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Economics 850 Fall, 2024

Assignment 3

Due: November 14, 2024

1. Suppose that a test statistic follows the Student's t distribution with 6 degrees of freedom under the null hypothesis that a coefficient equals zero. The realized value of the test statistic is -1.97.

a) What is the P value for a one-tailed test when the alternative is in the upper tail?

b) What is the P value for a one-tailed test when the alternative is in the lower tail?

c) What is the P value for a two-tailed test?

d) Generate 999 realizations of a random variable that follows the standard normal distribution, and plot the resulting empirical distribution function.

e) Let z_i denote the i^{th} realization of the random variable of part d). Use the z_i along with 999 realizations of another random variable to generate 999 realizations t_i of a random variable that follows the $t(6)$ distribution. Plot the resulting EDF on the same axes. Is there anything in the figure that is worth remarking on?

f) Based on the EDF of part e), what is the P value for the one-tailed test of part a) where the alternative is in the upper tail? What is the P value for the two-tailed test of part c)? How do these simulated P values compare with the ones you obtained previously?

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2. Recall that

$$\sinh(x) = \tfrac{1}{2}\big(\exp(x) - \exp(-x)\big), \text{ and}$$
$$\sinh^{-1}(y) = \log\big(y + (1 + y^2)^{1/2}\big).$$

Both sinh and its inverse are monotonically increasing functions. Suppose you estimate a nonlinear model and obtain an estimate $\hat{\beta} = 2.3364$ with a standard error of 0.3832. However, you are really interested in $\gamma = \sinh^{-1}(\beta)$.

a) Calculate $\hat{\gamma} = \sinh^{-1}(\hat{\beta})$ and its standard error using the delta method.

b) Construct a .95 confidence interval for γ based on inverting a t statistic for γ. Is this interval symmetric around $\hat{\gamma}$? Explain.

c) Construct a .95 confidence interval for γ based on inverting a t statistic for β. Is this interval symmetric around $\hat{\gamma}$? Explain.

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3. You wish to make inferences about a parameter θ. Your estimate is -0.15013126, and the standard error is 0.09521909. The file `bootstrap.csv` contains 999 bootstrap estimates of θ, along with an asymptotic standard error for each of them.

a) Theory suggests that t-statistics should in this case approximately follow the $t(11)$ distribution. Under this assumption, form a 95% confidence interval for θ.

b) Compute the bootstrap standard error based on the numbers given in `bootstrap.csv`, and use it to form a 95% confidence interval for θ. Is this interval longer or shorter than the one from part a)?

c) Use the numbers in `bootstrap.csv` to compute equal-tail and symmetric bootstrap P values for the hypothesis that $\theta = 0$.

d) Use the numbers in `bootstrap.csv` to compute a 95% studentized bootstrap confidence interval for θ. How does it compare with the intervals in parts a) and b)? Can you explain the differences?

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