Implementation Cycles, Growth and the Labour Market

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Abstract

We develop a theory of growth and cycles that endogenously relates job flows, worker flows and wages over the cycle to the processes of restructuring, innovation and implementation that drive long–run growth. Expansions are the result of clustered implementation of new ideas and recessions are the negative consequence of the restructuring that anticipates them. Due to incentive problems, production workers are employed via relational contracts and experience involuntary unemployment. Separation rates and firm turnover are counter-cyclical, but labour productivity growth and hiring rates are procyclical. Our framework also highlights the counter-cyclical forces on wages due to restructuring, and illustrates the relationship between the cyclicality of wages and long–run productivity growth.

Key Words: Endogenous cycles, endogenous growth, employment flows, wage rigidity.
JEL: E0, E3, O3, O4

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1 Introduction

Schumpeter's paradigm of "growth through creative destruction" has been central to many theories of long–run growth and has also strongly influenced research on employment flows over the business cycle.[1] In this article we bring together these two distinct areas in a model of innovation–driven growth that features endogenous cycles as part of its equilibrium growth path. By doing so, we link employment flows and wages over the business cycle to the endogenous processes of innovation, implementation and firm turnover that drive long run growth. We study the equilibrium of an endogenous growth model exhibiting two key features: (1) fluctuations are an intrinsic part of the growth process — expansions reflect the endogenous, clustered implementation of productivity improvements, and recessions are the negative side–product of the restructuring that anticipates them — and (2) the wages of production workers are endogenously determined by relational contracts designed to resolve incentive problems. We argue that the interactions between these two features play a crucial role in determining the joint cyclical movements in firm entry and exit, employment flows and wages.

Our analytical framework builds on the business cycle paradigm that we have developed elsewhere, by allowing for relational contracts in labor markets.[2] Output in each sector of the economy is produced using the most productive methods available combined with the services of managers and production workers. Managers' time may also be allocated away from supervising production workers in order to search for new ideas or ways of raising productivity. This ongoing search implies that the profits associated with productivity improvements are temporary. In order to protect the knowledge embedded in them, firms optimally delay implementation until macroeconomic conditions are most favorable. In the presence of imperfect competition, implementation by some firms increases the demand for others' products by raising aggregate demand, thereby creating such favorable conditions. This leads to mutually reinforcing endogenous clustering of implementation, causing booms in productivity.

As firms optimally allocate managers to searching for commercially viable ideas in anticipation of booms, there are downturns in aggregate output which are also self–reinforcing. During these recessions, the aggregate demand for complementary production workers falls. However, the implications for underlying firm turnover, worker flows and wages depends on the employment

[1] The recent Schumpeterian literature on long–run growth starts with Aghion and Howitt (1992), Grossman and Helpman (1991) and Segerstrom, Anant and Dinopoulos (1990). The recent literature on employment flows evolves from Davis and Haltiwanger (1992) and Caballero and Hammour (1994).

[2] Our earlier formalization of this approach is in Francois and Lloyd-Ellis (2003), which builds on the work of Shleifer (1986). We extended this to include capital in Francois and Lloyd-Ellis (2008). Pioneering work on the relational contract paradigm is that of Macleod and Malcomson (1989). An earlier, though less formally complete contribution is the efficiency wage model of Shapiro and Stiglitz (1986).

contract between firms and workers. Since the output of production workers is dependent on their imperfectly observable effort, firms must offer forward–looking implicit contracts to induce effort. However, the effectiveness of such contracts depends on workers' expectations of continued employment at a given firm. If, during a recession, it becomes clear that a firm is about to be made obsolete, its ability to motivate its workers is severely compromised. When this happens, firms may shut down immediately causing the rate of separation during recessions to rise endogenously.

Our theory has several implications for the movement of key macroeconomic variables over the business cycle. While these predictions are qualitative in nature, we argue that they are broadly consistent with those observed in US data and that our model offers a useful perspective on the joint determination of productivity, wages and employment flows during downturns:

• *Average labor productivity is procyclical*: Caballero and Hammour (1994) develop a partial equilibrium model in which exogenous declines in industry demand cause the least efficient units to shut down, thereby freeing up resources for more productive uses. Although their model has a similar Schumpeterian flavor to ours, it has a starkly counter–factual implication: average labor productivity is counter-cyclical (see Yi, 2004).[3] In our model, although new entrants optimally take over production during recessions, they delay implementation of their own improvements until the subsequent expansion. Unlike the previous incumbents, new entrants can credibly guarantee to honour contracts with workers and so can profitably produce. Consequently, our model predicts that labor productivity is procyclical.[4]

• *Both counter-cyclical job destruction and pro-cyclical job creation contribute to cyclical variation in employment growth* — This prediction is qualitatively consistent with various evidence starting with the work of Davis and Haltiwanger (1992) and Davis, Haltiwanger and Shuh (1996). Like these authors, we define "job destruction" as the sum of all declines in employment at firms where employment declines. In our model, job destruction rises quickly at the beginning of a recession then gradually increases until its end, when it falls at the boom. Job creation also rises during the recession, but at a much lower rate, as new firms replace old ones. Job-creation then rises rapidly at the boom as firms expand production again and hire more workers. These movements are qualitatively consistent with the recent observations of Faberman (2008), for example.

• *Increased job destruction in recessions is due to both a decline in the hiring rate and a rise in the separation rate* — In contrast to many models, our model explicitly distinguishes job flows from worker flows. In particular, an increase in the rate of job destruction could be due to a

[3] Moreover, they assume that unit wage costs are held fixed exogenously over the cycle. If labor markets were perfectly flexible, wages would fall allowing inefficient firms to remain in business.

[4] If labor productivity were correctly measured, it would remain constant during recessions. However, because the reallocation of skilled labor effort to innovative activities is likely to be unmeasured, it would look like labor hoarding, and measured labour productivity would fall.

decline in the rate of hiring or a rise in the rate of separation, both of which move endogenously in our model. Indeed, the initial rise in job destruction is associated with both a decline in hiring as aggregate demand falls and a rise in the separation rate as managerial effort is allocated away from production and workers are let go. Subsequently, however, the hiring rate rises again and separations become the dominant source of job destruction through most of the recession. Thus, as documented by Elsby et al. (2009) and Fujita and Ramey (2009), our model predicts that cyclical variation in both outflows and inflows to unemployment play an important role.[5]

• *Real wages are only mildly procyclical*: Average real wages are commonly characterized as being mildly pro–cyclical (e.g. Stock and Watson, 1998). Within reasonable parameters ranges, the canonical Real Business Cycle (RBC) model predicts excessively pro–cyclical real wages (see Chang, 2000). Efficiency wage models have been suggested by some as a way of offsetting this pro–cyclical wage behavior (e.g. Romer, 2001). However, Gomme (1999) finds that, while the variability of the average wage is reduced, it remains just as pro-cyclical. His analysis does not, however, allow for the fact that increases in the rate of job destruction require a compensating increase in current wages to maintain incentive compatibility.[6] In our framework these effects can offset, and even outweigh, the downward pressure on wages due to falling demand. Once again, it is the fact that this restructuring is concentrated during recessions which drives the counter–cyclical effect on wages. The wage behavior implied here would not arise in a model where productivity movements were treated exogenously.[7]

• *Real wages were more procyclical during the productivity slowdown (1970-1993) than either before or afterwards:* Until 1970, average real wages were acyclical whereas between the early 1970s and the early 1990s wages became more pro-cyclical (see Abraham and Haltiwanger, 1995). The pro-cyclicality of wages fell again after 1994 (see Table 3). The apparent increase in the procyclicality of average wages coincided with the 'productivity slowdown' which extended from the early 1970's until the mid 1990's. Was this timing a mere coincidence, or were both a function of changes in the same underlying factors? Our model offers one explanation for why these two phenomena may occur simultaneously: any factor that induces greater innovation on average, thereby fueling faster long–run productivity growth, also induces more obsolescence and more restructuring to occur during recessions. This implies that the counter–cyclical force on wages

[5] Many recent models of labor market fluctuations treat unemployment inflows (separation) as acyclical. Shimer (2005) and Hall (2005) have offered evidence in supporting this assumption. However, Fujita and Ramey (2009) and Elsby et al. (2009) re-examine the evidence and conclude that separation rates are strongly countercyclical .

[6] In his model, firms do not shut down to be replaced by new ones, but rather adjust their rate of hiring downwards as demand falls. Other efficiency wage models can dampen the procyclicality of wages (see Alexopolous, 2004 and Danthine and Kurman, 2004).

[7] The pro–cyclicality of average wages can be offset by a composition bias (see Bils (1985) and Solon, Barsky, and Parker (1994)). This effect is also present in our model because only production workers face employment fluctuations.

due to turnover tends to be greatest when average productivity growth is high, and lowest during productivity slowdowns.

It is worth re–iterating that these joint predictions are crucially driven by the interaction between the endogeneity of the economy's cyclical process and the dynamic contractual relationship between firms and workers. Forward–looking relational contracts are needed to motivate workers, but a firm's ability to commit to future employment varies over the cycle. These contracts are complicated by uncertainty over the firm's future competitive position (due to randomness in the innovation process) and the anticipated state of the macroeconomy in the future (through its impact on demand for the firm's products). However, the contractual outcomes between workers and firms also impact upon the aggregate economy by dictating incentive compatible wages, and hence the level of production, over the cycle and the incentives to innovate. Consequently, a general equilibrium analysis here involves simultaneously determining a sequence of aggregate fluctuations and time-varying self-enforcing contracts between workers and firms.

Our framework is related to that of Ramey and Watson (1997), who explore the effects of exogenous transitory shocks in inducing permanent increases in separations, and to den Haan, Ramey and Watson (1999), who also explore relational contracting in a similar framework. In their framework, the macroeconomy impinges on worker–firm relationships by reducing the surplus to maintaining matches and rendering incentive compatibility infeasible. This can lead to break-ups which propagate shocks. Our framework, in contrast, emphasizes the Schumpeterian necessity of break-up as part of the economy's rejuvenation process. Worker reallocations which accompany contractual break up are also an integral part of the economy's endogenous cycle, and aggregate productivity changes here. Our paper also bears some relation to the search/matching literature on labor flows (for a recent survey see Rogerson, Shimer, and Wright, 2004). These models can accommodate numerous labor market responses to aggregate shocks depending on the model specification (see Mortensen and Pissarides 1994). However, like all previous work on labor market flows, these authors treat aggregate demand, and its fluctuations, as exogenous.

The remainder of the paper is laid out as follows. Section 2 sets up the building blocks of the model. Section 3 posits and describes behavior in the cyclical equilibrium and elaborates the dynamics over the phases of the cycle, with particular emphasis on labor flows. Section 4 derives sufficient conditions for existence to be met. Section 5 demonstrates existence of the equilibrium for various sets of parameter values and explores the equilibrium's qualitative characteristics. The model's comparative statics are also examined, and applied to the productivity slowdown. Section 6 concludes.

2 The Model and Optimal Behavior

2.1 Final Goods Production

Final output is produced by competitive firms according to a Cobb–Douglas production function utilizing intermediates, x, indexed by i, over the unit interval:

$$Y(t) = \exp\left(\phi t + \int_0^1 \ln x_i(t) di\right). \tag{1}$$

Final goods production is subject to exogenous productivity growth at a constant rate ϕ.[8] Final output is costlessly storable, but cannot be converted back into an input for use in production. We let p_i denote the price of intermediate i. Final goods producers choose intermediates to minimize costs. The implied demand for intermediate i is then

$$x_i^d(t) = \frac{Y(t)}{p_i(t)} \tag{2}$$

2.2 Intermediate Goods Production

The output of intermediate i depends upon a productivity level, $a_i(t)$, and on the labor allocated to production. There are two types of labor — managers who earn a salary $s(t)$, and production workers who are paid a wage $w_i(t)$ in sector i. There are two distinct modes of production which intermediate firms can use:

• Large scale production — an incumbent firm operates a constant returns to scale technology that requires both managers and production workers. Provided there is no shirking, the firm uses $n_i(t)$ production workers to produce output according to:

$$x_i^s(t) = a_i(t) n_i(t). \tag{3}$$

We assume that managers have a fixed "span of control" with one manager required to supervise $\theta L > 1$ production workers, where $\theta < 1$. It follows that the number of managers required is given by

$$m_i(t) = \frac{n_i(t)}{\theta L} \tag{4}$$

• Small scale production — a manager can set up production using only her own labor. Since this individual works alone, there are no incentive issues, and the unit cost is simply $s(t)/a_i(t)$. Any individual holding the state of the art technology could produce using this method but, because of the small scale of production, the profit would be negligible.

[8] A positive value of ϕ is not necessary to generate equilibrium cycles. With $\phi = 0$ there would be no growth during the first phase of the cycle that we describe below, but nothing else of qualitative difference.

2.3 Entrepreneurial Search

Commercially viable productivity improvements are introduced into the economy via a process of "entrepreneurial search". Competitive entrepreneurs in each sector allocate skilled labor effort to searching for ideas, and finance this by selling claims. The rate of success from search is $\delta h_i(t)$, where δ is a parameter, and h_i represents the labor effort allocated to search in sector i. At each date, entrepreneurs decide whether or not to allocate skilled labor to search, and if they do so, how much. The aggregate labor effort allocated to search is given by

$$H(t) = \int_0^1 h_i(t)dt. \tag{5}$$

New ideas and innovations dominate old ones in terms of productivity by a factor e^{γ}, where $\gamma > 0$. This process is therefore formally identical to the innovation process in the quality–ladder model of Grossman and Helpman (1991). However, we explicitly do not interpret this activity as R&D. Although it is common to do so, this Poisson process is, in fact, a very bad description of R&D. Typically R&D is a knowledge-intensive (and often capital-intensive) activity, which involves accumulation of knowledge over time. In sharp contrast, the search activity described here is a skill-intensive one, which we interpret as a form of entrepreneurship. In our view this entrepreneurial function is the central player in economic activity, with R&D playing a supportive role that is not modeled here.[9] This activity could be undertaken by independent entrepreneurs, but in modern production it is often a role taken on by managers and other skilled workers *within* firms.

Entrepreneurs with commercially viable productivity improvements must make two decisions: (1) they must choose the timing of their entry into production, and (2) they must choose whether or not to implement immediately upon entering production, or delay until a later date. Once they implement, the profitability of this idea in this particular sector becomes publicly known and can be built upon by rival entrepreneurs. However, prior to implementation, this knowledge is privately held by the entrepreneur. By delaying implementation the entrepreneur loses some profits, but gains by delaying the rate of entry of more productive rivals.

An entrepreneur must have control of the productive resources of the firm prior to implementation. This is intended to capture the idea that some degree of reorganization is required to take advantage of new approaches or innovations. Once an innovation has been implemented,

[9] This view was shared by Schumpeter (1950, p.132): "...The function of entrepreneurs is to reform or revolutionize the pattern of production by exploiting an invention or, more generally, an untried technological possibility ... This function does not essentially consist in either inventing anything or otherwise creating the conditions which the enterprise exploits. It consists in getting things done". More recently, Comin (2002) estimates the contribution of R&D to US productivity growth to be very small. He notes that a larger contribution is likely to come from unpatented managerial and organizational innovations.

the entrepreneur with the knowledge of how to implement can costlessly enter or exit production at any time. The information embedded in a new productive technology will be used by future entrepreneurs to search for improvements that will make the current incumbent's technology redundant.

We let the indicator function $Z_i(t)$ take on the value 1 if a commercially viable idea has been identified in sector i but it has not yet been implemented, and 0 otherwise. The set of instances at which ideas are implemented in sector i is denoted by Ψ_i. We let $V_i^I(t)$ denote the expected present value of profits from implementing an innovation at time t, and $V_i^D(t)$ denote that of delaying implementation from time t until the most profitable time in future.

2.4 The Labor Market

In aggregate there is a unit measure of managers and a measure L of potential production workers. Firm owners can perfectly contract with managers so that these individuals are hired in a fully competitive labor market.[10] In contrast, production workers are more difficult to monitor, and may choose to "shirk" by providing zero effort, while potentially retaining their jobs. If they do not shirk, workers in sector i are subject to a rate of separation $\mu_i(t)$. Workers and firms may separate for two reasons. First, there is a constant, exogenous "normal" rate of within–firm job turnover, $\bar{\mu}$, which is independent of the business cycle. Second, there may be endogenous separation due to firms' exit decisions.[11] If a worker does shirk, the rate of separation increases to $\mu_i(t) + q$, where q depends on the ability of the firm to detect shirking. Since detection is imperfect, firms must pay workers a sufficiently high wage to ensure incentive compatibility.

The rate of separation may vary across sectors. We denote the equilibrium average rate of separation by $\mu^A(t)$. If workers do lose their job, they enter a pool of unemployed workers who are viewed as homogeneous by firms and, hence, face an equal probability of being re–hired. We denote the fraction exiting the unemployment pool each instant by $d\Lambda(t)$. In aggregate, the change in the level of employment, $n(t)$, must equal the number of hires less the number of separations:

$$dn(t) = (L - n(t))\, d\Lambda(t) - n(t)\mu^A(t)dt \tag{6}$$

In the absence of discontinuous jumps in employment levels, the hiring rate is given by the derivative $\lambda(t) = d\Lambda(t)/dt$.

[10] Introducing a moral hazard problem at this level as well would add unnecessary complexity without changing the qualitative implications.

[11] We assume throughout that incoming firms do not hire workers directly from the incumbents they replace. Allowing for partial transitions directly to new employers would de-couple plant level destruction from job destruction but changes nothing qualitatively.

2.5 Goods Market Competition

In order to extract rent from the leading edge technology, intermediate producers must utilize the large scale mode of production and hire workers.[12] Given the unit elasticity of demand, the producer holding the state of the art technology optimally limit prices at the marginal cost of his next best competitors. We assume that intermediates are completely used up in production, but can be stored for use at a later date. Incumbent intermediate producers must therefore decide whether to sell now, or store and sell later.

2.6 Households

The economy is populated by a unit measure of infinitely–lived "large" households. Each household consists of a unit measure of managers and a measure $L > 1$ of potential production workers. Managers supply labor inelastically, but the supply of production worker effort is determined by the household in response to the employment opportunities and wages offered. Households are assumed to have preferences given by

$$U(t) = \int_t^\infty e^{-\rho(\tau - t)} u(c(\tau), n(\tau)) d\tau, \tag{7}$$

where ρ denotes the rate of time preference, $c(t)$ denotes household consumption and $n(t)$ denotes the measure of production workers that are exerting effort. That is

$$n(t) = \int_0^L \varepsilon_j(t) dj, \tag{8}$$

where $\varepsilon_j(t) \in \{0, 1\}$ denotes the effort level exerted by worker j in period t. We assume that the instantaneous utility function takes the Cobb–Douglas form

$$u(c(t), n(t)) = c(t)^{1-\sigma} (L - n(t))^{\sigma}. \tag{9}$$

We focus on preferences of this form for expositional simplicity, but they can be generalized to allow for homogeneity of degree other than one without changing our qualitative conclusions.[13] Note that we assume throughout that $\rho > (1 - \sigma)\phi$, which is necessary, though not sufficient, for a household's present discounted utility to be bounded in equilibrium.

[12] In order to sustain meaningful innovation, there must exist some way for successful innovators to extract, at least temporarily, rents from their knowledge. We leave unspecified the precise means by which this occurs and simply assume that the leading technology is the exclusive province of the innovator. Once the innovation has been superceded, others may enter and use it at will, since, as it can no longer generate rents, the innovator controlling it no longer has incentive to limit its use elsewhere.

[13] Note that the implied intertemporal elasticity of substitution exceeds unity. This feature can be relaxed in a model with physical capital (see Francois and Lloyd–Ellis, 2008), but is essential here.

Workers within a household do not have independent preferences. The household chooses the supply of labor effort for each worker so as to maximize his/her marginal contribution to household utility. The relational contract that we analyze here is an equilibrium of the repeated game played between a firm in sector i and the firm's worker. We proceed to compute incentive compatible wages, which will be an equilibrium condition of the model, under the assumption that a worker who is found shirking is dismissed and not re-hired by that firm.[14]

For any production worker who is offered employment in sector i at the beginning of time t, the household chooses whether he/she should accept the offer or remain unemployed. If the offer is accepted, the household chooses whether or not the worker should exert effort. If the production wage in sector i is $w_i(t)$, and the worker supplies one unit of labor effort, the marginal contribution to household utility in period t is

$$w_i(t)u_c(t) + u_n(t), \tag{10}$$

where

$$u_c(t) = (1-\sigma)c(t)^{-\sigma}(L-n(t))^{\sigma} > 0 \tag{11}$$
$$u_n(t) = -\sigma c(t)^{1-\sigma}(L-n(t))^{\sigma-1} < 0. \tag{12}$$

Note that the household's valuation of the wage earned by each worker takes as given aggregate household consumption and employment.

The marginal utility contributed to the household by worker j if he/she is offered employment in sector i can be written as

$$\psi^*_{ij}(t) = \max\left[\psi^E_{ij}(t), \psi^S_{ij}(t), \psi^U_j(t)\right] \tag{13}$$

where $\psi^E_{ij}(t)$ represents the value of being employed in sector i and supplying effort $\varepsilon_j(t) = 1$, $\psi^S_{ij}(t)$ represents the value of being employed in sector i and shirking, $\varepsilon_j(t) = 0$, and $\psi^U_j(t)$ represents the value of being unemployed. For workers who are not offered employment at time t the household also receives $\psi^U_j(t)$.

The "large" household assumption effectively implies that total household wage income, $\omega(t) = \int_0^L w_j(t)dj$, is certain and identical across households. Each household chooses consumption over time to maximize (7) subject to the intertemporal budget constraint

$$\int_t^\infty e^{-[R(\tau)-R(t)]}c(\tau)d\tau \leq S(t) + \int_t^\infty e^{-[R(\tau)-R(t)]}\left[s(\tau)+\omega(\tau)\right]d\tau \tag{14}$$

where $S(t)$ denotes the household's stock of assets at time t and $R(t)$ denotes the market discount factor from time zero to t. The stock of assets could potentially include claims to the profits of

[14] Appendix B formally states this game's information sets, timing and strategies, and shows that the posited behavior between the worker and firm are equilibrium strategies.

intermediate firms and stored output. The first–order conditions of the household's dynamic optimization require that

$$dR(t) = \rho dt + \sigma \left[\frac{dc(t)}{c(t)} + \frac{dn(t)}{L - n(t)} \right] \qquad \forall \ t \tag{15}$$

and that (14) holds with equality. The Euler equation is expressed in the form above to allow for the possibility of discontinuous jumps. Over intervals during which neither the discount factor nor employment jumps, household consumption satisfies

$$\frac{\dot{c}(t)}{c(t)} + \frac{\dot{n}(t)}{L - n(t)} = \frac{r(t) - \rho}{\sigma}, \tag{16}$$

where $r(t) = \dot{R}(t)$.

2.7 General Equilibrium

Given an initial stock of implemented innovations represented by a cross–sectoral distribution of productivities $\{a_i(0)\}_{i=0}^{1}$ and an initial distribution of unimplemented innovations, $\{Z_i(0)\}_{i=0}^{1}$, an equilibrium for this economy satisfies the following conditions:

• Households allocate consumption optimally over time, (15).

• Households only accept employment offers for worker j if the contribution to household utility for that worker is no less than that of remaining unemployed:

$$\max \left[\psi_{ij}^{E}(t), \psi_{ij}^{S}(t) \right] \geq \psi_{j}^{U}(t). \tag{17}$$

• Final goods producers choose intermediates to minimize costs, (2).

• Given demand, in (2), intermediate producers set prices so as to maximize profits.

• Intermediate producers choose the mode of production which maximizes their profits.

• The skilled–labor market clears:

$$\int_0^1 m_i(t) di + H(t) = 1. \tag{18}$$

• In the face of unemployment, intermediate producers offer a path of production wages so as to maximize profits subject to the participation constraint and the incentive compatibility condition:

$$\psi_{ij}^{E}(t) \geq \psi_{ij}^{S}(t). \tag{19}$$

• There is free entry into arbitrage. For all assets that are held in strictly positive amounts by households, the rate of return between must be equal.

• There is free entry into innovation. Managerial innovative effort is allocated to the sector which maximizes the expected present value of the innovation. Also

$$\delta \max[V_i^D(t), V_i^I(t)] \leq s(t), \quad h_i(t) \geq 0 \qquad \text{with at least one equality} \tag{20}$$

• Entrepreneurs with innovations choose whether to enter production using the previous technology.

• In periods where there is implementation, entrepreneurs with innovations must prefer to implement rather that delay until a later date

$$V_i^I(t) \geq V_i^D(t) \quad \forall\ t \in \Psi_i \tag{21}$$

• In periods where there is no implementation, either there must be no innovations available to implement, or entrepreneurs with innovations must prefer to delay rather than implement:

$$\begin{aligned} \text{Either } Z_i(t) &= 0, \\ \text{or if } Z_i(t) &= 1,\ V_i^I(t) \leq V_i^D(t) \quad \forall\ t \notin \Psi_i. \end{aligned} \tag{22}$$

3 The Cyclical Equilibrium

Although there exists an acyclical equilibrium growth path that satisfies the conditions stated above, our focus here is on a cyclical equilibrium growth path. In this section, we start by positing a temporal pattern of entrepreneurial behavior in innovation, entry into production, and implementation of productivity improvements. We then derive the implications of this posited pattern for relative returns between entrepreneurship and management, innovation levels, and firms' choice of production mode and evolution of aggregate variables. Section 4 then derives a set of sufficient conditions under which the implied evolution of aggregate variables, and market clearing, yield optimal entrepreneurial behavior corresponding with the originally posited behavior.

3.1 Posited Entrepreneurial Behavior

Suppose implementation occurs at discrete dates denoted by T_ν where $v \in \{1, 2, ..., \infty\}$. We adopt the convention that the vth cycle starts at time T_{v-1} and ends at time T_ν. The posited behavior of entrepreneurs over this cycle is illustrated in Figure 1. After implementation at date T_{v-1} there is an interval during which there is no entrepreneurial search and consequently all managers are used to supervise production. At some time T_v^E, search commences again as the economy begins to decline. Innovative activity is allocated symmetrically across sectors which have not yet had a success in the current cycle. Once a success occurs, all innovative activity ceases in the sector and the successful entrepreneur enters production displacing the existing incumbent. Implementation of the productivity improvement is, however, withheld until time T_v.

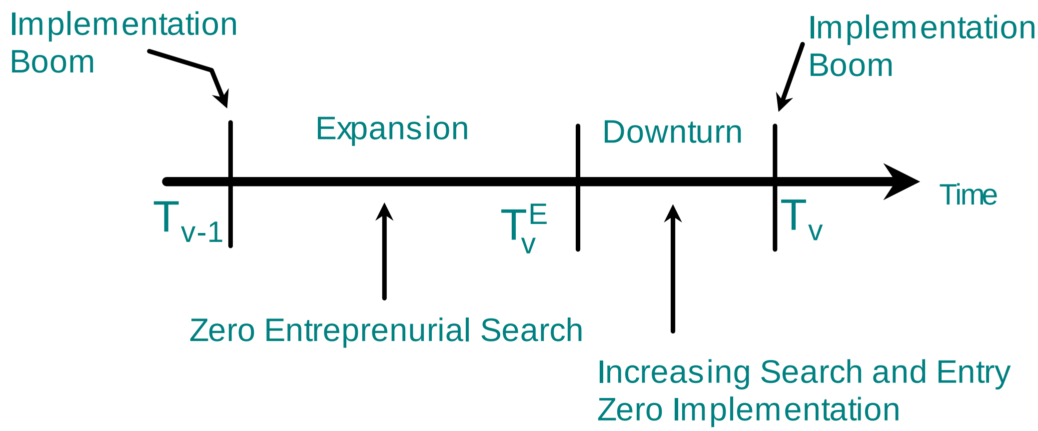

Figure 1: Search, entry and implementation over the cycle

3.2 Within–Cycle Implications

Lemma 1 *An incumbent firm using a technology which is certain to be replaced at the end of the cycle cannot profitably produce using the large scale mode of production.*

A firm in a sector where a new idea is about to be implemented with probability 1 cannot maintain a relational contract with production workers since it cannot promise employment into the future. Its production is thus taken over by the newly successful entrepreneur whose termination date, though not yet known with certainty, will at least not occur before the next cyclical downturn. Its lowest cost competitors are the competitive fringe who, though producing at small scale, can in aggregate, steal the new incumbent's whole market if too high a price is charged. These competitors avoid the costs of setting up large scale production and hiring multiple workers, but can only produce using the previous state of the art technology. The limit price charged by intermediate producer i is therefore

$$p_i(t) = \frac{s(t)}{e^{-\gamma}a_i(t)}. \tag{23}$$

It follows from (2), (4) and (23), that the skilled work force receives a constant share of output:

$$s(t)(1 - H(t)) = \frac{e^{-\gamma}}{\theta L}Y(t). \tag{24}$$

It also follows that the profits of an intermediate producer can be expressed as

$$\pi\left(w_i(t),t\right)=\left(1-\frac{e^{-\gamma}}{\theta L}-e^{-\gamma}\frac{w_i(t)}{s(t)}\right)Y(t). \tag{25}$$

Note that, since all sectors face the same skilled salary $s\left(t\right)$ and revenue shares are symmetric, profits vary across sectors only due to differences in the production wage, $w_i(t)$. A final implication is that, in equilibrium, the managerial salary is tied to the level of technology. Since within the posited cycle, the state of intermediate technology in use is unchanging, it follows that

Lemma 2 *Within the posited cycle, the managerial salary grows at the rate of technological change in the final output sector:*

$$\frac{\dot{s}(t)}{s(t)}=\phi. \tag{26}$$

Utilizing the incentive compatibility condition (19) , and the time-varying net present value of each of the three possible employment states $\psi_{ij}^{E}(t),\psi_{ij}^{S}(t)$ and $\psi_{j}^{U}(t)$ yields the following binding incentive compatible wage for production workers within the posited cycle.

Lemma 3 *: The production wage in sector i is given by*

$$w_i(t)=-\frac{u_n(t)}{u_c(t)}\left[1+\frac{1}{q}\left(\rho+\mu_i(t)+\lambda(t)-\frac{\dot{u}_n(t)}{u_n(t)}\right)\right]. \tag{27}$$

This expression summarizes the key forces acting on the incentive compatible production wage. If $q\to\infty$, then detection of shirking becomes perfect, so that the incentive problem disappears, and the expression simplifies to the standard first–order condition for household labor supply: $w_i(t)=-u_n(t)/u_c(t)$. In this case, the wage is determined by two forces, both of which are pro–cyclical. As $n(t)$ rises during an expansion the marginal utility cost to the household of supplying additional workers rises, so that a higher wage is needed to induce this supply. Also, as consumption $c(t)$ rises the marginal benefit of supplying additional labor falls, so that firms must raise wages to induce labor effort.

However, imperfect detection (q finite) of shirking introduces three other forces. One of these is pro–cyclical — as the hiring rate, $\lambda(t)$, rises, being fired is a less costly threat, so that firms must raise the wage to provide greater incentives not to shirk. However, the other two forces are counter-cyclical. First, a higher rate of separation, $\mu_i(t)$, implies that workers must be compensated for the increased likelihood of job loss.[15] Second, with negative employment growth, the marginal cost of supplying effort is lower tomorrow than today $\dot{u}_n(t)/u_n(t)>0$. Other things

[15] This positive impact on incentive compatible wages of increased job loss chances has been noticed before in similar relational contract frameworks by Saint-Paul (1996), and Fella (2000), with evidence consistent with this being found in Abowd and Ashenfelter (1981), Adams (1985) and Li (1986).

equal, this makes workers more willing to risk unemployment by shirking, so firms must raise wages to compensate.

3.3 The Expansion ($T_{v-1} \rightarrow T_v^E$)

Since all managers are used in production in the expansion of the posited cycle, it must be the case that production worker employment is at its maximum:

$$n(t) = \theta L. \tag{28}$$

Let the level of output immediately following the boom be given by $Y_0(T_{v-1})$.[16] With a constant level of employment it follows that output grows during the expansion at the rate ϕ, so that $Y(t) = e^{\phi(t-T_{v-1})}Y_0(T_{v-1})$. Since the economy is closed, and there is no incentive to store either intermediate or final output across periods (provided $r(t) \geq 0$), it must be the case that consumption grows at this rate too:

$$\frac{\dot{c}(t)}{c(t)} = \frac{\dot{Y}(t)}{Y(t)} = \phi. \tag{29}$$

Substituting these facts into (16) yields the implied interest rate during the expansion,

$$r(t) = \rho + \sigma\phi. \tag{30}$$

Since there is no firm turnover, flows out of employment are given by the exogenous separation rate $\bar{\mu}$. Using (6) and (28), it follows that the rate at which workers are hired from the unemployment pool is then given by

$$\bar{\lambda} = \frac{\bar{\mu}\theta}{1-\theta} \tag{31}$$

Substituting these facts into (27) implies that the production wage also grows at the rate ϕ during the expansion:

Proposition 1 *: During the expansion of the posited cycle unemployment is constant at $(1-\theta)L$ and the production wage is given by*

$$w^A(t) = \frac{A}{L}e^{\phi(t-T_{v-1})}Y_0(T_{v-1}) \tag{32}$$

where $A = \sigma\left(\rho - (1-\sigma)\phi + q + \frac{\bar{\mu}}{1-\theta}\right)\Big/\left[(1-\sigma)(1-\theta)q\right].$

[16] Throughout, we use the subscript 0 to denote the value of a variable immediately after the boom. Formally, $X_0(T) = \lim_{t\rightarrow T^+} X(t)$.

During this phase, the discounted value of a commercially viable idea whose implementation is delayed until the subsequent boom, $V^D(t)$, would grow at the rate of interest, $r(t) = \rho + \sigma\phi$, as the date of implementation draws closer. If there is no search, it must be the case that $\delta V^D(t) < s(t)$. However, since $\rho > (1-\sigma)\phi$, $\delta V^D(t)$ must eventually equal $s(t)$. At this point, entrepreneurial search commences. The following Lemma demonstrates that it does so smoothly:

Lemma 4 *At time T_v^E, when entrepreneurial search first commences in the posited cycle, $s(T_v^E) = \delta V^D(t)$ and $H\left(T_v^E\right) = 0$.*

3.4 The Contraction ($T_v^E \rightarrow T_v$)

For the positive search to occur under free entry, it must be that $s(t) = \delta V^D(t)$ while it does so. Since the skilled wage grows at rate ϕ throughout the posited cycle, $\delta V^D(t)$ must also grow at the same rate during this phase. Since the time until implementation is falling and there is no stream of profits, the interest rate over this phase is given by

$$r(t) = \frac{\dot{V}^D(t)}{V^D(t)} = \frac{\dot{s}(t)}{s(t)} = \phi. \tag{33}$$

The household's Euler equation can be expressed as

$$\frac{\dot{c}(t)}{c(t)} + \frac{\dot{n}(t)}{L - n(t)} = -\left(\frac{\rho - \phi}{\sigma}\right). \tag{34}$$

Since the economy is closed, it follows once again that, because there is no incentive to store output, $c(t) = Y(t)$. Hence, consumption growth must equal the sum of the growth in final goods productivity, ϕ, and the growth (negative) in employment. Solving the implied first order differential equation in $n(t)$ we have:

Lemma 5 *: Employment during the contraction of the posited cycle evolves according to*

$$n(t) = \frac{\theta L e^{-\frac{(\rho-(1-\sigma)\phi)}{\sigma}(t-T_v^E)}}{1-\theta+\theta e^{-\frac{(\rho-(1-\sigma)\phi)}{\sigma}(t-T_v^E)}}. \tag{35}$$

Note that $n(T_v^E) = \theta L$. This expression implies that $n(t)$ must be declining during the downturn because, as high–skilled labor flows out of production and into search, the demand for production workers falls in proportion. Note that these entrepreneurial flows out of production into future oriented productivity improving tasks would look like firms scaling back production in response to a decline in aggregate demand. The implied skilled labor that flows into search is therefore

$$H(t) = 1 - \frac{n(t)}{\theta L} = \frac{(1-\theta)\left(1 - e^{-\frac{(\rho-(1-\sigma)\phi)}{\sigma}(t-T_v^E)}\right)}{1-\theta+\theta e^{-\frac{(\rho-(1-\sigma)\phi)}{\sigma}(t-T_v^E)}}. \tag{36}$$

The proportion of sectors in which no viable productivity improvement has been identified by time $t \in (T_v^E, T_v)$ is given by

$$P(t) = \exp\left(-\int_{T_v^E}^{t} \delta h(\tau) d\tau\right). \tag{37}$$

Recalling that labor is only devoted to search in sectors where no ideas has been identified, the labor allocated to search in those sectors is

$$h(t) = \frac{H(t)}{P(t)}. \tag{38}$$

In the measure $(1-P(t))$ of sectors where new employers have already entered, the only source of separation is normal turnover, $\bar{\mu}$. However, in sectors where no such restructuring has occurred yet, the rate of separation also includes the probability of a restructuring occurring, $\delta h(t)$, which increases as the downturn proceeds. It follows that the aggregate rate of job destruction is given by

$$\mu^A(t) = (1 - P(t))\bar{\mu} + P(t)\left[\bar{\mu} + \delta \frac{H(t)}{P(t)}\right] = \bar{\mu} + \delta H(t) \tag{39}$$

In sectors where entry has already occurred, wages are lower, denoted by $w^L(t)$, as there is no chance of further restructuring in the contraction, and so

$$w^L(t) = \frac{\sigma c(t)}{(1-\sigma)(L-n(t))}\left[1 + \frac{1}{q}\left(\rho + \bar{\mu} + \lambda(t) - \frac{\dot{u}_n(t)}{u_n(t)}\right)\right]. \tag{40}$$

When a restructuring has not yet occurred, there is higher probability of job destruction and a higher wage is required to ensure incentive compatibility:

$$w^H(t) = \frac{\sigma c(t)}{(1-\sigma)(L-n(t))}\left[1 + \frac{1}{q}\left(\rho + \bar{\mu} + \delta H(t) + \lambda(t) - \frac{\dot{u}_n(t)}{u_n(t)}\right)\right]. \tag{41}$$

Intuitively, this is because new entrants have a longer expected duration of incumbency than existing firms, and can thus promise non-shirking workers a longer expected span of employment. Recall that free mobility of labor does not equalize wages across these sectors as incentive compatibility binds.

Since the production wage in each sector is linearly related to the rate of job destruction, it follows that the average production wage is simply given by (27) with $\mu(t) = \mu^A(t)$. Substituting in for the endogenous variables yields the following result:

Proposition 2 : *During the contraction of the posited cycle, the average production wage is given by*

$$w^A(t) = \left[Be^{-\frac{\rho-(1-\sigma)\phi}{\sigma}(t-T_v^E)} - Ce^{-\frac{2(\rho-(1-\sigma)\phi)}{\sigma}(t-T_v^E)} + \frac{De^{-\frac{\rho-(1-\sigma)\phi}{\sigma}(t-T_v^E)}}{1-\theta+\theta e^{-\frac{\rho-(1-\sigma)\phi}{\sigma}(t-T_v^E)}}\right]\frac{e^{\phi(t-T_{v-1})}Y_0}{L} \tag{42}$$

where $B = \frac{\sigma(q+\delta+\bar{\mu})}{(1-\sigma)(1-\theta)q}$, $C = \frac{\sigma\left(\delta - \frac{\bar{\mu}\theta}{1-\theta}\right)}{(1-\sigma)(1-\theta)q}$ *and* $D = \frac{\rho-(1-\sigma)\phi}{(1-\sigma)q}$.

The evolution of the production wage during downturns is ambiguous due to the interaction of the various pro– and counter– cyclical forces mentioned earlier. However, as we document below, in general the wage traces out a hump–shaped pattern. If counter-cyclical forces dominate, the wage rises to begin with, largely reflecting the rising rate of separation. However, as the downturn proceeds, more and more sectors are taken over by new entrants who then pay the relatively low wage, $w_L(t)$, and fewer sectors are left facing imminent exit and paying the wage $w_H(t)$. If the downturn continues for long enough, this change in sectoral composition, eventually drives down the average wage.

3.5 The Boom

The growth in aggregate productivity during period T_v is given by

$$\Gamma_v = \int_0^1 [\ln a_i(T_v) - \ln a_i(T_{v-1})]\, di \tag{43}$$

Productivity growth at the boom is given by $\Gamma_v = (1 - P(T_v))\gamma$, where $P(T_v)$ is defined by (37). Substituting in the allocation of labor to entrepreneurship through the downturn given by (36) and integrating over the interval

$$\Delta_v^E = T_v - T_v^E. \tag{44}$$

yields the following implication:

Proposition 3 *The growth in productivity during the boom of the posited cycle is given by*

$$\Gamma_v = \delta\gamma\Delta_v^E + \frac{\sigma\delta\gamma}{(\rho - (1-\sigma)\phi)\theta} \ln\left(1 - \theta\left(1 - e^{-\frac{(\rho-(1-\sigma)\phi)}{\sigma}\Delta_v^E}\right)\right). \tag{45}$$

Equation (45) tells us how the size of the productivity boom depends positively on the amount of time the economy is in the search phase, Δ_v^E. The amount of search in that phase is determined by the movements in the interest rate, so once the length of the contraction is known, the growth rate over the cycle is pinned down. The size of the boom is convex in Δ_v^E, reflecting the fact that as the boom approaches, search effort increases.

During the boom, productivity and production employment both rise rapidly. Since it is the product of these two, consumption also increases discontinuously. For this to be consistent with optimal household behavior, it follows that the discount factor must also rise discontinuously. The long run discount factor during the boom is given by the household's Euler equation

$$R_0(T_v) - R(T_v) = \sigma \ln \frac{c_0(T_v)}{c(T_v)} - \sigma \ln \frac{L - n_0(T_v)}{L - n(T_v)}. \tag{46}$$

Since consumption growth at the boom results both from implementation and the reallocation of labor into production it follows that

$$R_0(T_v) - R(T_v) = \sigma\Gamma_v + \sigma \ln \frac{n_0(T_v)}{n(T_v)} - \sigma \ln \frac{L - n_0(T_v)}{L - n(T_v)}. \tag{47}$$

Now, $n_0(T_v) = \theta L$ and using (35) to determine $n(T_v)$, we get

$$R_0(T_v) - R(T_v) = \sigma\Gamma_v + (\rho - (1-\sigma)\phi)\,\Delta_v^E. \tag{48}$$

Over the boom, the asset market must simultaneously ensure that entrepreneurs holding viable ideas are willing to implement immediately (and no earlier) and that, for households, holding equity in firms dominates holding claims to alternative assets (particularly stored intermediates). The following Proposition demonstrates that these conditions imply that during the boom, the discount factor must equal productivity growth:

Proposition 4 *: Asset market clearing at the boom of the posited cycle requires that*

$$R_0(T_v) - R(T_v) = \Gamma_v \tag{49}$$

Combining (48) with (49) yields

$$\Gamma_v = \frac{(\rho - (1-\sigma)\phi)\Delta_v^E}{1-\sigma}. \tag{50}$$

Combining (45) and (50) yields a unique (non–zero) equilibrium pair (Γ, Δ^E) that is consistent with the within–cycle dynamics and asset market clearing. Note that although we did not impose any stationarity on the cycles, the equilibrium conditions imply stationarity of the size of the boom and the length of the downturn. Existence of such a pair requires that $0 < \rho-(1-\sigma)\phi < \delta\gamma(1-\sigma)$.

4 Optimal Entrepreneurial Behavior

This section derives a set of sufficient conditions under which the behavior posited in Figure 1 is optimal given the implied behavior of aggregates derived above.

4.1 Optimal Restructuring

The present value of profits earned in a sector where no future entry/exit is anticipated up to the end of the current cycle is

$$V^*(t) = \int_t^{T_{v+1}} e^{-\int_t^\tau r(s)ds} \pi(w^L(\tau), \tau) d\tau$$

In the cyclical equilibrium considered here, secrecy can be a valuable option.[17] Since ideas are withheld until a common implementation time, simultaneous implementation is feasible. However, as the following proposition demonstrates, such duplications do not arise in the cyclical equilibrium because successful entrepreneurs enter production to displace previous incumbents, sending a credible signal that directs subsequent entrepreneurial efforts to sectors other than their own. The formal details of the signalling game between incumbent, successful entrants, and other potential innovators are relegated to the appendix.

Proposition 5 *: Given that innovations are implemented at the subsequent boom, a successful entrepreneur transfers $V^*(t)$ to use the incumbent's technology until T_{v+1}, and takes over production in that sector. All search in their sector then stops until the next cycle.*

The payment $V^*(t)$ by a successful entrepreneur to the previous incumbent acts as a credible signal that this entrepreneur has had an innovation, but does not reveal the content of that success.[18] If an entrepreneur's announcement is credible, other entrepreneurs will exert their efforts in sectors where they have a chance of becoming the sole dominant entrepreneur. One might imagine that unsuccessful entrepreneurs would have an incentive to mimic successful ones by falsely announcing success to deter others from entering the sector. But doing this yields a flow of profits for the interval $t \to T_{v+1}$ which is ε less than paid for it, and is thus not worthwhile.

It follows that the value of an incumbent firm in a sector where no innovation has occurred by time t during the vth cycle can be expressed as

$$V^I(t) = \int_t^{T_{v+1}} e^{-\int_t^\tau [r(s)+\delta h_i(s)]ds} \left[\pi(w^H(\tau),\tau) + \delta h(\tau) V^*(\tau)\right] d\tau + \frac{P(T_{v+1})}{P(t)} e^{-[R_0(T_v)-R(t)]} V_0^I(T_{v+1}). \tag{51}$$

The first term here represents the expected discounted profit stream that accrues to the entrepreneur during the current cycle, and the second term is the expected discounted value of being an incumbent thereafter. Note that due to symmetry, the probability that no innovation arises between time t and the end of the cycle in any given sector, $\frac{P(T_{v+1})}{P(t)}$, is equal to the fraction of sectors that innovate between t and T_{v+1}.

[17] As Cohen, Nelson and Walsh (2000) document, delaying implementation to protect knowledge is a widely followed practice in reality. Graham (2004) also describes a similar critical role of secrecy in protecting value. Scarromozino, Temple and Vulkan (2005) suggest indirect evidence of delay can be gleaned from Hobijn and Jovanovic's (2001) evidence of major changes in US stock valuations anticipating productivity changes. Also, Pastor and Veronesi (2005) provide evidence arguing that the state of the macro economy determines decisions on IPO timing. This is also in line with delay playing a key role.

[18] The payment does not have to be a literal transfer for technology but would instead more realistically take the form of the entrant renting the incumbent's machines, plant and production methods for the remainder of the recession. For example, this could occur at a fire-sale over the incumbent firm's assets. After that, the entrant will implement his own methods so that the rental rate on such equipment is zero anyway. The important point is that the purchase of the assets at a price that would only be worthwhile to an entrant with a valuable innovation sends a credible signal to other innovators that profits will be higher if innovations are targetted elsewhere.

4.2 Optimal Search and Implementation

The willingness of entrepreneurs to delay implementation until the boom and to just start engaging in search at exactly T_v^E depends crucially on the expected value of monopoly rents relative to the current skilled labor returns. This is a forward looking condition: given Γ and Δ^E, the present value of these rents depend on the length of the subsequent cycle, $T_{v+1} - T_v$, which we denote by the term Δ_{v+1}.

The expected value of an entrepreneurial success occurring at some time $t \in \left(T_v^E, T_v\right)$ but whose implementation is delayed until time T_v is thus:

$$V^D(t) = e^{-[R_0(T_v) - R(t)]} V_0^I(T_v). \tag{52}$$

Since Lemma 4 implies that entrepreneurship starts at T_v^E, free entry into entrepreneurship, requires that

$$\delta V^D(T_v^E) = \delta e^{-[R_0(T_v) - R(T_v^E)]} V_0^I(T_v) = s(T_v^E). \tag{53}$$

The increase in the wage across cycles reflects the improvement in overall productivity: $s(T_{v+1}^E) = e^{\Gamma + \Delta_v \phi} s(T_v^E)$, and from the asset market clearing conditions, we know that $R_0(T_v) - R(T_v^E) = \Gamma + \phi \Delta^E$ is a constant across cycles. It immediately follows that the increase in the present value of monopoly profits from the beginning of one cycle to the next must satisfy:

$$V_0^I(T_v) = e^{\Gamma + \Delta_v \phi} V_0^I(T_{v-1}). \tag{54}$$

The following proposition demonstrates that (54) implies a unique cycle length:

Proposition 6 *Given the boom size, Γ, and the length of the downturn, Δ^E, there exists a unique cycle length, Δ, such that entrepreneurs are just willing to commence search Δ^E periods prior to the boom.*

The length of the cycle is given by

$$\Delta = \Delta^E + \frac{1}{b} \ln \left[1 + \alpha \Delta^E + \zeta_1 \left(1 - e^{-\frac{b}{\sigma}\Delta^E}\right) - \zeta_2 \left(1 - e^{-\frac{2b}{\sigma}\Delta^E}\right) - \frac{\zeta_3 \left(1 - e^{-\frac{b}{\sigma}\Delta^E}\right)}{1 - \theta \left(1 - e^{-\frac{b}{\sigma}\Delta^E}\right)} \right], \tag{55}$$

where $b = \rho + (1 - \sigma)\phi$ and α, ζ_1, ζ_2 and ζ_3 are positive constants. Notice, once again that stationarity is not imposed, but is implied by the equilibrium conditions.

The equilibrium conditions (20), (21) and (22) on posited entrepreneurial behavior also impose the following requirements on our hypothesized cycle:

• Entrepreneurs with commercially viable ideas at time $t = T_v$, must prefer to implement immediately, rather than delay:

$$V_0^I(T_v) > V_0^D(T_v). \tag{56}$$

This condition is also sufficient to ensure that household utility is bounded in equilibrium, since it implies that[19]

$$\frac{1}{\Delta}\ln\left(\frac{c_0(T_v)}{c_0(T_{v-1})}\right) = \frac{\Gamma}{\Delta} + \phi < \frac{\rho}{1-\sigma}. \tag{57}$$

From (50) this condition must hold if $\Delta > \Delta^E$.

• Entrepreneurs who identify commercially viable ideas during the downturn must prefer to wait until the beginning of the next cycle rather than implement earlier:

$$V^I(t) < V^D(t) \qquad \forall\, t \in (T_v^E, T_v) \tag{58}$$

• No entrepreneur wants to innovate during the expansion of the cycle. Since in this phase of the cycle $\delta V^D(t) < s(t)$, this condition requires that

$$\delta V^I(t) < s(t) \qquad \forall\, t \in (T_{v-1}, T_v^E) \tag{59}$$

In constructing the equilibrium we have implicitly imposed two additional requirements:

• The downturn is not long enough that all sectors innovate:

$$P(T_v) > 0. \tag{60}$$

• Firm operating profits are always positive:

$$\pi_i(t) > 0 \qquad \forall\, i,\, t \tag{61}$$

5 Baseline Example

In this section we demonstrate that the triple $(\Delta^E, \Delta, \Gamma) > 0$ solving (45), (50), (55) exists, and the conditions (E1) through (E5) are satisfied. For these values, posited entrepreneurial behavior is optimal given the implied cyclical behavior of aggregates that are generated by this behavior, so that the cycling steady state exists. We do this by first solving the model for a baseline set

[19] To see this observe that (E1) can be expressed as $V_0^I(T_v) > e^{-[R_0(T_{v+1})-R_0(T_v)]}e^{\Gamma+\Delta\phi}V_0^I(T_v)$, which holds only if $R_0(T_{v+1}) - R_0(T_v) = \sigma(\Gamma + \Delta\phi) + \rho\Delta > \Gamma + \Delta\phi$. Re-arranging yields (57).

of parameters, and then vary these. This exercise is not an attempt to assess the quantitative significance of the model. Rather, our aim here is instead to establish that the existence conditions can be simultaneously satisfied for values that are within reasonable bounds. The second aim is to gain some understanding of the model's comparative static properties by varying parameters around this baseline case.

Table 1 documents the parameters used for our baseline case. These parameters were chosen so that the model generates a long–run growth rate of close to 2%, a cycle length of about 10 years and a correlation of de–trended log average wages and log output close to zero. The length of an expansion, Δ^X, is 7.10 years and of a contraction, Δ^E, is 2.88 years in this baseline. Figure 2 illustrates the evolution of key aggregates over a cycle implied by simulating the baseline example. After rising gradually during the expansion, output falls rapidly in the contraction. However, correctly measured GDP should include the payments made to labour used in entrepreneurial search. As illustrated, GDP also falls during the downturn, but less rapidly. The reason GDP falls is that skilled labor used in production is being paid below its marginal product. As labour effort is transferred into search, the marginal cost in terms of lost output exceeds the marginal benefit of search. Because it is offset by the fall in this intangible investment, the rise in GDP at the boom is also less dramatic than otherwise.

Table 1: Baseline Parameters

Parameter	Value
ρ	0.03
ϕ	0.01
σ	0.30
θ	0.40
L	3.00
γ	0.40
$\bar{\mu}$	0.10
δ	1.25
q	0.40

Figure 3 illustrates the evolution of the relevant value functions and the productivity adjusted wage $s(t)/\delta$ in the baseline example. At the beginning of the cycle $s(t) = \delta V^I(T_v) > \delta V^D(T_v)$. The value $\delta V^D(t)$ grows while $\delta V^I(t)$ declines during the first phase of the cycle, this condition implies that $\delta V^D(t)$ and $\delta V^I(t)$ must intersect before $\delta V^D(t)$ reaches $s(t)$. It follows that when entrepreneurship starts, it is optimal to delay implementation, $V^D(T_v^E) > V^I(T_v^E)$. During the contraction, the probability of not being displaced at the boom if implementing early declines so that $V^I(t)$ rises. Eventually, an instant prior to the boom, $V^I(T_{v+1}) = V^D(T_{v+1})$, but until that point it continues to be optimal to delay. At the boom, the value of immediate implementation rises, while the value of delayed implementation falls, so that all existing innovations are implemented. However, since the skilled wage increases by as much as $V^I(t)$, search ceases and the cycle begins again.

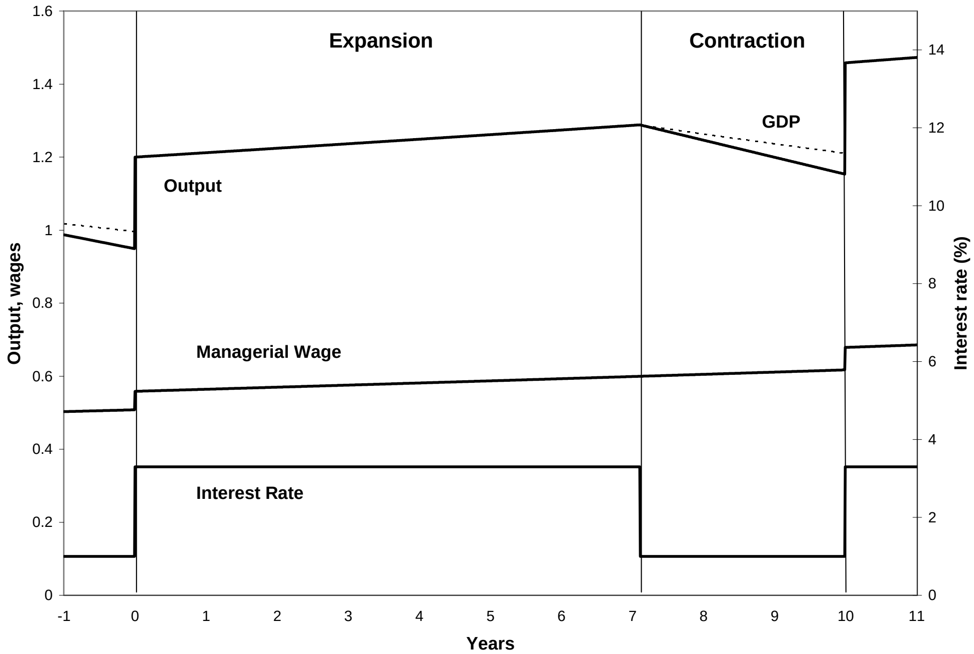

Figure 2: Evolution of some key variables over the cycle

5.1 Restructuring: Intensive and Extensive Margin Adjustment

The pattern of average hiring and separation rates implied by this baseline case are plotted in Figure 4. Both are stable through the expansion and only occur because of turnover on the intensive margin, $\bar{\mu}$. Upon entering the contractionary phase, the hiring rate falls as firms cut back on the intensive margin in response to falling aggregate demand. This is generated by the endogenous recession occurring through increased restructuring. Simultaneously, the steady increase in entrepreneurial search leads to increasing job separation through this phase on the extensive margin; old firms are being driven out of production by new ones. Entry is also reflected in the gradual but steady pick up in the hiring rate that starts to occur in anticipation of the forthcoming boom. At the boom, existing firms then adjust employment on the intensive margin, increasing employment to meet the increased aggregate demand. This leads to a surge in hiring and an increase in employment which starts off the next cycle.[20] As Figure 4 suggests, the variance

[20] As in the data, we measure hiring and separation at monthly intervals. Consequently, the discontinuity in the hiring rate at the boom shows up as a dramatic, but finite increase in hiring.

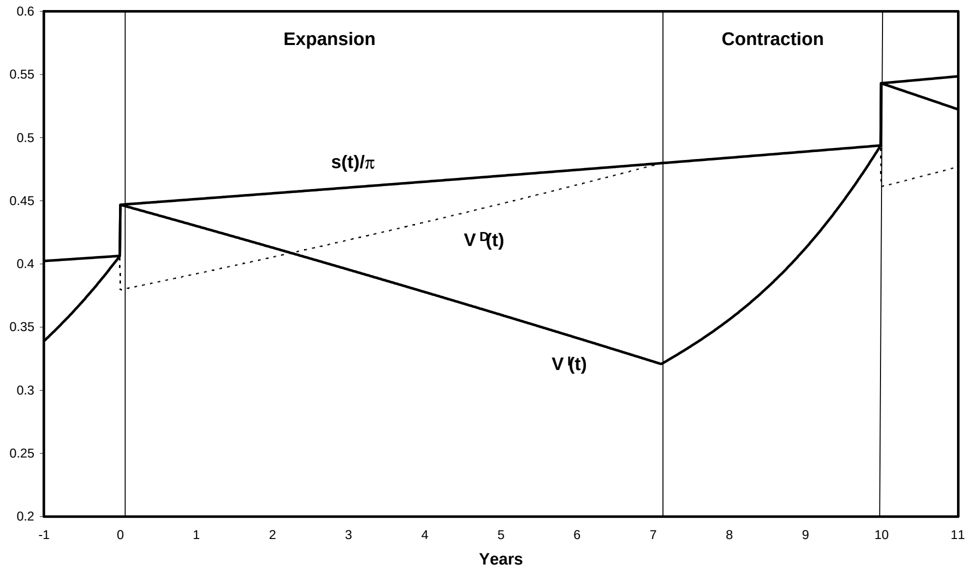

Figure 3: Evolution of value functions

of the hiring rate is somewhat greater than that of the separation rate, but both contribute substantially to variation in employment growth.

Davis, Haltiwanger and Schuh (1996) define "job destruction" as the "sum of declines in employment across plants where employment declines". As Figure 5 illustrates, the reduction in the hiring rate on the intensive margin at the beginning of the recession shows up as a sharp increase in job destruction.[21] Job destruction and job creation then rise together (though the latter is at a lower rate) as old firms are replaced by new ones. In our baseline example, the variance of job destruction is somewhat greater than that of job destruction, but both contribute substantially to variation in employment growth.

5.2 Low Wage Pro-Cyclicality

The production wage grows at the rate ϕ through the expansion and thus tracks output through this phase, it does, however, move very differently through the recession and subsequent boom. In the baseline case, illustrated in Figure 6, the production wage rises at the start of the recession and then follows a hump shape through the downturn.[22] Moreover, in this example, the production wage actually falls at the beginning of the expansion reflecting the sudden increased stability of employment relationships. For other parameter values (see below), this effect is offset by the

[21] During the recession, job destruction is measured as $n(t)\,\delta H(t) - \dot{n}(t)$, and job creation as $n(t)\,\delta H(t)$.

[22] The wage never declines in the baseline case, but for other parameter values it does.

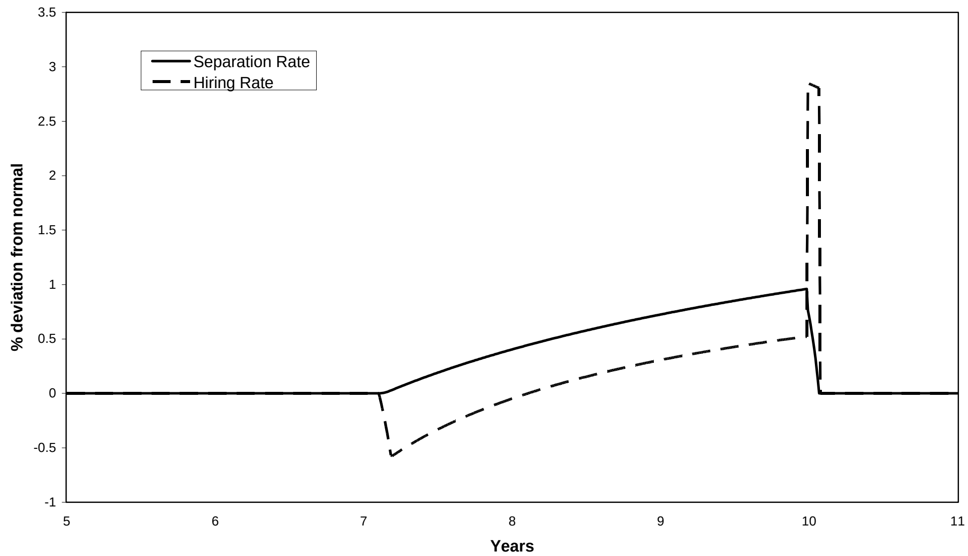

Figure 4: Hiring and separation rates

increased demand at the boom so that the production wage rises. Overall, as we discuss in the next section, the production wage may be counter-cyclical or procyclical, depending on the relative strength of the various forces discussed earlier.

Although the skilled wage is procyclical, the weighted average of the two, illustrated in Figure 6, inherits features of the production wage and consequently can also exhibit various degrees of cyclicality. In general, although it rises at the boom, the average wage tends to grow more rapidly than trend during the recession. Thus, although through the expansion wages and output are positively correlated, this may be offset by the negative correlation during the downturn. In the baseline example, these two almost exactly offset each other, so that the correlation of log output with log average wages (both de–trended) is approximately zero overall.

Note that average wages increase upon entering the recession because household employment growth, which was previously zero, becomes negative. Consequently, the marginal cost of providing effort is expected to be lower tomorrow than today which, on the margin, increases the willingness of workers to risk unemployment by shirking today. To compensate for this effect, firms must raise the wage in order to maintain incentives.

5.3 Comparative Statics

Table 2 shows the variation in output/wage correlation, growth, and cycle length for changes in each of the underlying parameters. The table lists the single parameter varied, and its value in

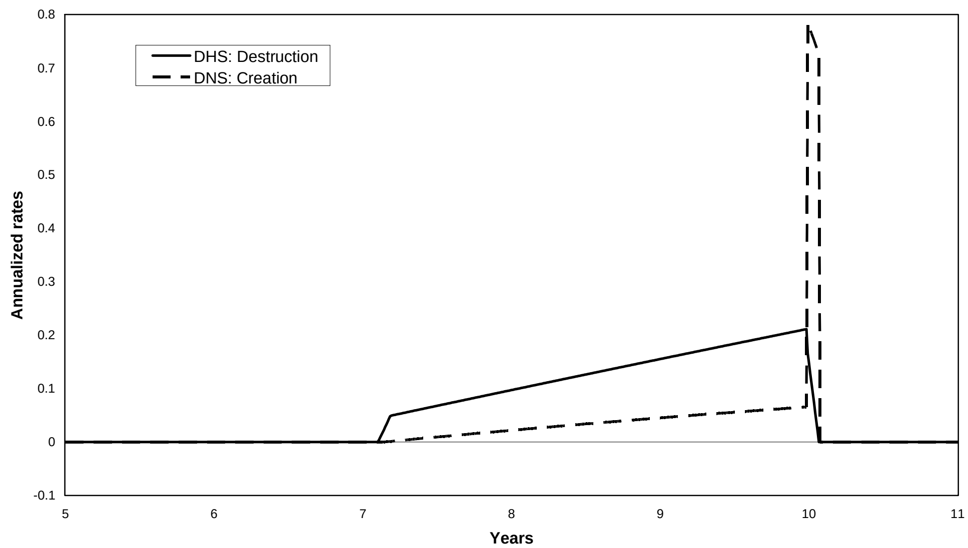

Figure 5: Job Creation and Destruction using definition of Davis et al. (1996)

the first column with the endogenous results in the columns to the immediate right. The intuition for most of the comparative static results in the table is relatively straightforward. Changes in parameters that reduce incentives to engage in entrepreneurial search: lowering δ or γ (which have direct effect on returns to search effort); lowering q and raising $\overline{\mu}$ (which raises the efficiency wage, thereby lowering profits); raising σ and ρ (which makes consumers less willing to delay consumption) all lower the growth rate, as one would expect in a model of endogenous growth. Most of these changes also leave the length of the economy's contraction relatively unchanged, but imply (sometimes large) changes in the length of expansions. This is because with weaker fundamental incentives to search for productivity improvements, longer expansionary phases, and therefore a longer reign of incumbency and profit, are required to provide sufficient incentives for entrepreneurship.

Table 2: Growth, Wage Cyclicality, and Cycle Lengths

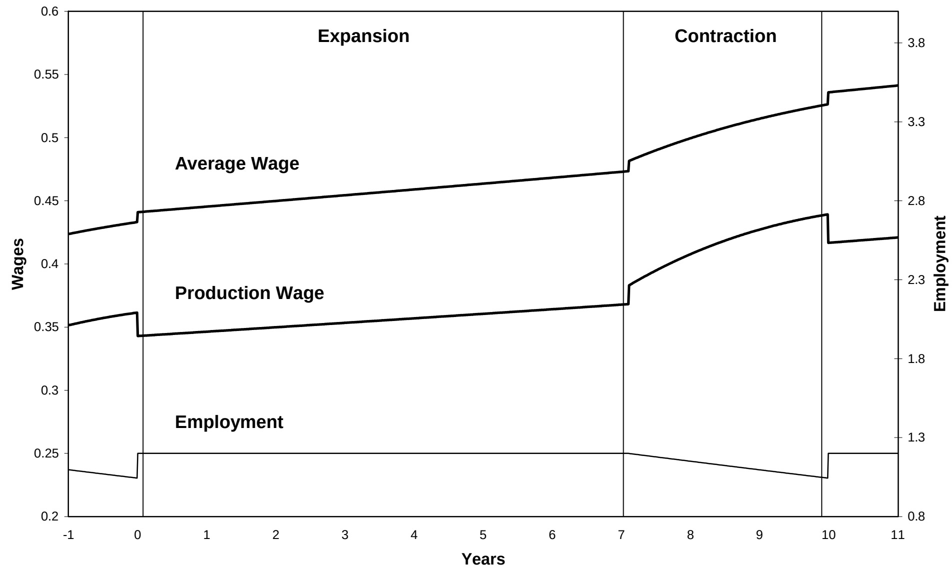

Figure 6: Wages in the baseline example

Parameter Values	$g(\%)$	Corr$(\hat{w}, \hat{y})$	Δ^X	Δ^E
Baseline Case	1.95	-0.01	7.10	2.88
Variation: $\delta = 1.400$	1.96	-0.32	6.25	2.58
$\delta = 1.100$	1.94	0.35	8.14	3.29
$\phi = 0.015$	2.57	-0.25	4.63	2.87
$\phi = 0.005$	1.33	0.18	10.28	2.90
$\sigma = 0.305$	1.52	0.27	15.80	2.96
$\sigma = 0.295$	2.37	-0.25	3.86	2.82
$\bar{\mu} = 0.110$	1.46	0.32	17.91	2.88
$\bar{\mu} = 0.090$	2.41	-0.28	3.81	2.88
$q = 0.410$	2.11	-0.03	5.65	2.88
$q = 0.390$	1.79	0.03	9.16	2.88
$\gamma = 0.410$	2.22	-0.15	4.76	2.82
$\gamma = 0.390$	1.70	0.14	11.02	2.97
$\rho = 0.035$	1.77	0.24	12.09	2.90
$\rho = 0.025$	2.12	-0.37	3.72	2.87
$\theta = 0.405$	1.53	0.24	15.15	2.91
$\theta = 0.395$	2.37	-0.19	4.00	2.87

5.4 The Productivity Slowdown and Wage Pro-Cyclicality

As mentioned in the introduction, the correlation between real wages and US industrial production has varied throughout the post-war period. Table 3, column 1, shows the correlation between the cyclical components of U.S. real manufacturing wages and industrial production over various periods (extracted using a HP filter). The last column reports annualized U.S. non-farm productivity growth (from the Bureau of Labor Statistics) over the sub-periods.

Table 3: U.S. Wage Cyclicality and Productivity Growth

Period	Correlation	Productivity Growth
1948:1–2005:4	0.27	1.8%
1948:1–1969:4	0.12	2.2%
1970:1–1993:4	0.43	1.1%
1994:1–2007:4	0.01	2.5%

From the second column in Table 2, compared with the baseline wage/output correlation of approximately zero, changes in any or all of the following variables lead to an increase in the correlation between wages and production: lowering δ, ϕ, q, γ or increasing $\bar{\mu}, \rho, \sigma$. Interestingly, for every one of these changes the growth rate, reported in column 1 of Table 2, also falls from the baseline case of 1.95%. Consequently, the pattern of co–movement between productivity and wage cyclicality generated by the model is consistent with that observed in post-war US data for **all** of the models' comparative statics. Each one of these changes in parameters lowers the relative returns to innovation and lowers entrepreneurial search on average. This lowers long-run productivity growth but also reduces firm obsolescence and implies less restructuring in recessions. This implies that the counter–cyclical force on wages due to turnover tends to be low when average productivity growth is also low, and high during phases of rapid productivity growth. This explains a pattern like that observed in Table 3 and is a remarkably robust implication of the model.

6 Concluding Remarks

A Schumpeterian process of creative destruction implies a cyclical pattern of firm turnover, employment flows, wage movements and aggregate demand, that is qualitatively consistent with several key features of US data. Specifically, it can generate counter–cyclical restructuring, pro–cyclical productivity, and wage fluctuations which, depending on parameters, may exhibit any form of cyclicality. These patterns are derived in a model where the underlying source of productivity growth is partially endogenous, as is the clustering of activities across disparate sectors.

Some features of our model's prediction are clearly at odds with the facts, and elements of the model are too abstract to provide direct linkage to actual real-world phenomena. However, we believe it is possible to extend the model in various ways to address some of these issues. In particular, the productivity boom and the associated jump in job creation are rather abrupt. As we show in Francois and Lloyd–Ellis (2008), adding capital can help to smooth out the boom to some extent. If capital and production labor are strong complements in the short run, job creation may then be relatively slow following the boom. Another unrealistic feature of the cyclical process that we generate is that every cycle is the same and all fluctuations are deterministic. Extending the model to allow for some stochastic elements would relax some of these strong predictions. One approach that we are exploring is to allow the exogenous component of productivity growth to be subject to temporary i.i.d. shocks. This can change the length and amplitude of each cycle without changing the basic story.

In this paper, the search for commercially viable ideas and productivity improvements is a counter–cyclical form of innovative activity. Indeed, there is some evidence that this kind of innovative activity is undertaken by managers during periods of slack demand. For example, Nickell, Nicolitsas and Patterson (2001) find that "managerial innovations" — changes in structure; organization leaner as result of change; significant changes resulting in more decentralized organization; significant changes in human resources management practices and industrial relations; the implementation of just in time technologies.— are concentrated during in downturns. In contrast, recent evidence suggests that R&D is procyclical, even for firms that are not obviously cash–constrained during downturns (see Barlevy, 2007, and Walde and Woitek, 2004). In Francois and Lloyd-Ellis (2009) we introduce endogenous R&D as a separate, knowledge-intensive activity that generates ideas whose commercial viability is unclear. The resulting stock of ideas can then be drawn upon by manager/entrepreneurs in their search for productivity improvements, and matched with specific markets. As in the current paper, entrepreneurial search is counter-cyclical, but R&D investment is *pro-cyclical*.

Appendix

Proof of Lemma 1: Consider a firm holding an obsolete technology at time t. For there to exist a technology better than the firm's, an innovator must have allocated h to innovation in the firm's sector. But only firms planning to implement an innovation will find it worthwhile to undertake innovative effort, consequently, for the new technology, there exists some planned optimal date at which the technology will be implemented. Denote this date by t^*. Using the large scale of production requires hiring workers who do not shirk. This is only possible if the wage is incentive compatible. Clearly, at t^* the firm using the obsolete technology is not able to compete in production, so the large scale of production cannot be used. However it is also the case that incentive compatibility cannot hold, and the large scale of production not be used, if employment terminates with probability one in the next instant. Consequently at time $t^* - dt$ it is also not possible to use the large scale technology. However, the same argument applies at time $t^* - 2dt$ since the worker then knows that at $t^* - dt$ employment will be terminated. The same reasoning applies for each instant until time t.

Proof of Lemma 2: From the production function we have $\ln y(t) = \phi t + \int_0^1 \ln \frac{y(t)}{p_i(t)} di$. Substituting for $p_i(t)$ using (23) re-arranges to

$$s(t) = e^{-\gamma} \exp\left(\phi t + \int_0^1 \ln a_i(T_{v-1}) di\right) = e^{\phi(t-T_{v-1})} s\left(T_{v-1}\right). \tag{62}$$

Proof of Lemma 3: Profit maximization implies that $\psi_i^E(t) = \psi_i^S(t) \geq \psi^U(t)$ and so $\psi_i^*(t) = \psi_i^E(t)$. It follows that the Bellman equation associated with a worker who is employed and not shirking in sector i is given by

$$\rho\psi_i^E(t) = (w_i(t)u_c(t) + u_n(t)) - \mu_i(t)\left[\psi_i^E(t) - \psi^U(t)\right] + \dot{\psi}_i^E(t) \tag{63}$$

Similarly the Bellman equations associated with workers who are shirking in sector i and unemployed, respectively, can be expressed as

$$\rho\psi_i^S(t) = w_i(t)u_c(t) - [\mu_i(t) + q]\left[\psi_i^S(t) - \psi^U(t)\right] + \dot{\psi}_i^S(t) \tag{64}$$

$$\rho\psi^U(t) = \lambda(t)\left[\psi_i^E(t) - \psi^U(t)\right] + \dot{\psi}^U(t) \tag{65}$$

Profit maximization subject to the incentive compatibility condition implies that $\psi_i^E(t) = \psi_i^S(t)$ and so subtracting (63) from (64) we get

$$\psi_i^E(t) - \psi^U(t) = -\frac{u_n(t)}{q} \tag{66}$$

It follows that $\psi_i^E(t) = \psi^E(t)\ \forall i$ and that

$$\dot{\psi}^E(t) - \dot{\psi}^U(t) = -\frac{\dot{u}_n(t)}{q} \tag{67}$$

Subtracting (65) from (63), substituting using (66) and re-writing yields (27) .■

Proof of Proposition 1: Substituting for $c(t)$ from (29), setting $\frac{\dot{u}_n}{u_n} = (1-\sigma)\phi$, and noting that when $dn = 0$ and $\mu^A = \overline{\mu}$, $\lambda = \frac{\overline{\mu}\theta}{1-\theta}$, equation (27) rearranges to (32).

Proof of Lemma 4: Note that in any preceding no-entrepreneurship phase, $r(t) = \rho + \sigma\phi$. Thus, since, in a cycling equilibrium, the date of the next implementation is fixed at T_v, the expected value of entrepreneurship, δV^D, also grows at the rate $\rho + \sigma\phi > 0$. Thus, if under $H(T_v^E) = 0$, $\delta V^D(T_v^E) > s(T_v^E)$, then the same inequality is also true the instant before, i.e. at $t \to T_v^E$, since $s(t)$ grows at the slower rate, $\phi < \rho + \sigma\phi$, within the cycle. But this violates the assertion that entrepreneurship commences at T_v^E. Thus necessarily, $\delta V^D(T_v^E) = s(T_v^E)$ at $H\left(T_v^E\right) = 0$.

Proof of Proposition 2: From (6) we can express the rate of job creation as

$$\lambda(t) = \frac{\mu^A(t)n(t) + \dot{n}(t)}{L - n(t)}. \tag{68}$$

Using the fact that $\mu^A(t) = \bar{\mu} + \delta(1 - n(t)/\theta L)$ and substituting into (42) yields

$$w^A(t) = \frac{\sigma c(t)}{(1-\sigma)\left(L - n(t)\right)q}\left[\rho + q + \frac{\delta}{\theta} - \frac{\frac{\delta(1-\theta)}{\theta}L - \bar{\mu}L}{L - n(t)} + \frac{\dot{n}(t)}{L - n(t)} - \frac{\dot{u}_n(t)}{u_n(t)}\right] \tag{69}$$

Differentiating (35) yields

$$\dot{n}(t) = \frac{-\left(1-\theta\right)\frac{\rho-(1-\sigma)\phi}{\sigma}\theta L e^{-\frac{\rho-(1-\sigma)\phi}{\sigma}(t-T_v^E)}}{\left[1-\theta+\theta e^{-\frac{\rho-(1-\sigma)\phi}{\sigma}(t-T_v^E)}\right]^2}. \tag{70}$$

Using (83) and (35) we have that

$$L - n(t) = \frac{\left(1-\theta\right)L}{1-\theta+\theta e^{-\frac{\rho-(1-\sigma)\phi}{\sigma}(t-T_v^E)}} \tag{71}$$

Differentiating (12) w.r.t. time and using (34) to substitute yields

$$\frac{\dot{u}_n(t)}{u_n(t)} = \rho - \frac{\rho - (1-\sigma)\phi}{\sigma} \tag{72}$$

Noting that $c(t) = Y_0(T_{v-1})e^{\phi(t-T_{v-1})}n(t)/\theta L$, substituting into (69) using (70), (71) and (72) and re–arranging yields (42).

Proof of Proposition 3: Long–run productivity growth is given by

$$\Gamma_v = \gamma(1 - P(T_v)) = \delta\gamma \int_{T_v^E}^{T_v} H(\tau) d\tau \tag{73}$$

Substituting using (36) and integrating yields (45).

Proof of Proposition 4: Just prior to the boom, when the probability of displacement is negligible, the value of implementing immediately must equal that of delaying until the boom:

$$\delta V^I(T_v) = \delta V^D(T_v) = s(T_v). \tag{74}$$

During the boom $V_0^I(T_v) > V_0^D(T_v)$. Thus the return to innovation at the boom is the value of immediate incumbency. It follows that free entry into entrepreneurship at the boom requires that

$$\delta V_0^I(T_v) \leq s_0(T_v). \tag{75}$$

The opportunity cost to financing entrepreneurship is the rate of return on shares in incumbent firms in sectors where no innovation has occurred. Just prior to the boom, this is given by the capital gains in sectors where no innovations have occurred. Combined with (74) and (75) it follows that asset market clearing at the boom requires

$$R_0(T_v) - R(T_v) = \log\left(\frac{V_0^I(T_v)}{V^I(T_v)}\right) \leq \log\left(\frac{s_0(T_v)}{s(T_v)}\right) = \Gamma_v. \tag{76}$$

Provided that $R_0(T_v) - R(t) > 0$, households will not store final output from within a cycle to the beginning of the next. However, the return on stored intermediate output in sectors with no innovations is strictly positive because its price increases at the boom. If innovative activities are to be financed at time t, households cannot be strictly better off buying claims to stored intermediate goods. Consider a sector where no innovation has occurred just prior to the boom. Since the cost of production is the same whether the good is stored or not, the rate of return on claims to stored intermediates in sector i is $\log p_{i,v+1}/p_{i,v} = \Gamma_v$. It follows that the long run rate of return on claims to firm profits an instant prior to the boom must satisfy

$$R_0(T_v) - R(T_v) \geq \log p_{i,v+1}/p_{i,v} = \Gamma_v. \tag{77}$$

Combining (76) and (77) yields the equality in the statement of proposition.

Proof of Proposition 5:

Message space: At $\varepsilon \to 0$ cost, an innovator targeting sector i, which has not yet had a signal of search success, can either send a public signal of success $Z_i(t) = 1$, or send no signal in which case $Z_i(t) = 0$. Since search occurs in continuous time we ignore the zero probability event of more than one innovation arriving simultaneously.

Description of equilibrium behavior: If, and only if, an entrepreneur succeeds he sends a signal of success immediately. If, and only if, an incumbent sees $Z_i(t) = 1$, he shuts down production. If, and only if, another entrepreneur targeting sector i sees $Z_i(t) = 1$ he subsequently targets another sector j in which $Z_j(t) = 0$.

Optimality of signal sender's equilibrium behavior: During the phase in which search occurs, $V^I(t) < V^D(t)$. Consequently, an entrepreneur would prefer delaying implementation of innovation to time T_0. Suppose that at time t an incumbent sees $Z_i(t) = 1$. Then either an entrepreneur has succeeded and sent a signal, or has not succeeded and is falsely sending a signal of success. If an entrepreneur has been successful, expected returns when sending the signal are: $V^D(t) - \varepsilon$. This exceeds the return to immediate implementation, $V^I(t)$.

If the entrepreneur has not been successful, but is sending a signal falsely, then expected returns to innovation are $\delta V^D(t) - \varepsilon$. However, if the false sender of the signal were to follow the posited equilibrium behavior instead, and delay sending a signal until if and when a success arrives, then expected returns to search would be $\delta\left(V^D(t) - \varepsilon\right) > \delta V^D(t) - \varepsilon$. Consequently, an unsuccessful entrepreneur would never falsely send a signal of success, and a successful entrepreneur strictly prefers sending a signal of success.

Optimality of recipient's equilibrium behavior: Given equilibrium described by strategies $\widehat{\sigma}$ for workers and incumbents in production, when $Z_i(t) = 1$, the incumbent firm knows that, in all subsequent periods, any positive wage offer to any worker j will lead to shirking. Consequently the firm strictly prefers to shut down production.

Optimality of unsuccessful entrepreneur's behavior: Given that $Z_i(t) = 1$, a non-innovator conjectures that an innovation success has arrived in sector i. Consequently, if the entrepreneur were to subsequently succeed in sector i, $V_i^D(t)$ would equal 0, since the entrepreneur would Bertrand compete with an incumbent holding an identical technology. Consequently, provided there exists another sector j, with $V_j^D(t) > 0$, and with $Z_j(t) = 0$, the entrepreneur will target sector j subsequently.

Proof of Proposition 6: Although, during the downturn, the realized profits of firm differ across sectors, the fact that profits are linear in the production wage implies that expected profits at time T_v^E are equivalent to the profits of a firm that pays the average wage for $t \in \left[T_v^E, T_v\right]$ (see Appendix B for details) This implies that the value of a firm at time T_{v-1} can be expressed as

$$V_0^I(T_{v-1}) = \frac{1}{\left(1 - P(T_v)e^{\Gamma_v - [R_0(T_v) - R(T_{v-1})]}\right)} \int_{T_{v-1}}^{T_v} e^{-\int_{T_{v-1}}^{\tau} r(s)ds} \left(1 - \frac{e^{-\gamma}}{\theta L} - e^{-\gamma}\frac{w^A(\tau)}{s(\tau)}\right) Y(\tau)d\tau$$

Substituting in for the endogenous variables, integrating and dividing through by $Y_0(T_{v-1})$ yields

$$\begin{aligned}
&\left(1 - P(T_v)e^{\Gamma - [R_0(T_v) - R_0(T_{v-1})]}\right) \frac{V_0^I(T_{v-1})}{Y_0(T_{v-1})} \\
= \quad &\left(1 - \frac{e^{-\gamma}}{\theta L}\right) \left[\frac{1 - e^{-b(T_v^E - T_{v-1})}}{b} - e^{-\rho(T_v^E - T_{v-1})} \left(\frac{\sigma}{b\theta} \ln\left(1 - \theta\left(1 - e^{-\frac{b}{\sigma}\Delta^E}\right)\right)\right)\right] \\
&- \left(\frac{1 - e^{-\rho(T_v^E - T_{v-1})}}{\rho}\right) A\theta \qquad (78) \\
&-e^{-b(T_v^E - T_{v-1})} \frac{\sigma}{b} \left[\begin{array}{c} \left(B + \frac{1-\theta}{\theta}C\right)\left(1 - e^{-\frac{b}{\sigma}\Delta^E} + \frac{1-\theta}{\theta}\ln\left(1 - \theta\left(1 - e^{-\frac{b}{\sigma}\Delta^E}\right)\right)\right) \\ -\frac{1}{2}C\left(1 - e^{-\frac{2b}{\sigma}\Delta^E}\right) - D\left(\frac{1}{\theta}\ln\left(1 - \theta\left(1 - e^{-\frac{\rho}{\sigma}\Delta^E}\right)\right) + \frac{(1-\theta)\left(1 - e^{-\frac{b}{\sigma}\Delta^E}\right)}{1 - \theta\left(1 - e^{-\frac{b}{\sigma}\Delta^E}\right)}\right) \end{array}\right]
\end{aligned}$$

where $b = \rho - (1 - \sigma)\phi$. Noting that $\delta V_0^I(T_{v-1}) = w_0(T_{v-1}) = e^{-\gamma}Y_0(T_{v-1})/\theta L$ and substituting using (45) and (50) we can derive

$$\begin{aligned}
&\left(e^{b(\Delta - \Delta^E)} - 1\right)\left[\frac{e^{-\gamma}}{\delta\theta L} - \frac{1}{b}\left(1 - \frac{e^{-\gamma}}{\theta L}\right) + \frac{A\theta}{b}\right] \qquad (79) \\
= \quad &\left[(1 - \frac{e^{-\gamma}}{\theta L}) - \frac{b}{\delta\gamma(1-\sigma)} + \left[\left(B + \frac{1-\theta}{\theta}C\right)(1-\theta) - D\right]\left(1 - \frac{b}{\delta\gamma(1-\sigma)}\right)\right]\Delta^E \\
&-\frac{\sigma}{b}\left[\left(B + \frac{1-\theta}{\theta}C\right)\left(1 - e^{-\frac{b}{\sigma}\Delta^E}\right) - \frac{1}{2}C\left(1 - e^{-\frac{2b}{\sigma}\Delta^E}\right) - D\frac{(1-\theta)\left(1 - e^{-\frac{b}{\sigma}\Delta^E}\right)}{1 - \theta\left(1 - e^{-\frac{b}{\sigma}\Delta^E}\right)}\right]
\end{aligned}$$

Dividing through by $\left[\frac{e^{-\gamma}}{\delta\theta L} - \frac{1}{b}\left(1 - \frac{e^{-\gamma}}{\theta L}\right) + \frac{A\theta}{b}\right]$ and solving for Δ yields (55), where

$$\alpha = \frac{\frac{b}{\delta\gamma(1-\sigma)} - (1 - \frac{e^{-\gamma}}{\theta L}) - \left[\left(B + \frac{1-\theta}{\theta}C\right)(1-\theta) - D\right]\left(1 - \frac{b}{\delta\gamma(1-\sigma)}\right)}{\frac{1}{b}\left(1 - \frac{e^{-\gamma}}{\theta L}\right) - \frac{e^{-\gamma}}{\delta\theta L} - \frac{A\theta}{b}}, \qquad (80)$$

$$\begin{aligned}
\zeta_1 \quad &= \quad \frac{\frac{\sigma}{b}\left(B + \frac{1-\theta}{\theta}C\right)}{\frac{1}{b}\left(1 - \frac{e^{-\gamma}}{\theta L}\right) - \frac{e^{-\gamma}}{\delta\theta L} - \frac{A\theta}{b}},\ \zeta_2 = \frac{\frac{\sigma}{b}\frac{1}{2}C}{\frac{1}{b}\left(1 - \frac{e^{-\gamma}}{\theta L}\right) - \frac{e^{-\gamma}}{\delta\theta L} - \frac{A\theta}{b}} \qquad (81) \\
\text{and } \zeta_3 \quad &= \quad \frac{\frac{\sigma}{(\rho - (1-\sigma)\phi)}D(1-\theta)}{\frac{1}{b}\left(1 - \frac{e^{-\gamma}}{\theta L}\right) - \frac{e^{-\gamma}}{\delta\theta L} - \frac{A\theta}{b}}.
\end{aligned}$$

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Appendix B: Not–for–Publication Appendix

Formal Statement of Incumbent Firm / Worker game:

When considering the worker/firm game we proceed under the assumption that any signal of success in an incumbent firm's sector is a credible signal of a successful innovation on the part of the signal sender. This is an equilibrium condition of the model, which we take as given for now, but the formal proof of this is shown in the proof of Proposition 5.

Consider an incumbent firm in sector i and a worker j. The labor market is anonymous in the sense that, the history of a worker's effort choices prior to being employed at the current firm is not known to the firm. The incumbent firm only knows the history of this worker's effort choices while employed with the current incumbent firm. The history of worker j, denoted $H_j(t)$, summarizes the effort decision of worker j in each instant from the time he was employed in the firm t_j to the present, t. That is, it summarizes the value of $\varepsilon_j(t)\ \forall t \in [t_j, t]$. Equilibrium strategies will vary conditionally on this history in one of two ways. Case 1: The worker did not work for the firm in any previous period, or the worker did work for the firm and set $\varepsilon_j(t) = 1$ $\forall t \in [t_j, t]$. We denote this case as history $H_j(t) = 1$. Case 2: $\exists$ at least one $t \in [t_j, t]$ such that $\varepsilon_j(t) = 0$. We denote this by $H_j(t) = 0$.

Public Signals: $Z_i(t) = 1$, if, at time t, there has not been a signal of an innovation success in sector i subsequent to the success signal of the current incumbent firm i. $Z_i(t) = 0$ otherwise.

Actions: Incumbent Firm: Conditional on no exogenous seperation between incumbent firm, i and worker j,at time t, firm decides whether to rehire worker j, $R_{ij}(t) = 1$, or dismiss, $R_{ij}(t) = 0$. If rehired, firm decides on wage $w_{ij}(t)$.

Worker: Conditional on no exogenous seperation between incumbent firm, i and worker j,at time t, worker observes wage and decides whether to remain with the firm $R_{ji}(t) = 1$, or leave $R_{ji}(t) = 0$, and, whether to set $\varepsilon_j(t) = 1$ or 0.

Strategies: Incumbent firm: $\sigma_{ij}^{I}(H_j(t), Z_i(t))$ is a mapping from the worker's history H_j and the public signals Z_i received in that sector, to the firm's actions $(R_{ij}(t), w_{ij}(t))$.

Worker: $\sigma_{ji}^{W}(H_j(t), Z_i(t), w_{ij}(t))$ is a mapping from the worker's history H_j and the public signals Z_i received in that sector, to the worker's action set $(R_{ji}(t), \varepsilon_j(t))$.

Equilibrium: Firm Behavior: At time t, given that worker j has not shirked in any previous period of employment with the firm and given that there has not been a signal of a new innovation success in sector i, the firm offers the worker a binding incentive compatible wage, $w_i^*(t)$ computed in equation (30). For any other history, or if a success has been signalled in the sector, the firm does not offer worker j employment subsequently.

Worker Behavior: At time t, given that worker j has not shirked in any previous period of employment with the firm and given that there has not been a signal of a new innovation success in sector i, and given that the offered wage is at least equal to $w_i^*(t)$, the worker accepts a position in the firm and does not shirk. For any other history, or if an innovation success has been signalled, the worker accepts the employment offer and shirks.

Specification of equilibrium strategies: Firms, i, follow $\widehat{\sigma}_{ij}^I(H_j(t), Z_i(t))$, in dealing with any worker j and workers j follow $\widehat{\sigma}_{ji}^W(H_j(t), Z_i(t), w)$ dealing with any firm i.

Denote $w_i(t)$ defined in equation (30) as $w_i^*(t)$.

Incumbent firm: $\widehat{\sigma}_{ij}^I(H_j(t), Z_i(t))$ such that,

$$\begin{aligned}
\widehat{\sigma}_{ij}^I(1,1) &= (1, w_i^*(t)), \\
\widehat{\sigma}_{ij}^I(H_j(t), Z_i(t)) &= (0,0), \text{ for } H_j(t).Z_i(t) \neq 1.
\end{aligned}$$

Worker: $\widehat{\sigma}_{ji}^W(H_j(t), Z_i(t), w)$ such that

$$\begin{aligned}
\widehat{\sigma}_{ji}^W(1,1,w) &= (1,1) \ \ if \ w \geq w_i^*(t), \\
\widehat{\sigma}_{ji}^W(1,1,w) &= (1,0) \ \ if \ w < w_i^*(t), \\
\widehat{\sigma}_{ji}^W(H_j(t), Z_i(t), w) &= (1,0) \, \forall w \ for \ H_j(t).Z_i(t) \neq 1.
\end{aligned}$$

Proof that these strategies constitute a sub-game perfect equilibrium.

From the worker's perspective given that $(H_j(t), Z_i(t)) = (1,1)$, $w_i^*(t)$ is the incentive compatible wage, since the incumbent firm's strategy,$\widehat{\sigma}_{ij}^I$, specifies re-hiring if workers do not shirk. Consequently workers will not deviate to shirking along the equilibrium path of play.

Consider a deviation by the firm. Specifically, suppose that $(H_j(t), Z_i(t)) = (1,1)$ but the firm sets $w_i(t) \neq w_i^*(t)$. Clearly, given σ_{ji}^W, any wage below $w_i^*(t)$ will lead the worker to shirk, and wages strictly above $w_i^*(t)$ reduce profit, so, conditional up re-hiring the worker, $w_i^*(t)$ is optimal for the firm. Consider a once off deviation given history $(H_j(t), Z_i(t)) = (1,1)$ in which the firm does not re-hire the worker, and hires another worker instead. The firm then draws a worker randomly from the employment pool. Such a worker also has a history $H_j(t) = 1$, so that assuming that the firm continues with the equilibrium strategies thereafter, (this will be optimal given these are a best response) the firm sets $w = w_i^*(t)$ for this worker. Then, since the newly hired worker has $H_j(t) = 1$, and is also following $\widehat{\sigma}_{ji}^W$, this worker will not shirk if and only if $w \geq w_i^*(t)$ so that profits cannot increase under this deviation. Clearly if $(H_j(t), Z_i(t)) = (1,1)$ and the firm neither rehires nor replaces the worker, profit falls, since output falls and this is also not a worthwhile deviation. Now suppose that $H_j(t).Z_i(t) \neq 1$. According to $\widehat{\sigma}^W_{ji}$, worker j will shirk for any w. This is also a best response of workers in this sub-game because, under $\widehat{\sigma}^I_{ij}$, the worker correctly conjectures that he will not be hired subsequent to the present instant. Consequently, his best response in any subgame where he is offered a positive wage and where $H_j(t).Z_i(t) \neq 1$ is to shirk. Consequently, from the perspective of the firm, a deviation to rehiring and offering any positive wage yields strictly lower profit. Since w^* has already been calculated to satisfy worker incentive compatibility, $\widehat{\sigma}^W$ is also a best response for the worker.

Proof of Lemma 5: The Euler equation can be expressed as

$$\left(\frac{\dot{n}(t)}{n(t)} + \phi\right) + \frac{\dot{n}(t)}{L - n(t)} = -\left(\frac{\rho - \phi}{\sigma}\right). \tag{82}$$

Re–arranging and integrating yields

$$\ln \frac{n(t)}{\theta L} - \ln\left(\frac{L - n(t)}{L - \theta L}\right) = -\frac{\rho - (1-\sigma)\phi}{\sigma}(t - T^E_v) \tag{83}$$

Solving for $n(t)$ yields (35).

Comment on Proof of Proposition 4: In sectors with unimplemented innovations, entrepreneurs who hold innovations and are currently producing with the previous technology have the option of implementing the new technology before the boom, storing and then selling at the boom. Any such path of implementation will, however, affect the incentive compatible wage stream offered to production workers. Intuitively, producing and storing today for sale tomorrow implies a higher rate of layoffs tomorrow and hence a higher efficiency wage today. This upward effect on incentive compatible wages is what rules out such storage as a profitable option. Since the stream of revenue is unaffected by such storage, in order for firms' expected discounted profits to rise it must be that the discounted wage bill for production workers falls. To see this clearly, consider the expected value of the firm with an innovation at the beginning of the cycle, $V^I(T_v)$. Since $Y(\cdot), s, P(\cdot)$ and all discount rates are taken as given by the firm, V^I can only rise if the value of being employed at the firm $\psi^E(T_v)$ falls. But at all instants, the incentive compatible wage, $w^L(\tau)$ is given by the solution to $\psi^E(t) - \psi^U(t) = -\frac{u_n(t)}{q}$. Thus any such storage which raises profits for the firm will necessarily violate incentive compatibility for unskilled workers and will lead to shirking. The same argument rules out altering the production stream to benefit from discrete jumps in the wage anywhere along the cycle.

Further details of Proposition 6: The value of an incumbent firm over a small interval dt during the cycle is

$$V^I_i(t) = \pi_i(w_H(t), t)dt + e^{-r(t)dt}\left[e^{-\delta h_i dt}V^I(t+dt) + (1 - e^{-\delta h_i dt})V^*(t+dt)\right]$$

Subtracting $e^{-(r(t)+\delta h_i)dt}$ from both sides, dividing by dt and letting $dt \to 0$ yields

$$(r(t) + \delta h_i(t))V_i^I(t) = \pi_i(w_H(t), t) + \delta h_i(t)V^*(t) + \dot{V}_i^I(t) \tag{84}$$

Given some initial t, and T_v, the solution to this first order differential equation is

$$V_i^I(t) = \int_t^{T_v} e^{-\int_t^s r(\tau)d\tau} \frac{P(s)}{P(t)} \left[\pi_i(w_H(s), s) + \delta h_i(s)V^*(s)\right] ds + e^{-[R_0(T_v)-R(t)]} \frac{P(T_v)}{P(t)} V_0^I(T_v) \tag{85}$$

For $t \geq T_v^E$ $r(\tau) = \phi$ and $h(s)P(s) = H(s)$. Hence

$$V_i^I(t) = \frac{1}{P(t)} \int_t^{T_v} e^{-\phi(s-t)} \left[P(s)\pi_i(w_H(s), s) + \delta H(s)V^*(s)\right] ds + e^{-[R_0(T_v)-R(t)]} \frac{P(T_v)}{P(t)} V_0^I(T_v). \tag{86}$$

Note that by partial integration

$$\int_t^{T_v} e^{-\phi(s-t)} \delta H(s)V^*(s)ds = \int_t^{T_v} e^{-\phi(s-t)} V^*(s)d(1 - P(s)) \tag{87}$$

$$= \left[e^{-\phi(s-t)}V^*(s)(1 - P(s))\right]_t^{T_v} - \int_t^{T_v} (1 - P(s))de^{-\phi(s-t)}V^*(s) \tag{88}$$

But $V^*(T_v) = 0$ and $de^{-\phi(s-t)}V^*(s) = -e^{-\phi(s-t)}\pi_i(w_L(s), s)ds$, so that

$$\int_t^{T_v} e^{-\phi(s-t)} \delta H(s)V^*(s)ds = \int_t^{T_v} (1 - P(s))e^{-\phi(s-t)}\pi(w_L(s), s)ds - V^*(t)(1 - P(t)) \tag{89}$$

Since profits are a linear function of the production wage we get $\forall\ t \geq T_v^E$:

$$V_i^I(t) = \frac{1}{P(t)} \int_t^{T_v} e^{-\phi(s-t)}\pi(w^A(s), s)ds + e^{-[R_0(T_v)-R(t)]} \frac{P(T_v)}{P(t)} V_0^I(T_v) - V^*(t) \left(\frac{1 - P(t)}{P(t)}\right) \tag{90}$$

Now if $t = T_v^E$, $P(T_v^E) = 1$ and this can be expressed as

$$V_i^I(T_v^E) = \int_{T_v^E}^{T_v} e^{-\phi(s-T_v^E)}\pi(w^A(s), s)ds + e^{-\left[R_0(T_v)-R(T_v^E)\right]} P(T_v)V_0^I(T_v). \tag{91}$$

This confirms that the expected profits at time T_v^E are equivalent to the profits of a fim that pays the average wage for $t \in \left[T_v^E, T_v\right]$. This implies that the value of a firm at time T_{v-1} can be expressed as

$$
\begin{aligned}
& V_0^I(T_{v-1}) && (92)\\
= & \int_{T_{v-1}}^{T_v} e^{-\int_{T_{v-1}}^{\tau} r(s)ds}\left(1-\frac{e^{-\gamma}}{\theta L}\right)Y(\tau)d\tau \\
& -\int_{T_{v-1}}^{T_v} e^{-\int_{T_{v-1}}^{\tau} r(s)ds} e^{-\gamma}\frac{w^A(\tau)}{s(\tau)}Y(\tau)d\tau + P(T_v)e^{-[R_0(T_v)-R(T_{v-1})]}V_0^I(T_v) && (93)\\
= & \left(1-e^{-\gamma}\right)Y_0(T_{v-1})\left(\int_{T_{v-1}}^{T_v^E} e^{-(\rho-(1-\sigma)\phi)(\tau-T_{v-1})}d\tau + e^{-(\rho-(1-\sigma)\phi)(T_v^E-T_{v-1})}\int_{T_v^E}^{T_v}\frac{n(\tau)}{\theta L}d\tau\right) && (94)\\
& -\left(\int_{T_{v-1}}^{T_v^E} e^{-(\rho+\sigma\phi)(\tau-T_{v-1})}w^A(\tau)\theta L d\tau + e^{-(\rho+\sigma\phi)(T_v^E-T_{v-1})}\int_{T_v^E}^{T_v} e^{-\phi(t-T_v^E)}w^A(\tau)n(\tau)d\tau\right)\\
& +P(T_v)e^{-[R_0(T_v)-R(T_{v-1})]}V_0^I(T_v)\\
= & \left(1-\frac{e^{-\gamma}}{\theta L}\right)Y_0(T_{v-1})\left(\int_{T_{v-1}}^{T_v^E} e^{-(\rho-(1-\sigma)\phi)(\tau-T_{v-1})}d\tau + e^{-(\rho-(1-\sigma)\phi)(T_v^E-T_{v-1})}\int_{T_v^E}^{T_v}\frac{n(\tau)}{\theta L}d\tau\right)\\
& -\left(\int_{T_{v-1}}^{T_v^E} e^{-(\rho-(1-\sigma)\phi)(\tau-T_{v-1})}A\theta Y_0(T_{v-1})d\tau + e^{-(\rho+\sigma\phi)(T_v^E-T_{v-1})}\int_{T_v^E}^{T_v} e^{-\phi(t-T_v^E)}w^A(\tau)n(\tau)d\tau\right)\\
& +P(T_v)e^{-[R_0(T_v)-R(T_{v-1})]}V_0^I(T_v)
\end{aligned}
$$

Using (54) and re–arranging we get

$$
\begin{aligned}
& \left(1-P(T_v)e^{\Gamma-[R_0(T_v)-R(T_{v-1})]}\right)V_0^I(T_{v-1})\\
= & \left(1-\frac{e^{-\gamma}}{\theta L}\right)Y_0(T_{v-1})\left(\frac{1-e^{-b(T_v^E-T_{v-1})}}{b} + e^{-\rho(T_v^E-T_{v-1})}\int_{T_v^E}^{T_v}\frac{e^{-\frac{b}{\sigma}(t-T_v^E)}}{1-\theta+\theta e^{-\frac{b}{\sigma}(t-T_v^E)}}dt\right)\\
& -\left(\frac{1-e^{-b(T_v^E-T_{v-1})}}{b}\right)A\theta Y_0(T_{v-1}) && (95)\\
& -e^{-b(T_v^E-T_{v-1})}Y_0(T_{v-1})\theta\int_{T_v^E}^{T_v}\left[\frac{Be^{-2\frac{b}{\sigma}(t-T_v^E)}-Ce^{-3\frac{b}{\sigma}(t-T_v^E)}}{1-\theta+\theta e^{-\frac{b}{\sigma}(t-T_v^E)}} + \frac{De^{-2\frac{b}{\sigma}(t-T_v^E)}}{\left[1-\theta+\theta e^{-\frac{b}{\sigma}(t-T_v^E)}\right]^2}\right]dt,
\end{aligned}
$$

where it is analytically convenient to let

$$b=\rho-(1-\sigma)\phi.$$

Integrating and dividing through by $Y_0(T_{v-1})$ yields

$$
\begin{aligned}
& \left(1 - P(T_v) e^{\Gamma - [R_0(T_v) - R_0(T_{v-1})]}\right) \frac{V_0^I(T_{v-1})}{Y_0(T_{v-1})} \\
= \; & \left(1 - \frac{e^{-\gamma}}{\theta L}\right) \left[\frac{1 - e^{-b(T_v^E - T_{v-1})}}{b} - e^{-\rho(T_v^E - T_{v-1})} \left(\frac{\sigma}{b\theta} \ln\left(1 - \theta\left(1 - e^{-\frac{b}{\sigma}\Delta^E}\right)\right)\right)\right] \\
& - \left(\frac{1 - e^{-\rho(T_v^E - T_{v-1})}}{\rho}\right) A\theta \qquad (96) \\
& - e^{-b(T_v^E - T_{v-1})} \frac{\sigma}{b} \left[\begin{array}{c} \left(B + \frac{1-\theta}{\theta} C\right) \left(1 - e^{-\frac{b}{\sigma}\Delta^E} + \frac{1-\theta}{\theta} \ln\left(1 - \theta\left(1 - e^{-\frac{b}{\sigma}\Delta^E}\right)\right)\right) \\ -\frac{1}{2} C \left(1 - e^{-\frac{2b}{\sigma}\Delta^E}\right) - D \left(\frac{1}{\theta} \ln\left(1 - \theta\left(1 - e^{-\frac{\rho}{\sigma}\Delta^E}\right)\right) + \frac{(1-\theta)\left(1 - e^{-\frac{b}{\sigma}\Delta^E}\right)}{1 - \theta\left(1 - e^{-\frac{b}{\sigma}\Delta^E}\right)}\right) \end{array} \right]
\end{aligned}
$$

Collecting terms

$$
\begin{aligned}
& \left(1 - P(T_v) e^{\Gamma - [R_0(T_v) - R_0(T_{v-1})]}\right) \frac{V_0^I(T_{v-1})}{Y_0(T_{v-1})} \\
= \; & \left(1 - \frac{e^{-\gamma}}{\theta L}\right) \frac{1 - e^{-b(T_v^E - T_{v-1})}}{b} \\
& - e^{-(\rho - (1-\sigma)\phi)(T_v^E - T_{v-1})} \left(\frac{\sigma}{b\theta} \ln\left(1 - \theta\left(1 - e^{-\frac{b}{\sigma}\Delta^E}\right)\right)\right) \left[\left(1 - \frac{e^{-\gamma}}{\theta L}\right) + \left(B + \frac{1-\theta}{\theta} C\right)(1-\theta) - D\right] \\
& - \left(\frac{1 - e^{-b(T_v^E - T_{v-1})}}{b}\right) A\theta \qquad (97) \\
& - e^{-\rho(T_v^E - T_{v-1})} \frac{\sigma}{b} \left[\left(B + \frac{1-\theta}{\theta} C\right)\left(1 - e^{-\frac{b}{\sigma}\Delta^E}\right) - \frac{1}{2} C \left(1 - e^{-\frac{2b}{\sigma}\Delta^E}\right) - D \frac{(1-\theta)\left(1 - e^{-\frac{b}{\sigma}\Delta^E}\right)}{1 - \theta\left(1 - e^{-\frac{b}{\sigma}\Delta^E}\right)}\right]
\end{aligned}
$$

Substituting using (45) and (50) we get

$$
\begin{aligned}
& \left(1 - \left(1 - \frac{b\Delta^E}{\gamma(1-\sigma)}\right) e^{-b(\Delta - \Delta^E)}\right) \frac{e^{-\gamma}}{\delta\theta L} \\
= \; & \left(1 - \frac{e^{-\gamma}}{\theta L}\right) \frac{1 - e^{-b(\Delta - \Delta^E)}}{b} \\
& + e^{-\rho(\Delta - \Delta^E)} \left[\left(1 - \frac{e^{-\gamma}}{\theta L}\right) + \left(B + \frac{1-\theta}{\theta} C\right)(1-\theta) - D\right] \left(1 - \frac{\rho}{\delta\gamma(1-\sigma)}\right) \Delta^E \\
& - \left(\frac{1 - e^{-b(\Delta - \Delta^E)}}{b}\right) A \qquad (98) \\
& - e^{-b(\Delta - \Delta^E)} \frac{\sigma}{b} \left[\left(B + \frac{1-\theta}{\theta} C\right)\left(1 - e^{-\frac{b}{\sigma}\Delta^E}\right) - \frac{1}{2} C \left(1 - e^{-\frac{2b}{\sigma}\Delta^E}\right) - D \frac{(1-\theta)\left(1 - e^{-\frac{b}{\sigma}\Delta^E}\right)}{1 - \theta\left(1 - e^{-\frac{b}{\sigma}\Delta^E}\right)}\right]
\end{aligned}
$$

$$
\begin{aligned}
&\left(e^{b(\Delta-\Delta^E)} - 1 + \frac{b\Delta^E}{\gamma(1-\sigma)}\right)\frac{e^{-\gamma}}{\delta\theta L} \\
= \quad &\left(1-\frac{e^{-\gamma}}{\theta L}\right)\frac{e^{b(\Delta-\Delta^E)}-1}{b} + \left[\left(1-\frac{e^{-\gamma}}{\theta L}\right) + \left(B+\frac{1-\theta}{\theta}C\right)(1-\theta) - D\right]\left(1-\frac{b}{\delta\gamma(1-\sigma)}\right)\Delta^E \\
&-\left(\frac{e^{b(\Delta-\Delta^E)}-1}{b}\right)A\theta \qquad (99) \\
&-\frac{\sigma}{b}\left[\left(B+\frac{1-\theta}{\theta}C\right)\left(1-e^{-\frac{b}{\sigma}\Delta^E}\right) - \frac{1}{2}C\left(1-e^{-\frac{2b}{\sigma}\Delta^E}\right) - D\frac{(1-\theta)\left(1-e^{-\frac{b}{\sigma}\Delta^E}\right)}{1-\theta\left(1-e^{-\frac{b}{\sigma}\Delta^E}\right)}\right]
\end{aligned}
$$

Collecting terms we get (79).