Queen's University
School of Graduate Studies and Research
Department of Economics

Economics 850 **Fall, 2019**

Professor James MacKinnon

Midterm Examination

October 29, 2019. Time Limit: 80 minutes

Please answer all questions. The marks for each question are shown in square brackets. All answers should be as short as it is reasonably possible to make them.

1. You estimate a regression model and obtain a vector of OLS estimates $\hat{\boldsymbol{\beta}}$. The disturbances u_t are assumed to be independent but possibly heteroskedastic, with $\mathrm{E}(u_t^2) = \omega_t^2$, where the ω_t^2 are unknown.

a) How would you estimate $\mathrm{Var}(\hat{\boldsymbol{\beta}})$, the covariance matrix of $\hat{\boldsymbol{\beta}}$? For the purposes of this question, any method that is asymptotically valid is fine.

b) Let $\gamma = \beta_4 - \beta_5$. Using the estimate of $\mathrm{Var}(\hat{\boldsymbol{\beta}})$ from part a), construct a 90% confidence interval for γ. Note that 2.7055 is the .90 quantile of the $\chi^2(1)$ distribution. Your interval should depend on this number.

c) Using the estimate of $\mathrm{Var}(\hat{\boldsymbol{\beta}})$ from part a), write down a Wald statistic for testing the joint null hypothesis that $\beta_3 = 0$ and $\gamma = 0.5$. What distribution will this test statistic follow asymptotically under the null?

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2. Consider the linear regression model

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{Z\gamma} + \boldsymbol{u}, \tag{1}$$

where the $n \times k$ and $n \times l$ matrices $\boldsymbol{X}$ and $\boldsymbol{Z}$ satisfy standard assumptions for consistency and asymptotic normality, and the elements of the n-vector $\boldsymbol{u}$ are independent and identically distributed.

a) Write down an FWL regression and use it to obtain an expression for the OLS estimator $\hat{\boldsymbol{\gamma}}$.

b) Write down a test statistic for $\boldsymbol{\gamma} = \boldsymbol{0}$. Does it have a known distribution in finite samples under the stated assumptions? Does it have a known asymptotic distribution? In each case, what is the distribution if it is known?

c) Under the null hypothesis that $\boldsymbol{\gamma} = \boldsymbol{0}$, the test statistic of part b) is $O_p(n^p)$. Under the alternative that $\boldsymbol{\gamma} = \boldsymbol{\gamma}_0 \neq \boldsymbol{0}$, it is $O_p(n^q)$. What are the values of p and q?

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d) Suppose that you reject the null hypothesis $\boldsymbol{\gamma} = \mathbf{0}$ whenever the test statistic exceeds a critical value C. Now imagine graphing the probability that the test rejects the null against the sample size n, for given $\boldsymbol{\gamma}_0 \neq \mathbf{0}$. What can you say about the shape of this graph?

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3. Consider the linear regression model

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{u}, \tag{2}$$

where $\boldsymbol{X}$ is exogenous and the data are assumed to be generated by a special case of this model with $\boldsymbol{\beta} = \boldsymbol{\beta}_0$. Let $\hat{\boldsymbol{\beta}}$ denote the OLS estimator.

a) Show that $\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}_0 = \boldsymbol{D}\boldsymbol{u}$ for a particular choice of the matrix $\boldsymbol{D}$.

b) What is the finite-sample distribution of $\hat{\boldsymbol{\beta}}$ if the vector $\boldsymbol{u}$ follows the multivariate normal distribution with mean vector $\mathbf{0}$ and covariance matrix $\boldsymbol{\Omega}$?

c) What restrictions must be imposed on the matrix $\boldsymbol{\Omega}$ for $\hat{\boldsymbol{\beta}}$ to be more efficient than any other unbiased linear estimator of $\boldsymbol{\beta}$?

d) Now consider another unbiased linear estimator $\tilde{\boldsymbol{\beta}}$, which can be written as $\boldsymbol{A}\boldsymbol{y}$ with $\boldsymbol{A}\boldsymbol{X} = \mathbf{I}$. We can always write $\tilde{\boldsymbol{\beta}} = \hat{\boldsymbol{\beta}} + \boldsymbol{v}$. Show that the covariance matrix of $\boldsymbol{v}$ and $\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}_0$ is a zero matrix when the restrictions of part c) are satisfied.

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