Queen's University
School of Graduate Studies and Research
Department of Economics

Economics 850 | **Econometrics I** | **Fall, 2018**

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Final Examination

December 13, 2018. Time Limit: 3 hours

Please answer any **four (4)** of the following **six (6)** questions. Each question has **four parts** and is worth 25% of the final mark. Before deciding which questions to answer, it would be a very good idea to read every question carefully.

Note: A table with some critical values of the χ^2 distribution appears at the end of the examination.

1. Consider the linear regression model

$$\boldsymbol{y} = \boldsymbol{X}_1\boldsymbol{\beta}_1 + \boldsymbol{X}_2\boldsymbol{\beta}_2 + \boldsymbol{u}, \tag{1}$$

where there are n observations and k exogenous regressors, with k_1 regressors in the matrix $\boldsymbol{X}_1$ and k_2 regressors in the matrix $\boldsymbol{X}_2$. Assume initially that the elements of $\boldsymbol{u}$ are uncorrelated and have the same variance.

a) Write down the F statistic for the hypothesis that $\boldsymbol{\beta}_2 = \boldsymbol{0}$ as a function of $\boldsymbol{y}$, $\boldsymbol{X}_1$, and $\boldsymbol{X}_2$, using projection matrices to simplify the algebra. How is this statistic distributed asymptotically under the null hypothesis? Would it follow a known distribution in finite samples? Would it follow a known distribution in finite samples if you made additional assumptions? Explain briefly.

b) Write down the Wald statistic for the hypothesis that $\boldsymbol{\beta}_2 = \boldsymbol{0}$ as a function of $\boldsymbol{y}$, $\boldsymbol{X}_1$, and $\boldsymbol{X}_2$. How is this statistic distributed asymptotically under the null hypothesis? Explain how it is related to the F statistic you derived in part a).

c) Now suppose that the elements of $\boldsymbol{u}$ are independent but not identically distributed, with $\mathrm{E}(u_i^2) = \sigma_i^2$ for observation i, where the σ_i^2 are unknown. If $\boldsymbol{\beta}_2^0$ denotes the true value of $\boldsymbol{\beta}_2$, how is the vector $n^{1/2}(\hat{\boldsymbol{\beta}}_2 - \boldsymbol{\beta}_2^0)$ distributed asymptotically?

d) Using the results of part c), write down the Wald statistic for the hypothesis that $\boldsymbol{\beta}_2 = \boldsymbol{0}$. Explain how you would perform a bootstrap test of this hypothesis based on your Wald statistic, a suitable bootstrap generating process, and the appropriate method for computing a bootstrap P value.

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2. Consider the linear regression model

$$y_i = \beta_1 + \beta_2 t_i + u_i, \quad i = 1, \ldots, n, \tag{2}$$

where t_i is a dummy variable that equals 1 for treated observations and 0 for untreated ones. Assume that n is fairly large, say, $n \geq 2000$.

a) Explain how, without using bootstrap methods, you could test the hypothesis that $\beta_2 = 0$ if the u_i were assumed to be uncorrelated but possibly heteroskedastic, with unknown variances σ_i^2.

b) Now suppose that the n observations fall into G groups, where $\mathrm{E}(u_i u_j) \neq 0$ whenever i and j belong to the same group, but $\mathrm{E}(u_i u_j) = 0$ whenever i and j belong to different groups. Suppose further that treatment occurs at the group level, so that, for all observations within each group, either $t_i = 0$ or $t_i = 1$. Explain how, without using bootstrap methods, you could test the hypothesis that $\beta_2 = 0$ under these assumptions.

c) The procedure you proposed in part b) should yield a test statistic that follows the standard normal distribution when G is infinite. Suppose the value of this statistic is 4.37. What would you conclude if there were 17 groups, but only one of them was treated? What would you conclude if there were 17 groups and 6 of them were treated? Explain briefly.

d) Suppose you averaged the data within each group and ran the regression

$$\bar{y}_g = \beta_1 + \beta_2 t_g + u_g, \quad g = 1, \ldots, G, \tag{3}$$

where $\bar{y}_g$ is the average value of y_i for group g, and t_g is a dummy variable that equals 1 for treated groups and 0 for untreated ones. You would like to test the hypothesis $\beta_2 = 0$ using the *ordinary* t statistic for $\beta_2 = 0$ in equation (3). Are there any assumptions you can make about G, the u_i in equation (2), and the numbers of observations in each group under which this would be a sensible thing to do? Explain briefly.

3. You are interested in the coefficient on $\boldsymbol{y}_2$ in the linear equation

$$\boldsymbol{y}_1 = \beta \boldsymbol{y}_2 + \boldsymbol{Z}\boldsymbol{\gamma} + \boldsymbol{u}, \tag{4}$$

where there are 1244 observations. As the notation implies, $\boldsymbol{y}_2$ is endogenous. The matrix $\boldsymbol{Z}$ is 1244×5 and contains observations on exogenous variables, including a vector of ones. The vector $\boldsymbol{y}_2$ is determined by the equation

$$\boldsymbol{y}_2 = \boldsymbol{W}\boldsymbol{\pi} + \boldsymbol{v} = \boldsymbol{W}_1\boldsymbol{\pi}_1 + \boldsymbol{Z}\boldsymbol{\pi}_2 + \boldsymbol{v}, \tag{5}$$

where $\boldsymbol{W}$ is a 1244×9 matrix of instruments, some of which also belong to $\boldsymbol{Z}$. In both equations (4) and (5), the disturbances are assumed to be IID. The correlation between a typical element of $\boldsymbol{u}$ and the corresponding element of $\boldsymbol{v}$ is ρ.

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a) Will the OLS estimate $\hat{\beta}$ from (4) be unbiased? Will it be consistent? Does your answer depend on the value of ρ? Explain briefly.

b) How would you obtain a consistent estimate of β? Would this estimate, say $\check{\beta}$, be unbiased? Explain briefly.

c) For the estimation method you used in part b), the regression package reports that the SSR is 827.76 and that

$$(\boldsymbol{y} - \check{\beta}\boldsymbol{y}_2 + \boldsymbol{Z}\check{\boldsymbol{\gamma}})^\top \boldsymbol{P}_{\boldsymbol{W}}(\boldsymbol{y} - \check{\beta}\boldsymbol{y}_2 + \boldsymbol{Z}\check{\boldsymbol{\gamma}}) = 7.34, \tag{6}$$

where $\boldsymbol{P}_{\boldsymbol{W}}$ projects orthogonally onto the space spanned by the columns of $\boldsymbol{W}$. How many overidentifying restrictions are there? Calculate a test statistic for the hypothesis that these overidentifying restrictions are valid. Based on the asymptotic distribution of this test statistic, would you reject these restrictions at the .01 level?

d) Suppose that the R^2 from regression (5) is 0.3254 and the R^2 from regressing $\boldsymbol{y}_2$ on $\boldsymbol{Z}$ is 0.3142. Based on these numbers, would you expect $\check{\beta}$ to be well approximated by its asymptotic distribution? Explain briefly.

4. Consider the linear regression model

$$y_t = \beta_1 + \beta_2 x_{1t} + \beta_3 x_{2t} + u_t, \tag{7}$$

which is to be estimated using a sample of 83 observations. The regressors x_{1t} and x_{2t} are assumed to be exogenous, and the disturbances u_t are assumed to be independent but possibly heteroskedastic. You are interested in the parameter $\gamma \equiv \beta_2/\beta_3$.

a) Explain how you would obtain a consistent estimate $\hat{\gamma}$. Would $\hat{\gamma}$ be biased or unbiased? Explain briefly.

b) Explain how you would obtain both a standard error $s(\hat{\gamma})$ based on asymptotic theory and a bootstrap standard error $s^*(\hat{\gamma})$.

c) Explain how you would perform a bootstrap test of the hypothesis that $\gamma = 2$ at the .01 level.

d) Briefly discuss two ways to obtain a 95% bootstrap confidence interval for γ. Only one of them should make use of $s^*(\hat{\gamma})$.

5. Suppose you set out to gather samples from two different populations, with the samples having n_1 and n_2 observations, respectively. Observations from the first population have mean μ_1, and observations from the second population have mean μ_2. Your objective is to test the hypothesis that $\mu_1 = \mu_2$.

a) Explain how you could test the hypothesis that $\mu_1 = \mu_2$ by running a single linear regression. Assume that all observations have the same variance.

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b) If you can only afford to gather 250 observations, how would you choose n_1 and n_2 to maximize the power of the test under the assumptions of part a)?

c) Suppose you notice that observations in the first sample are less variable than observations in the second sample. You believe it is reasonable to assume that $\sigma_1/\sigma_2 = \gamma$, where γ is an unknown parameter. Explain how you could obtain more efficient estimates of μ_1 and μ_2 than the ones you obtained in part a).

d) Explain how you could test the hypothesis that $\mu_1 = \mu_2$ using the estimates from part c). The values of n_1 and n_2 chosen in part b) evidently do not maximize the power of this test. Explain why not. Should the ratio of n_1 to n_2 be larger or smaller than the one you chose in part b)?

6. This question deals with the nonlinear regression model

$$y_t = \alpha + \gamma \sum_{j=0}^{5} (\beta^j x_{t-j}) + u_t, \tag{8}$$

where it is assumed that $\mathrm{E}(u_t \,|\, x_t, x_{t-1}, \ldots) = 0$, that $\mathrm{E}(u_t u_s) = 0$ for $s \neq t$, and that $\mathrm{E}(u_t^2) = \sigma_t^2$, with the σ_t^2 unknown. You wish to estimate the parameters α, β, and γ using 324 monthly observations. Note that β^j means β raised to the power j.

a) Would nonlinear least squares estimates of (8) be unbiased? Would they be consistent? Explain briefly.

b) The model (8) involves nonlinear restrictions on a linear regression model. What is the unrestricted model, and how many restrictions are there? How would you test these restrictions in the context of NLS estimation under the stated assumptions?

c) Explain how you could obtain estimates of the three parameters in (8) that are consistent and asymptotically more efficient than the NLS estimates.

d) How you would test the restrictions of part b) when equation (8) is estimated by the method you proposed in part c)? Explain briefly.

Table 1. Some Critical Values of the χ^2 Distribution

D.F. / Level	.10	.05	.025	.01
1	2.706	3.841	5.024	6.635
2	4.605	5.991	7.378	9.210
3	6.251	7.815	9.348	11.345
4	7.779	9.488	11.143	13.277
5	9.236	11.070	12.833	15.086
6	10.645	12.592	14.449	16.812