# Queen's University
# School of Graduate Studies and Research
# Department of Economics

**Economics 850** **Fall, 2018**

**Professor James MacKinnon**

## Midterm Examination

October 23, 2018. Time Limit: 80 minutes

**Please answer all questions. The marks for each question are shown in square brackets. All answers should be as short as it is reasonably possible to make them.**

**1.** Consider the linear regression model

$$y_t = \beta_1 + \beta_2 x_t + \beta_3 y_{t-1} + u_t, \quad u_t \sim \mathrm{IID}(0, \sigma^2),$$

where the regressor $x_t$ is assumed to be exogenous. Let $\hat{\beta}_j$ denote the OLS estimates of $\beta_j$ for $j = 1, 2, 3$.

a) Is $\hat{\beta}_3$ unbiased? Briefly explain why or why not.

b) Is $\hat{\beta}_2$ unbiased? Briefly explain why or why not.

c) Under standard conditions, the following statement is true:

$$n^{1/2}(\hat{\beta}_3 - \beta_3) \overset{a}{\sim} \mathrm{N}(q_1, q_2).$$

What are the values of the scalars $q_1$ and $q_2$ here? [30]

**2.** Consider the linear regression model

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{u}, \quad \mathrm{E}(\boldsymbol{u}\boldsymbol{u}^\top) = \sigma^2\mathbf{I},$$

where the regressors in $\boldsymbol{X}$ are exogenous, and the data are assumed to be generated by a special case of this model with $\boldsymbol{\beta} = \boldsymbol{\beta}_0$. Let $\hat{\boldsymbol{\beta}}$ denote the OLS estimator.

a) Is $\hat{\boldsymbol{\beta}}$ biased or unbiased? Show that $\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}_0 = \boldsymbol{Du}$ for a particular choice of the matrix $\boldsymbol{D}$.

b) Now consider another unbiased linear estimator $\tilde{\boldsymbol{\beta}}$, which can be written as $\boldsymbol{Ay}$ with $\boldsymbol{AX} = \mathbf{I}$. We can always write $\tilde{\boldsymbol{\beta}} = \hat{\boldsymbol{\beta}} + \boldsymbol{v}$. Show that the covariance matrix of $\boldsymbol{v}$ and $\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}_0$ is a zero matrix. What does this imply about the efficiency of $\tilde{\boldsymbol{\beta}}$ relative to $\hat{\boldsymbol{\beta}}$?

c) Now suppose that $\mathrm{E}(\boldsymbol{u}\boldsymbol{u}^\top) = \boldsymbol{\Omega}$, where $\boldsymbol{\Omega}$ is a diagonal, positive definite matrix. Is the result of part b) still true? Explain briefly. [30]

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**3.** Consider the linear regression model

$$\boldsymbol{y} = \boldsymbol{X}_1\boldsymbol{\beta}_1 + \boldsymbol{X}_2\boldsymbol{\beta}_2 + \boldsymbol{u}, \quad \mathrm{E}(\boldsymbol{u}\boldsymbol{u}^\top) = \boldsymbol{\Omega},$$

where the $k_1$ regressors in $\boldsymbol{X}_1$ and the $k_2$ regressors in $\boldsymbol{X}_2$ are assumed to be exogenous. There are $n$ observations, and $k = k_1 + k_2 < n$.

a) Write down the FWL regression that can be used to estimate $\boldsymbol{\beta}_2$. Use it to obtain expressions for the OLS estimate $\hat{\boldsymbol{\beta}}_2$ and for its (true) covariance matrix.

b) Suppose the matrix $\boldsymbol{X}_1$ in the above regression is replaced by $\boldsymbol{X}_1\boldsymbol{A}$, where $\boldsymbol{A}$ is a $k_1 \times k_1$ matrix with full rank. How will this affect the OLS estimates of $\boldsymbol{\beta}_2$? Explain very briefly.

c) Assume that the matrix $\boldsymbol{\Omega}$ is diagonal, with diagonal elements that vary across observations. How would you estimate the covariance matrix of $\hat{\boldsymbol{\beta}}_2$?

d) Write down the Wald statistic for testing the hypothesis that $\boldsymbol{\beta}_2 = \mathbf{0}$ under the assumptions of part c). How will this statistic be distributed asymptotically?

[40]