ONLINE APPENDIX:
Testing the Present-Value Model of the Exchange Rate with Commodity Currencies

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A. Data Sources

1. Canada

x: The commodity price x is the log of the Bank of Canada's monthly commodity price index, BCPI (label M.BCPI) expressed in real terms by division by the US CPI, series CPIAUCSL from FRED. The BCPI is a chain Fisher index. As of 2019, the largest components are prices of two grades of oil: West Texas Intermediate (24.8%) and Western Canada Select (21.1%). The main component x_m is the energy component of this index (label M.ENER).

d: The main monetary policy indicator is the difference between the overnight interest rate in Canada and the effective federal funds rate in the US. At the US ZLB from December 2008 to November 2015 we instead use the shadow rate constructed by Wu and Xia (2016). At the Canadian ZLB from April 2009 to June 2010 we use the shadow rate constructed by MacDonald and Popiel (2020). Each of these is based on a term structure model. The estimation instruments Δd_t and so defends against some forms of measurement error.

s: The exchange rate is the log of the monthly average price of the CAD in USD, the inverse of series EXCAUS from FRED.

2. Australia

x: The commodity price x is the log of the Reserve Bank of Australia's monthly commodity price index, in USD, (series GRCPAIUSD), expressed in real terms by division by the US CPI. The Reserve Bank of Australia's Index of Commodity Prices (ICP) is a Laspeyres index, with weights periodically updated. The largest components are the prices of iron ore (with a weight of 28%) and metallurgical coal (with a weight of 16.6%). The main component x_m is the base metals component of this index (series GRCPBMUSD).

d: The policy indicator for Australia is the monthly average of the cash rate target after August 1990 when it was introduced. Before that we use the interbank overnight cash rate. The source is `f01hist.xls` from the RBA. Then d subtracts the effective federal funds rate described above.

s: The exchange rate is the log of the monthly average price of the AUD in USD, the series EXUSAL from FRED

3. New Zealand

x: The commodity price x is the log of the ANZ commodity price index, in USD, expressed in real terms by division by the US CPI. Weights in the ANZ index are based on shares of commodity exports. For 2019 the largest components are the prices of dairy products (38%), lamb (12.1%), and beef (9.9%). The main component x_m thus is the dairy products component of this index. The source is `www.anz.co.nz/about-us/economic-markets-research/commodity-price-index/`

d: The policy rate is the official cash rate (OCR) from March 1999 when it was introduced. Before that we use the overnight interbank cash rate. The source is `hb2-monthly.xls` from the RBNZ. Then d subtracts the effective federal funds rate described above.

s: The exchange rate is the log of the monthly average price of the NZD in USD, the series EXUSNZ from FRED.

Note: Norway adopted inflation targeting in 2001, after stabilizing the exchange rate to varying degrees during the 1990s. We omit it from our study because this shorter time span will limit the power of statistical tests.

B. Correlations

The correlation coefficients between Δs_t and Δx_t are positive: 0.45 for Canada, 0.34 for Australia, and 0.17 for New Zealand. The same is true of the correlations between Δs_t and $\Delta x_{m,t}$: 0.36 for Canada, 0.48 for Australia, and 0.23 for New Zealand. Whether measured with the overall index or its main component, a commodity-price increase thus is associated with a nominal appreciation. We report these correlations for growth rates to allow for the possibility that the commodity prices and exchange rates are nonstationary.

C. Figures

In figures 1–3 the upper panels show the monthly values of the export commodity price indexes and values of the local currencies in USD (S_t) since 1985 for Canada and Australia and since 1986 for New Zealand. The middle panels instead graph the the value of the currency with the main component of each commodity price index. The lower panels show the interest differentials.

D. Monetary Model

To derive the model in the text, begin with the traditional, monetary model of the exchange rate. There are two interest rates: i_m is the market rate and affects the demand for money and the forward exchange rate; i is policy rate set by the central bank and applies to banks and their reserves.

Call e the nominal exchange rate in domestic currency units. Let λ be the semi-elasticity of money demand with respect to the market interest rate:

$$m - p = -\lambda i_m \qquad\qquad m^* - p^* = -\lambda i_m^*. \tag{D1}$$

PPP and UIP hold:

$$e - p + p^* = 0 \qquad\qquad E_t e_{t+1} - e_t = i_m - i_m^*, \tag{D2}$$

so

$$e_t = \frac{1}{1-\lambda}(m_t - m_t^*) + \frac{\lambda}{1-\lambda} E_t e_{t+1}. \tag{D3}$$

Define $\beta \equiv \lambda/(1-\lambda)$ and change units from local currency to USD so $s_t \equiv -e_t$:

$$s_t = (1-\beta)(m_t^* - m_t) + \beta E_t s_{t+1}. \tag{D4}$$

Finally, α is the semi-elasticity of the money supply with respect to the policy rate i. Thus we have money supply curves:

$$m = -\alpha i \qquad\qquad m^* = -\alpha i^*, \tag{D5}$$

so that tightening policy raises i which reduces the money supply. Substitute (D5) in (D4) and recall $d \equiv i - i^*$:

$$s_t = (1-\beta)\alpha d_t + \beta E_t s_{t+1}. \tag{D6}$$

E. Cointegration

The text notes that a unit root in x_t is inherited by d_t and s_t. This section demonstrates this and shows the cointegrating relationships. Begin with the commodity price index following an autonomous random walk:

$$x_t = x_{t-1} + \epsilon_{xt}. \tag{E1}$$

Next, suppose that the international interest differential reacts to the commodity price and to the current value of the exchange rate:

$$d_t = \gamma_s s_t + \gamma_x x_t + \epsilon_{dt}, \tag{E2}$$

where ϵ_{dt} is a martingale difference series. This is not intended as a complete description of the relative policy rule, for it excludes other variables such as inflation and also excludes dynamics from interest-rate smoothing. And we do not try to identify the parameters of the interest-differential reaction function (E2). Rather, we use it to show that one can estimate and test the present-value model even though d_t reacts to s_t and x_t. At monthly frequency it makes sense to allow for monetary policy to react to both commodity prices and the exchange rate (among other variables). Thus s_t and d_t are determined simultaneously. We are assuming, but not testing, that policy reacts to these variables because they are indicators of future inflation, which is the true target of monetary policy. Bernanke, Gertler, and Watson (1997) argued that effects of oil-price shocks on macroeconomic variables are intermediated through reactions of monetary policy. Parsley and Popper (2014) show that monetary-policy reactions to the nominal exchange rate in Canada from 1999 to 2008 were consistent with an inflation target.

Using the present-value relation (1), the law of motion (E1), and the reaction function (E2), the guess-and-verify method shows that the projections of the two endogenous variables on x_t are:

$$\begin{aligned} P(s_t|x_t) &= \frac{\alpha\gamma_x}{1-\alpha\gamma_s} x_t \\ P(d_t|x_t) &= \frac{\gamma_x}{1-\alpha\gamma_s} x_t. \end{aligned} \tag{E3}$$

This simplest example illustrates two points. First, d_t and s_t are each cointegrated with x_t. The cointegrating vectors also are given by equations (E3) regardless of higher-order dynamics. For example, interest-rate smoothing might add a lagged, value d_{t-1} to equation (E2) but that would not affect the long-run relationships (E3).

Second, this long-run information can be used to identify α. The coefficients in (E3) show that α is given by the ratio of the two cointegrating vectors.

F: References

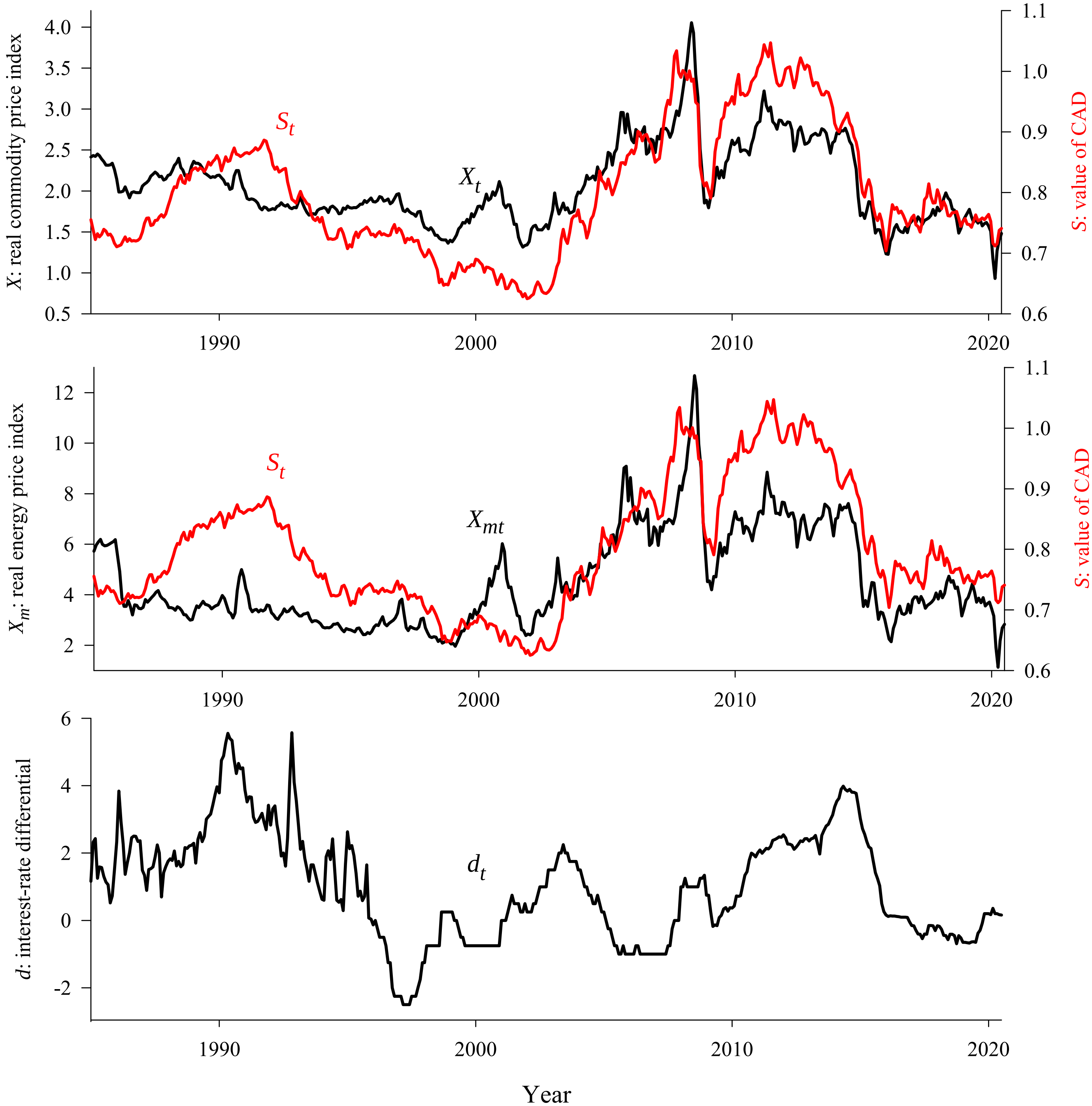
Bernanke, Ben S., Mark Gertler, and Mark Watson (1997) Systematic monetary policy and the effect of oil price shocks. *Brookings Papers on Economic Acitivity* 1, 91–157.

MacDonald, Margaux and Michal Popiel (2020) Unconventional monetary policy in a small open economy. *Open Economies Review* forthcoming

Parsley, David and Helen Popper (2014) Gauging exchange rate targeting. *Journal of International Money and Finance* 43, 155–166.

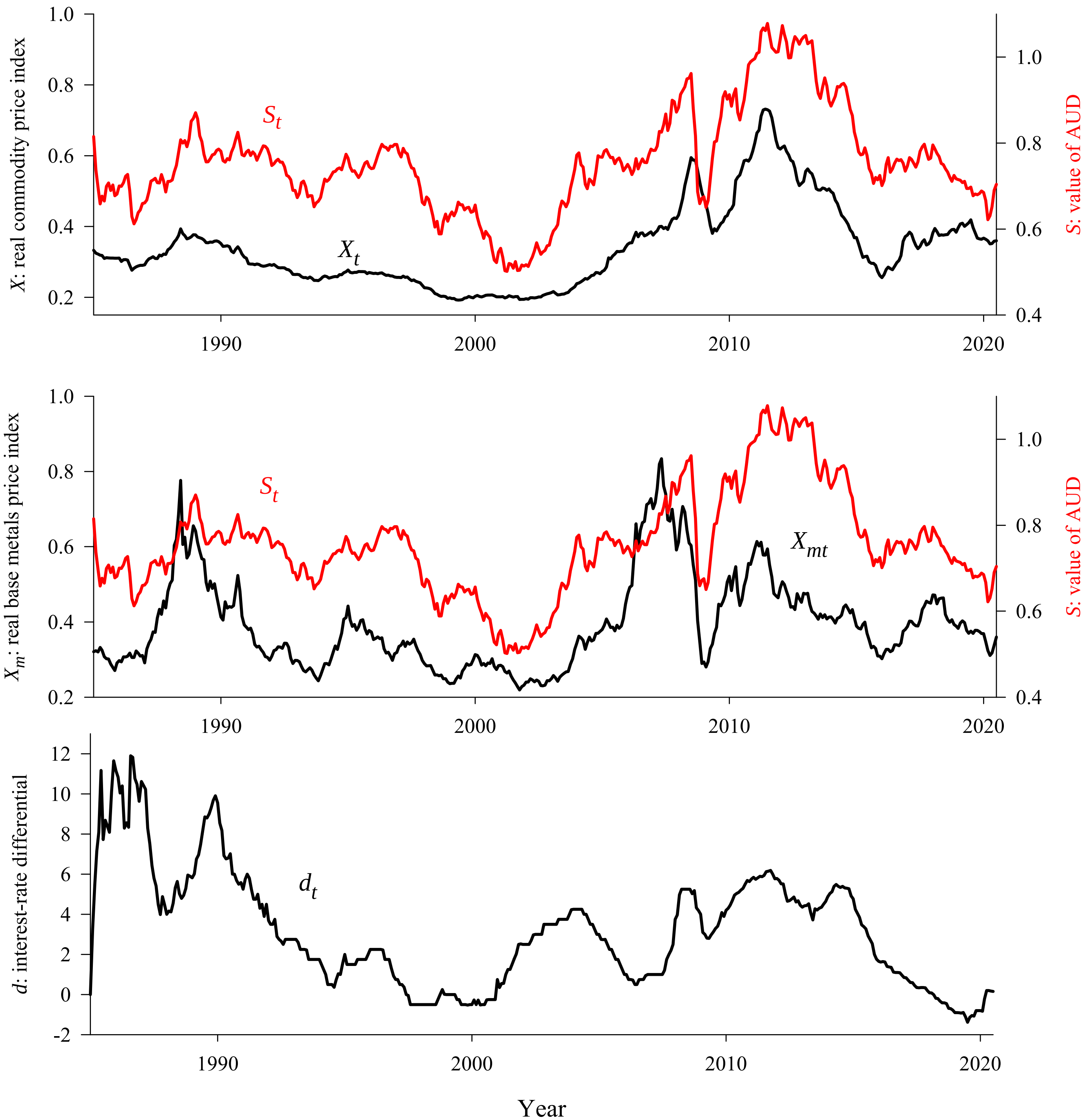
Wu, Jing Cynthia and Fan Dora Xia (2016) Measuring the macroeconomic impact of monetary policy at the zero lower bound. *Journal of Money, Credit and Banking* 48, 253–291.

Figure 1: Commodity Prices, Exchange Rates, and Interest Rates
Canada



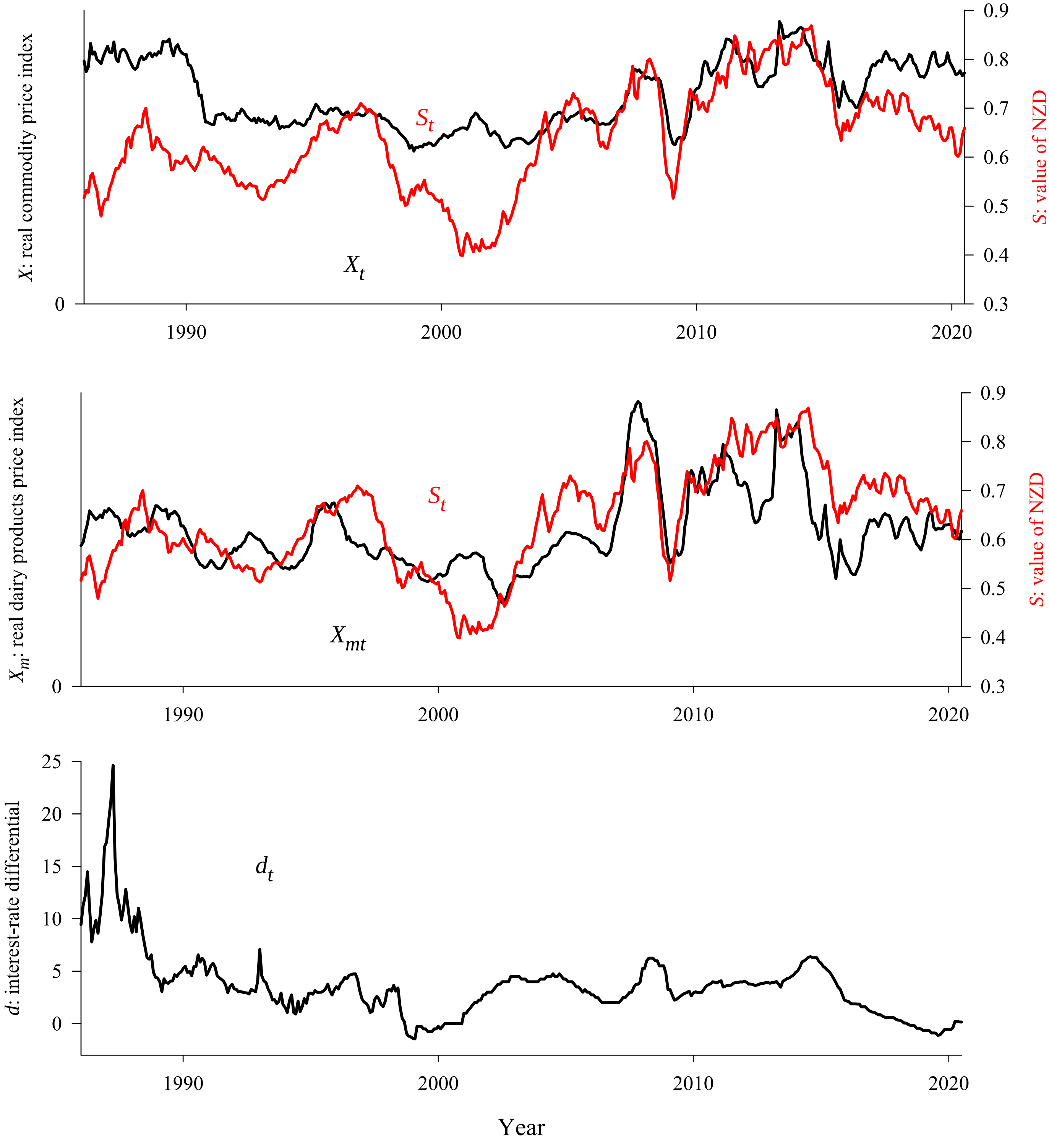
Notes: In the upper panel the black line (left axis) shows the Bank of Canada's commodity price index (in USD), divided by the US CPI, monthly. The red line (right axis) shows the monthly average value of the currency measured in US dollars. The middle panel instead includes the energy component of the commodity price index. The lower panel shows the policy interest-rate differential $d = i\text{-}i^*$.

Figure 2: Commodity Prices, Exchange Rates, and Interest Rates Australia



Notes: In the upper panel the black line (left axis) shows the Reserve Bank of Australia's commodity price index (in USD), divided by the US CPI, monthly. The red line (right axis) shows the monthly average value of the currency measured in US dollars. The middle panel instead graphs the base metals component. The lower panel shows the policy interest-rate differential $d = i\text{-}i^*$.

Figure 3: Commodity Prices, Exchange Rates, and Interest Rates
New Zealand



Notes: In the upper panel the black line (left axis) shows the ANZ commodity price index (in USD), divided by the US CPI, monthly. The red line (right axis) shows the monthly average value of the currency measured in US dollars. The middle panel instead uses the dairy products component. The lower panel shows the policy interest-rate differential $d = i\text{-}i^*$.