Queen's University
School of Graduate Studies and Research
Department of Economics

Economics 850 **Fall, 2016**

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Midterm Examination

October 26, 2016. Time Limit: 80 minutes

Please answer all 3 questions. The marks for each question are shown in square brackets. All answers should be as short as possible. In many cases, a formula, a single sentence, or a few ordered points will suffice.

1. Consider the linear regression model

$$\boldsymbol{y} = \boldsymbol{X}_1\boldsymbol{\beta}_1 + \boldsymbol{X}_2\boldsymbol{\beta}_2 + \boldsymbol{u}, \quad \boldsymbol{u} \sim \text{NID}(\mathbf{0}, \sigma^2\mathbf{I}), \tag{1}$$

where there are n observations and $k = k_1 + k_2$ regressors, with the regressors in $\boldsymbol{X}_1$ exogenous and the ones in $\boldsymbol{X}_2$ predetermined but not exogenous.

a) Write the F statistic for $\boldsymbol{\beta}_2 = \mathbf{0}$ explicitly as a function of n, k, k_2, $\boldsymbol{y}$, $\boldsymbol{X}_1$, and $\boldsymbol{X}_2$, and/or projection matrices that depend on the regressor matrices. What are the two degrees-of-freedom parameters for the F test?

b) Will the F statistic for $\boldsymbol{\beta}_2 = \mathbf{0}$ be exactly distributed according to the F distribution given in part a) under the null hypothesis? [YES or NO]

c) Write down the Wald statistic for $\boldsymbol{\beta}_2 = \mathbf{0}$ in the model (1). How is it related to the F statistic of part a)? How is it distributed asymptotically under the null hypothesis?

d) When the null hypothesis is true, the Wald statistic of part c) must be $O_p(n^a)$. When $\boldsymbol{\beta}_2 = \boldsymbol{\beta}_2^0 \neq \mathbf{0}$, the Wald statistic must be $O_p(n^b)$. Exactly what are the values of a and b?

[36]

2. Consider the linear regression model

$$\boldsymbol{y} = \boldsymbol{x}_1\beta_1 + \boldsymbol{X}_2\boldsymbol{\beta}_2 + \boldsymbol{u}, \quad \boldsymbol{u} \sim \text{IID}(\mathbf{0}, \sigma^2\mathbf{I}), \tag{2}$$

where $\boldsymbol{y}$, $\boldsymbol{x}_1$, and $\boldsymbol{u}$ are n–vectors, and $\boldsymbol{X}_2$ is an $n \times k_2$ matrix, with $k = k_2 + 1$. Assume that all regressors are exogenous.

a) If you impose the restriction that $\beta_1 = 1$ and it is false, will the estimates of $\boldsymbol{\beta}_2$ be biased or unbiased (in general)?

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b) If you know that $\beta_1 = 1$, you can usually obtain estimates of $\boldsymbol{\beta}_2$ that are more efficient than $\hat{\boldsymbol{\beta}}_2$, the ones obtained by applying OLS to regression (2). Briefly explain how to obtain the more efficient estimates using a regression package.

c) Is there any special case in which knowing that $\beta_1 = 1$ will *not* allow you to obtain estimates of $\boldsymbol{\beta}_2$ that are more efficient than $\hat{\boldsymbol{\beta}}_2$? Explain briefly. [27]

3. You are hired as a statistical consultant for a top-secret project. You are not allowed to see the data or the model. However, you can see estimates of the two key parameters, β and γ, along with various sets of bootstrap estimates.

a) Initially, you are told that $\hat{\beta} = 0.578$ and $\hat{\gamma} = 0.379$. You are also given 999 bootstrap estimates of β and γ based on an unrestricted bootstrap DGP. Briefly explain how you could estimate the standard errors of $\hat{\beta}$, $\hat{\gamma}$, and $\hat{\beta} + \hat{\gamma}$.

b) Do the results of part a) allow you to test the hypothesis that $\beta + \gamma = 1$? If so, briefly explain how to do so. If not, briefly explain why not.

c) The person who controls the data provides you with analytic standard errors for $\hat{\beta}$, $\hat{\gamma}$, and $\hat{\beta} + \hat{\gamma}$ for the actual sample and each of the 999 bootstrap samples. Briefly explain how to compute a bootstrap P value for the hypothesis that $\beta + \gamma = 1$ using the information you were given in part a) along with whatever parts of the new information you need.

d) The person who controls the data re-estimates the model under the restriction that $\beta + \gamma = 1$. The restricted estimates are $\tilde{\beta} = 0.602$ and $\tilde{\gamma} = 0.398$. You are also given the unrestricted estimates $\hat{\beta}^*_b$, $\hat{\gamma}^*_b$, and $\hat{\beta}^*_b + \hat{\gamma}^*_b$ and the corresponding standard errors from 999 new bootstrap samples indexed by b and based on the restricted estimates. Explain how to compute a bootstrap P value for the hypothesis that $\beta + \gamma = 1$ based on the same test statistic you used in part c) and the information from these new bootstrap samples. [37]