{t+1}$ we get:

$$\frac{K_{t+1}}{A_{t+1}N_{t+1}} = \frac{sY_t + (1-\delta)K_t}{A_{t+1}N_{t+1}}$$

$$\Rightarrow \frac{K_{t+1}}{A_{t+1}N_{t+1}} = \frac{sY_t + (1-\delta)K_t}{(1+g)A_t(1+n)N_t} \qquad \text{[Using equations (2) and (3)]}$$

$$\Rightarrow \frac{K_{t+1}}{A_{t+1}N_{t+1}} = \frac{1}{(1+n)(1+g)}\left(s\frac{Y_t}{A_t N_t} + (1-\delta)\frac{K_t}{A_t N_t}\right)$$

$$\Rightarrow \tilde{k}_{t+1} = \frac{1}{(1+n)(1+g)}\left(s\tilde{y}_t + (1-\delta)\tilde{k}_t\right) \quad \text{[Using the definitions of } \tilde{y}_t \text{ and } \tilde{k}_t\text{]}$$

$$\Rightarrow \tilde{k}_{t+1} = \frac{1}{(1+n)(1+g)}\left(s\tilde{k}_t^{\lambda} + (1-\delta)\tilde{k}_t\right) \qquad \text{[Substituting } \tilde{y}_t \text{ using (5)]}$$

Therefore, the transition equation for capital per effective worker is:

$$\tilde{k}_{t+1} = \frac{1}{(1+n)(1+g)}\left(s\tilde{k}_t^{\lambda} + (1-\delta)\tilde{k}_t\right) \qquad (6)$$

Figure B1(a) shows the transition equation as given by (6). This curve starts at (0,0) and is everywhere increasing. The 45^0 line, $k_{t+1} = k_t$, has also been drawn.

Differentiating (6) gives:

$$\frac{d\tilde{k}_{t+1}}{d\tilde{k}_t} = \frac{s\lambda A\tilde{k}_t^{\lambda-1} + (1-\delta)}{(1+n)(1+g)}.$$

This shows that the slope of the transition curve decreases monotonically from infinity to $(1-\delta)/(1+n)(1+g)$, as $\tilde{k}_t$ increases from zero to infinity. The latter slope is positive and less than one if $n + g + \delta + ng > 0$, which is empirically plausible. Hence the transition curve must have a unique intersection with the 45^0 line (which has slope one), to the right of $\tilde{k}_t = 0$.

Assume an initial level of capital per worker $\tilde{k}_0 > 0$, as drawn in Figure B1(a). Then $\tilde{k}_1$ will be the vertical distance from the horizontal axis up to the transition curve, and by going from the associated point on the transition curve horizontally to the 45^0 line, and then vertically down, $\tilde{k}_1$ will be taken to the horizontal axis as shown. Now, $\tilde{k}_2$ will be the vertical distance up to the transition curve. The dynamic evolution of the capital intensity (capital per worker) is given by the staircase broken line. It follows that over time, $\tilde{k}_t$ will converge to the specific value given by the intersection between the transition curve and the 45^0 line, and it will do so monotonically, getting closer and closer all the time and never transcending to the other side of the intersection point. Furthermore, the convergence is global in the sense that it holds for any strictly positive initial $\tilde{k}_0$.

The intersection between the transition curve and the 45^0 line is the unique positive solution, $\tilde{k}^*$, which is obtained by setting $\tilde{k}_{t+1} = \tilde{k}_t = \tilde{k}$ in (6) and solving for $\tilde{k}$. This $\tilde{k}^*$ is called the steady state value of capital per effective worker.

(b) Solve for the steady state equilibrium values of capital per effective worker, output per effective worker, consumption per effective worker, the real rental rate for capital and the real wage rate for labor. Illustrate the steady-state equilibrium values of capital per effective worker, output per effective worker and consumption per effective worker in a diagram. [7 marks]

To solve for the steady state equilibrium value of aggregate capital per effective worker we set $\tilde{k}_{t+1} = \tilde{k}_t = \tilde{k}$ in (6),

$$\tilde{k}_{t+1} = \frac{1}{(1+n)(1+g)}\left(s\tilde{k}_t^{\lambda} + (1-\delta)\tilde{k}_t\right)$$

$$\Rightarrow \tilde{k} = \frac{1}{(1+n)(1+g)}\left(s\tilde{k}^{\lambda} + (1-\delta)\tilde{k}\right)$$

$$\Rightarrow \tilde{k}(1+n)(1+g) = s\tilde{k}^{\lambda} + (1-\delta)\tilde{k}$$

$$\Rightarrow \tilde{k} + n\tilde{k} + g\tilde{k} + ng\tilde{k} = s\tilde{k}^{\lambda} + \tilde{k} - \delta\tilde{k}$$

$$\Rightarrow \tilde{k} + n\tilde{k} + g\tilde{k} + ng\tilde{k} - \tilde{k} + \delta\tilde{k} = s\tilde{k}^{\lambda}$$

$$\Rightarrow (n+g+\delta+ng)\tilde{k} = s\tilde{k}^{\lambda}$$

$$\Rightarrow \frac{\tilde{k}}{\tilde{k}^{\lambda}} = \frac{s}{(n+g+\delta+ng)}$$

$$\Rightarrow \tilde{k}^{(1-\lambda)} = \left(\frac{s}{n+g+\delta+ng}\right)$$

$$\Rightarrow \left(\tilde{k}^{(1-\lambda)}\right)^{\frac{1}{1-\lambda}} = \left(\frac{s}{n+g+\delta+ng}\right)^{\frac{1}{1-\lambda}}$$

$$\therefore \tilde{k}^{*} = \left(\frac{s}{n+g+\delta+ng}\right)^{\frac{1}{1-\lambda}} \qquad (7)$$

Therefore, the steady state equilibrium value of aggregate capital per effective worker is $\left(\frac{s}{n+g+\delta+ng}\right)^{\frac{1}{1-\lambda}}$.

The steady state equilibrium value of aggregate output per effective worker, $\tilde{y}^*$, is:

$$\tilde{y}^* = \tilde{k}^{*\lambda}$$

$$\Rightarrow \tilde{y}^* = \left[\left(\frac{s}{n+g+\delta+ng} \right)^{\frac{1}{1-\lambda}} \right]^{\lambda}$$

$$\therefore \tilde{y}^* = \left(\frac{s}{n+g+\delta+ng} \right)^{\frac{\lambda}{1-\lambda}}. \qquad (8)$$

Consumption per effective worker is $\tilde{c}_t = (1-s)\tilde{y}_t$ in any period t. So, the steady state equilibrium value of consumption per effective worker is:

$$\tilde{c}^* = (1-s)\tilde{y}^* = (1-s)\left(\frac{s}{n+g+\delta+ng} \right)^{\frac{\lambda}{1-\lambda}}. \qquad (9)$$

The real rental rate for capital, r_t, and the real wage rate for labor, w_t, are given by the following two profit maximizing conditions of the competitive firms in this model:

$$r_t = \text{Marginal product of capital} = \frac{dY_t}{dK_t} = \lambda K_t^{\lambda-1}(A_t N_t)^{1-\lambda} = \lambda \left(\frac{K_t}{A_t N_t} \right)^{\lambda-1} = \lambda \tilde{k}_t^{\lambda-1}$$

and

$$w_t = \text{Marginal product of labor}$$

$$= \frac{dY_t}{dN_t} = (1-\lambda) K_t^{\lambda} (A_t N_t)^{-\lambda} A_t = (1-\lambda) \left(\frac{K_t}{A_t N_t} \right)^{\lambda} A_t = (1-\lambda)\tilde{k}_t^{\lambda} A_t.$$

So, the steady state equilibrium values of r_t and w_t are:

$$r^* = \lambda(\tilde{k}^*)^{\lambda-1} = \lambda \left[\left(\frac{s}{n+g+\delta+ng} \right)^{\frac{1}{1-\lambda}} \right]^{\lambda-1} = \lambda \left(\frac{s}{n+g+\delta+ng} \right)^{-1}$$

and

$$w^* = (1-\lambda)(\tilde{k}^*)^{\lambda} A_t = (1-\lambda) \left[\left(\frac{s}{n+g+\delta+ng} \right)^{\frac{1}{1-\lambda}} \right]^{\lambda} A_t = (1-\lambda) \left(\frac{s}{n+g+\delta+ng} \right)^{\frac{\lambda}{1-\lambda}} A_t .$$

Figure B1(b) illustrates the steady state equilibrium levels of capital per effective worker, output per effective worker and consumption per effective worker.

(c) Find the growth rates of capital per effective worker, output per effective worker, output per labor, capital per labor, aggregate output and aggregate capital on the balanced growth path. [6 marks]

On the balanced growth path (at the steady-state equilibrium) the level of aggregate capital per effective worker is: $\tilde{k}^* = \left(\frac{s}{n+g+\delta+ng}\right)^{\frac{1}{1-\lambda}}$, which is a constant. So, the growth rate of $\tilde{k}^*$ is zero.

Similarly, on the balanced growth path (at the steady state equilibrium) the level of aggregate output per effective worker is: $\tilde{y}^* = \left(\frac{s}{n+g+\delta+ng}\right)^{\frac{\lambda}{1-\lambda}}$, which is a constant. So, the growth rate of $\tilde{y}^*$ is zero.

Define $y_t \equiv \frac{Y_t}{N_t}$ and $k_t \equiv \frac{K_t}{N_t}$. Thus, y_t denotes denote output per worker and k_t denote capital per worker.

Let the approximate growth rates of A_t, $\tilde{y}_t$, y_t, $\tilde{k}_t$ and k_t period $t-1$ to t be denoted by g_t^A, $g_t^{\tilde{y}}$, g_t^y, $g_t^{\tilde{k}}$ and g_t^k, respectively. Using the definitions of $\tilde{y}_t$ and y_t we can express output per worker as,

$$y_t = \tilde{y}_t A_t \qquad (10)$$

Taking logs on both sides of (10),

$$\ln y_t = \ln \tilde{y}_t + \ln A_t \qquad (11)$$

So, we can also write,

$$\ln y_{t-1} = \ln \tilde{y}_{t-1} + \ln A_{t-1} \qquad (12)$$

Subtracting (12) from (11) we get,

$$\ln y_t - \ln y_{t-1} = \ln \tilde{y}_t - \ln \tilde{y}_{t-1} + \ln A_t - \ln A_{t-1}$$

$$\Rightarrow g_t^y = g_t^{\tilde{y}} + g_t^A$$

$$\Rightarrow g_t^y = g_t^{\tilde{y}} + g$$

Since on the balanced growth path $g_t^{\tilde{y}}$ is zero, the growth rate of output per worker, g_t^y on the balanced growth path must be equal to growth in the effectiveness of labor, g.

Using the definition of $\tilde{k}_t$ and k_t we can express capital per worker as,

$$k_t = \tilde{k}_t A_t \qquad (13)$$

Taking logs on both sides of (13),

$$\ln k_t = \ln \tilde{k}_t + \ln A_t \qquad (14)$$

So, we can also write,

$$\ln k_{t-1} = \ln \tilde{k}_{t-1} + \ln A_{t-1} \qquad (15)$$

Subtracting (15) from (14) we get,

$$\ln k_t - \ln k_{t-1} = \ln \tilde{k}_t - \ln \tilde{k}_{t-1} + + \ln A_t - \ln A_{t-1}$$
$$\Rightarrow g_t^k = g_t^{\tilde{k}} + g_t^A$$
$$\Rightarrow g_t^k = g_t^{\tilde{k}} + g$$

Since on the balanced growth path $g_t^{\tilde{k}}$ is zero, the growth rate of capital per worker, g_t^k on the balanced growth path must be equal to growth in the effectiveness of labor, g.

Let the approximate growth rates of Y_t, K_t and N_t from period $t-1$ to t be denoted by g_t^Y, g_t^K and g_t^N. Using the definition of $\tilde{y}_t$ we can express aggregate output as,

$$Y_t = \tilde{y}_t A_t N_t \qquad (16)$$

Taking logs on both sides of (16),

$$\ln Y_t = \ln \tilde{y}_t + \ln A_t + \ln N_t \qquad (17)$$

So, we can also write,

$$\ln Y_{t-1} = \ln \tilde{y}_{t-1} + \ln A_{t-1} + \ln N_{t-1} \qquad (18)$$

Subtracting (18) from (17) we get,

$$\ln Y_t - \ln Y_{t-1} = \ln \tilde{y}_t - \ln \tilde{y}_{t-1} + \ln A_t - \ln A_{t-1} + \ln N_t - \ln N_{t-1}$$
$$\Rightarrow g_t^Y = g_t^{\tilde{y}} + g_t^A + g_t^N$$
$$\Rightarrow g_t^Y = g_t^{\tilde{y}} + g + n$$

Since on the balanced growth path $g_t^{\tilde{y}}$ is zero, the growth rate of aggregate output, g_t^Y, on the balanced growth path must be equal to the growth rate of labor plus the growth rate of the effectiveness of labor, $(n+g)$.

Using the definition of $\tilde{k}_t$ we can express the aggregate capital as,

$$K_t = \tilde{k}_t A_t N_t \qquad (19)$$

Taking logs on both sides of (19),

$$\ln K_t = \ln \tilde{k}_t + \ln A_t + \ln N_t \qquad (20)$$

So, we can also write,

$$\ln K_{t-1} = \ln \tilde{k}_{t-1} + \ln A_{t-1} + \ln N_{t-1} \qquad (21)$$

Subtracting (21) from (20) we get,

$$\ln K_t - \ln K_{t-1} = \ln \tilde{k}_t - \ln \tilde{k}_{t-1} + \ln A_t - \ln A_{t-1} + \ln N_t - \ln N_{t-1}$$

$$\Rightarrow g_t^K = g_t^{\tilde{k}} + g_t^A + g_t^N$$

$$\Rightarrow g_t^K = g_t^{\tilde{k}} + g + n$$

Since on the balanced growth path $g_t^{\tilde{k}}$ is zero, the growth rate of aggregate capital, g_t^K, on the balanced growth path must be equal to the growth rate of labor plus the growth rate of the effectiveness of labor, $(n+g)$.

(d) Find the elasticity of the steady state equilibrium value of output per worker with respect to the saving rate. For an empirically reasonable value of the capital share discuss whether the steady state prediction of the general Solow model about the influences of the saving rate on output per worker is consistent with the data.
[3 marks]

Using the definition of $\tilde{y}_t$, the steady state equilibrium value of output per worker, y_t^*, is:

$$y_t^* = A_t \tilde{y}^*$$

$$\therefore y_t^* = A_t \left(\frac{s}{n + g + \delta + ng} \right)^{\frac{\lambda}{1-\lambda}}. \qquad (22) \quad \text{[using equation (8)]}$$

Taking logs on both sides of (22) gives:

$$\ln y_t^* = \frac{1}{1-\lambda} \ln A_t + \frac{\lambda}{1-\lambda} \left[\ln s - \ln(n + g + \delta + ng) \right]. \qquad (23)$$

The elasticity of y_t^* with respect to the saving rate, $s \equiv \dfrac{d \ln y^*}{d \ln s} = \dfrac{\lambda}{1-\lambda}$.

See pages 140-141 of the textbook for an answer to the second part of Question **(d)**.

(e) Find the golden rule savings rate and the golden rule level of capital per effective worker. Solve for the steady state equilibrium values of output per effective worker and consumption per effective worker assuming that the exogenous saving rate is equal to the golden rule savings rate. Illustrate the golden rule levels of capital per effective worker, output per effective worker and consumption per effective worker in a diagram. [7 marks]

The steady state equilibrium value of consumption per effective worker is:

$$\tilde{c}^* = (1-s)\left(\frac{s}{n+g+\delta+ng}\right)^{\frac{\lambda}{1-\lambda}}. \qquad (9)$$

The golden rule saving rate, s^{**}, is the steady-state consumption per effective worker maximizing saving rate. So, to find s^{**} we have to maximize $\tilde{c}^*$ with respect to *s* using (9). We are allowed to take logs before maximizing. Taking logs on both sides of (9) gives:

$$\ln \tilde{c}^* = \ln(1-s) + \frac{\lambda}{1-\lambda}\ln s - \frac{\lambda}{1-\lambda}\ln(n+g+\delta+ng).$$

So, the first-order condition of this maximization problem:

$$\frac{\partial \ln \tilde{c}^*}{\partial s} = -\frac{1}{1-s} + \frac{\lambda}{1-\lambda}\frac{1}{s} = 0$$

$$\Rightarrow \frac{\lambda}{1-\lambda}\frac{1}{s} = \frac{1}{1-s}$$

$$\Rightarrow \lambda(1-s) = s(1-\lambda)$$

$$\Rightarrow \lambda - \lambda s = s - \lambda s$$

$$\therefore s^{**} = \lambda.$$

So, the golden rule saving rate is λ.

To find the golden rule level of capital per effective worker, $\tilde{k}^{**}$, we have to substitute *s* with λ in (7):

$$\tilde{k}^{**} = \left(\frac{\lambda}{n+g+\delta+ng}\right)^{\frac{1}{1-\lambda}}$$

To find the golden rule level of output per effective worker, $\tilde{y}^{**}$, we have to substitute *s* with λ in (8):

$$\tilde{y}^{**} = \left(\frac{\lambda}{n + g + \delta + ng} \right)^{\frac{\lambda}{1-\lambda}}.$$

To find the golden rule level of consumption per effective worker, $\tilde{c}^{**}$, we have to substitute *s* with λ in (9):

$$\tilde{c}^{**} = (1 - \lambda)\left(\frac{\lambda}{n + g + \delta + ng} \right)^{\frac{\lambda}{1-\lambda}}.$$

Figure B1(e) illustrates the golden rule levels of capital per worker, output per worker and consumption per worker.