Queen's University
School of Graduate Studies and Research
Department of Economics

Economics 850 | **Econometrics I** | **Fall, 2015**

Professor James MacKinnon

Final Examination

December 17, 2015. Time Limit: 3 hours

Please answer any **four (4)** of the following **six (6)** questions. Each question has **four parts** and is worth 25% of the final mark. Before deciding which questions to answer, it would be a very good idea to read every question carefully.

Note: A table with some critical values of the χ^2 distribution appears at the end of the examination.

1. Consider the linear regression model

$$\boldsymbol{y} = \boldsymbol{X}_1\boldsymbol{\beta}_1 + \boldsymbol{X}_2\boldsymbol{\beta}_2 + \boldsymbol{u},$$

where there are n observations and $k = k_1 + k_2$ regressors, which are assumed to be exogenous. The column vectors $\boldsymbol{y}$ and $\boldsymbol{u}$ have n elements, and the matrices $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ are $n \times k_1$ and $n \times k_2$, respectively.

a) Suppose the elements of $\boldsymbol{u}$ are believed to be independently distributed with unknown variances σ_t^2 for $t = 1, \ldots, n$. How would you estimate $\boldsymbol{\beta}_1$, $\boldsymbol{\beta}_2$, and the covariance matrix of your estimates? How would you then test the hypothesis that $\boldsymbol{\beta}_2 = \boldsymbol{0}$ using an asymptotic test?

b) Suppose the observations within your sample fall into G clusters. The elements of $\boldsymbol{u}$ are assumed to have unknown variances, to be correlated within each cluster, and to be uncorrelated across clusters. How would you estimate $\boldsymbol{\beta}_1$, $\boldsymbol{\beta}_2$, and the covariance matrix of your estimates? How would you then test the hypothesis that $\boldsymbol{\beta}_2 = \boldsymbol{0}$ using an asymptotic test?

c) Suppose you wish to perform a bootstrap test of the hypothesis that $\boldsymbol{\beta}_2 = \boldsymbol{0}$ under the assumptions of part b). How you would generate the B bootstrap samples that you need? If you had to choose between setting $B = 1000$ and $B = 999$, which value would you pick? Why?

d) After you have generated B bootstrap samples as described in part c), explain how you would use them to compute a bootstrap P value for the hypothesis that $\boldsymbol{\beta}_2 = \boldsymbol{0}$. Would you be surprised if this P value were substantially smaller than the P value for the test of part b)?

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2. This question deals with the nonlinear regression model

$$y_t = \alpha + \gamma \sum_{j=0}^{6} (\beta^j x_{t-j}) + u_t, \tag{1}$$

where it is assumed that $\mathrm{E}(u_t \,|\, x_t, x_{t-1}, \ldots) = 0$, that $\mathrm{E}(u_t u_s) = 0$ for $s \neq t$, and that $\mathrm{E}(u_t^2) = \sigma_t^2$, with the σ_t^2 unknown. You wish to estimate the parameters α, β, and γ using 456 monthly observations. Note that β^j means β raised to the power j.

a) Would nonlinear least squares estimates of (1) be unbiased? Would they be consistent? Explain briefly.

b) The model (1) involves nonlinear restrictions on a linear regression model. What is the unrestricted model, and how many restrictions are there? Explain briefly how you would test these restrictions in the context of NLS estimation under the stated assumptions.

c) Explain briefly how you could obtain estimates of the three parameters that are consistent and asymptotically more efficient than the NLS estimates.

d) Explain briefly how you would test the restrictions of part b) when (1) is estimated by the method you proposed in part c).

3. You have a sample of 5,137 households. The variable y_i is 1 if the household owns at least one automobile and 0 otherwise. This variable is believed to be related to a $1 \times k$ vector of exogenous variables $\boldsymbol{X}_i$.

a) Consider the model

$$\begin{aligned} &y_i^\circ = \boldsymbol{X}_i \boldsymbol{\beta} + u_i, \quad u_i \sim \mathrm{N}(0,1), \\ &y_i = 0 \ \text{ if } y_i^\circ \le 0, \ \ y_i = 1 \ \text{ if } y_i^\circ > 0, \end{aligned} \tag{2}$$

where the latent variable y_i° is not observed. According to this model, what is the probability P_i that $y_i = 1$? Explain how you would estimate the parameter vector $\boldsymbol{\beta}$. If your estimation method involves minimizing or maximizing some function, write out that function explicitly.

b) Suppose you are particularly interested in the relationship between automobile ownership and age. How would you decide whether to model y_i° as linear, quadratic, or cubic in age?

c) Suppose you wish to estimate $\bar{P} \equiv \Pr(y = 1 \,|\, \bar{\boldsymbol{X}})$, where $\bar{\boldsymbol{X}}$ is a $1 \times k$ vector of hypothetical values of the exogenous variables. Explain how you could use the delta method to obtain a 95% confidence interval for $\bar{P}$. Are the limits of this interval constrained to lie between 0 and 1?

d) Describe another method for obtaining a 95% confidence interval for $\bar{P}$ which will ensure that the limits of the interval lie between 0 and 1.

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4. Consider the linear simultaneous equations model

$$\boldsymbol{y}_1 = \beta \boldsymbol{y}_2 + \boldsymbol{Z}\boldsymbol{\gamma} + \boldsymbol{u}, \tag{3}$$

$$\boldsymbol{y}_2 = \boldsymbol{W}\boldsymbol{\pi} + \boldsymbol{v} = \boldsymbol{W}_1\boldsymbol{\pi}_1 + \boldsymbol{Z}\boldsymbol{\pi}_2 + \boldsymbol{v}, \tag{4}$$

where $\boldsymbol{y}_1$ and $\boldsymbol{y}_2$ are n–vectors of observations on endogenous variables, $\boldsymbol{Z}$ is an $n \times k$ matrix of observations on exogenous or predetermined variables, $\boldsymbol{W} \equiv [\boldsymbol{W}_1 \;\; \boldsymbol{Z}]$ is an $n \times l$ matrix of observations on exogenous or predetermined variables, and $\boldsymbol{u}$ and $\boldsymbol{v}$ are n–vectors of disturbances. Assume that $\mathrm{E}(u_t v_s) = 0$ for $t \neq s$ and $\mathrm{E}(u_t v_t) = \rho\sigma_u\sigma_v$, where u_t and v_s are elements of $\boldsymbol{u}$ and $\boldsymbol{v}$, and σ_u^2 and σ_v^2 are their variances. The rest of the notation should be obvious.

a) Is there any case in which the OLS estimate of β will be consistent? Explain.

b) Explain how to compute the generalized IV estimator $\hat{\beta}_{\mathrm{IV}}$. What can you say about the mean squared error of this estimator relative to the mean squared error of the OLS estimator $\hat{\beta}_{\mathrm{OLS}}$?

c) How many overidentifying restrictions are there? Explain how to test them using an asymptotic test. What would you conclude if $n = 350$, $k = 15$, $l = 18$, and the test statistic (in χ^2 form) were 5.76?

d) Suppose the F statistic for $\boldsymbol{\pi}_1 = \boldsymbol{0}$ in equation (4) were 2.34. Would you expect the standard error of $\hat{\beta}_{\mathrm{IV}}$ to be similar to the standard error of $\hat{\beta}_{\mathrm{OLS}}$ in this case? Would you expect the finite-sample distribution of $\hat{\beta}_{\mathrm{IV}}$ to be well approximated by its asymptotic distribution? Would your answers to either or both of these questions change if the value of this F statistic were 16.58?

5. This question concerns the linear regression model

$$\boldsymbol{y} = \boldsymbol{X}_1\boldsymbol{\beta}_1 + \beta_2\boldsymbol{x}_2 + \boldsymbol{u}, \quad \boldsymbol{u} \sim \mathrm{IID}(\boldsymbol{0}, \sigma^2\mathbf{I}), \tag{5}$$

where there are n observations and k regressors, the matrix $\boldsymbol{X}_1$ has k_1 columns, and $k = k_1 + 1$. All regressors are assumed to be predetermined but not necessarily exogenous.

a) Write down $\hat{\beta}_2$, the OLS estimate of β_2, as a function of $\boldsymbol{y}$, $\boldsymbol{X}_1$, and $\boldsymbol{x}_2$. Then replace $\boldsymbol{y}$ by $\boldsymbol{X}_1\boldsymbol{\beta}_1^0 + \beta_2^0\boldsymbol{x}_2 + \boldsymbol{u}$ and write $\hat{\beta}_2$ as a function of $\boldsymbol{u}$ and whatever else it depends on.

b) What is the asymptotic distribution of $n^{1/2}(\hat{\beta}_2 - \beta_2^0)$ under standard assumptions that allow conventional asymptotic results to be obtained? Briefly state these assumptions and show, without providing a formal proof, how they lead to your result.

c) Let $\acute{\beta}_2$ denote the estimator obtained by regressing $\boldsymbol{y}$ on $\boldsymbol{x}_2$. What would be the asymptotic distribution of $n^{1/2}(\acute{\beta}_2 - \beta_2^0)$ if $\boldsymbol{x}_2$ were random and generated in such a way that $\mathrm{E}(\boldsymbol{X}_1^\top\boldsymbol{x}_2) = \boldsymbol{0}$?

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d) Under the assumptions of part c) would the estimator $\acute{\beta}_2$ be as efficient, asymptotically, as the estimator $\hat{\beta}_2$? Explain why or why not.

6. Consider the dynamic linear regression model

$$y_t = \beta_1 + \beta_2 x_t + \beta_3 y_{t-1} + u_t, \tag{6}$$

where x_t is an exogenous variable that is not serially independent. There are 86 observations on x_t and 87 observations (including y_0) on y_t. The disturbances are assumed to be IID and to have at least four moments.

a) Explain how to test the hypothesis $\beta_3 = 0.75$ at the .05 level using a t statistic and employing a critical value from the Student's t distribution. Would you expect the rejection frequency of this test to be close to .05? Why or why not?

b) Explain how to test the hypothesis that $\beta_3 = 0.75$ using a residual bootstrap test at the .05 level. How would you generate the bootstrap samples? How would you decide whether or not to reject the null hypothesis based on 999 bootstrap samples? **Hint:** Would you expect this particular test statistic to be symmetrically distributed around zero?

c) Explain how you would generate 999 bootstrap samples using a residual bootstrap that does not impose any restrictions. Then explain how you would use them to construct a studentized bootstrap confidence interval at the .95 level for β_3. Would you expect this interval to be symmetric around $\hat{\beta}_3$? Why or why not?

d) The bootstrap samples of part c) can also be used to obtain a bias-corrected estimate of β_3, say $\acute{\beta}_3$. Explain how you could do this. Would the variance of $\acute{\beta}_3$ be greater or less than the variance of $\hat{\beta}_3$? Explain.

Table 1. Some Critical Values of the χ^2 Distribution

D.F. / Level	.10	.05	.025	.01
1	2.706	3.841	5.024	6.635
2	4.605	5.991	7.378	9.210
3	6.251	7.815	9.348	11.345
4	7.779	9.488	11.143	13.277
5	9.236	11.070	12.833	15.086
6	10.645	12.592	14.449	16.812