Queen's University
School of Graduate Studies and Research
Department of Economics

Economics 850 **Fall, 2017**

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Midterm Examination

October 25, 2017. Time Limit: 80 minutes

Please answer all questions. The marks for each question are shown in square brackets. All answers should be as short as possible. In many cases, a formula, a single sentence, or a few ordered points will suffice.

1. Consider the linear regression model

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{u}, \quad \boldsymbol{u} \sim \text{IID}(\mathbf{0}, \sigma^2\mathbf{I}), \tag{1}$$

where there are n observations and k regressors. Under standard assumptions about $\boldsymbol{X}$, by what power of n must each of the following expressions be multiplied in order to turn it into a quantity that is either $O(1)$ or $O_p(1)$? In each case, indicate whether the final quantity is deterministic – that is, $O(1)$ – or stochastic – that is, $O_p(1)$.

a) $\boldsymbol{X}^\top\boldsymbol{X}$

b) $\boldsymbol{X}^\top\boldsymbol{u}$

c) $(\boldsymbol{X}^\top\boldsymbol{X})^{-1}\boldsymbol{X}^\top\boldsymbol{y} - \boldsymbol{\beta}_0$

d) $\boldsymbol{y}^\top\big(\mathbf{I} - \boldsymbol{X}(\boldsymbol{X}^\top\boldsymbol{X})^{-1}\boldsymbol{X}^\top\big)\boldsymbol{y}$

e) $\boldsymbol{u}^\top\boldsymbol{X}(\boldsymbol{X}^\top\boldsymbol{X})^{-1}\boldsymbol{X}^\top\boldsymbol{u}$

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2. Consider the classical normal linear regression model

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{u}, \quad \boldsymbol{u} \sim \text{NID}(\mathbf{0}, \sigma^2\mathbf{I}), \tag{2}$$

where $\boldsymbol{X}$ contains 3 exogenous regressors, including a constant term, and there are 103 observations.

a) Write down the sum of squared residuals (SSR) as a function of the quantities on the right-hand side of equation (2).

b) How would you estimate σ? Is your estimator unbiased? Is it consistent?

c) How is the scalar SSR/σ^2 distributed?

Question continued on next page . . .

d) Explain how you would use the result of part c) to test the hypothesis that $\sigma = 3$. Is this an exact test or an asymptotic one?

e) Explain how you would use the result of part d) to construct a 95% confidence interval for σ. Will this interval be symmetric around the estimate you obtained in part b)?

[35]

3. Consider the linear regression model

$$\boldsymbol{y} = \boldsymbol{X}_1\boldsymbol{\beta}_1 + \beta_2\boldsymbol{x}_2 + \boldsymbol{u}, \quad \boldsymbol{u} \sim \mathrm{N}(\mathbf{0}, \sigma^2\mathbf{I}), \tag{3}$$

where there are n observations and $k = k_1 + 1$ exogenous regressors, k_1 of which are the columns of the matrix $\boldsymbol{X}_1$.

a) Write the t statistic for $\beta_2 = 0$ as a function of $\boldsymbol{y}$, $\boldsymbol{X}_1$, $\boldsymbol{x}_2$, n, and k. Precisely how is it distributed under the null hypothesis?

b) Write the F statistic for $\beta_2 = 0$ as a function of $\boldsymbol{y}$, $\boldsymbol{X}_1$, $\boldsymbol{x}_2$, n, and k. Precisely how is it distributed under the null hypothesis?

c) What is the finite-sample distribution of the t statistic for $\beta_2 = 0$ under the fixed alternative that $\beta_2 = \beta_2^0$?

d) Using the result of part c), show how to calculate the power of a two-tailed test for $\beta_2 = 0$ if you know β_2^0 and σ. How is power related to σ, n, k, and the matrix $\boldsymbol{X}$?

[35]