Collective Labor Supply: Heterogeneity and Nonparticipation*

Richard Blundell[†], Pierre-Andre Chiappori[‡], Thierry Magnac[§] and Costas Meghir[¶]

September 2002

Abstract

In this paper we extend the "collective" model of labour supply developed by Chiappori (1988) to allow for discrete choices, censoring and nonparticipation in employment. We derive the collective restrictions on labour supply functions that we contrast with restrictions implied by the usual "unitary" framework. We apply our results to the estimation of a collective labor supply model for married couples without children using UK data from 1979 to 1993. Taking into account unobserved heterogeneity, we use the log-linear labor supply framework. The implications of the unitary framework are rejected while those of the collective approach are not. The estimates of the sharing rule show that wages have a strong influence on bargaining power within couples.

---

*Acknowledgements: We are grateful to two anonymous referees, Francois Laisney, Jim Heckman, Tom MaCurdy, John Rust and Yoram Weiss for particularly helpful comments. We also thank participants at the AFSE, Carlos III, Chicago Econometrics seminar, Concordia University (Montreal), the IFS, Nantes University, SITE '98 - Stanford, and Paris-Jourdan, for their helpful comments. This research is part of the program of the ESRC Centre for the Micro-Economic Analysis of Fiscal Policy at IFS. The financial support of the Alliance program, ESRC, NSF and Commissariat general du Plan is gratefully acknowledged. Household data from the FES made available by the CSO through the ESRC Data Archive has been used by permission of the HMSO. Neither the CSO nor the ESRC Data Archive bear responsibility for the analysis or the interpretation of the data reported here. The usual disclaimer applies.

[†] University College London, Department of Economics, Gower Street, London, WC1E 6BT and Institute for Fiscal Studies.

[‡] University of Chicago, Department of Economics, 1126 E 59th Street Chicago ILL 60637 USA.

[§] INRA-LEA and CREST, 48 boulevard Jourdan, 75014 Paris, France.

[¶] University College London, Department of Economics, Gower Street, London, WC1E 6BT and Institute for Fiscal Studies.

1. Introduction

The standard unitary labour supply model is unable to explain a number of empirical facts. First, the assumption of income pooling, where the source of income does not matter for household behaviour is rejected (see Thomas, 1990). Second, the compensated substitution effects between male and female leisure, whenever compared are found not to be symmetric. These empirical failings lead directly to the question of how resources are allocated within households and how this is likely to change in response to changes in the environment.

These empirical failings can be addressed by a richer conceptual framework where individuals retain their identity within the household and where questions of individual welfare make sense. Knowing how resources are allocated within the household is a major policy issue, because it relates to targeting benefits and to the actual structure of the distribution of consumption. A lot of inequality may be hidden within households and the standard unitary model cannot handle this issue by construction.

Ideally we would like to estimate directly the rule governing the sharing of resources, as well as individual preferences. However, in practice we do not directly observe the resource allocation we would need for such an exercise. Thus in this paper we use and develop further the framework of Chiappori (1988) to estimate individual preferences and within household allocation rules based on observable labour supply decisions of the household.

The UK data displays two important features that we would wish our model to allow for. First a large proportion of women do not work. Hence we have to discuss the implications of the collective framework when a good is on the corner - something not done before. Second, male hours of work do not vary much at all. Although nonparticipation rates for men are large and approaching those for women over time, when men do work, they nearly always work full time. In our data set practically no men are seen to work for less than 35 hours a week and very few are seen to work for less than 52 weeks. As we illustrate in the empirical section, modelling the small variation of hours above 35 hours a week or below 52 weeks a year does not seem to us to be the most important issue to focus on. Consequently, we assume that the husband's choice is discrete (work or not) and that only this dimension of labor supply choice affects preferences. Female hours, on the other hand although censored from below are allowed to be continuous, in line with the data. The discreteness of male labour supply leads to complications which have led us to generalise the Chiappori (1988) framework so as to allow for these features.

The framework we develop allows identification of preferences without using information on preferences for singles; preferences are fully marriage specific, i.e. we allow all parameters of the utility function to depend on whether one is married or not. A natural interpretation is that (i) marriage has a 'value' (in utility terms) *per se*, in the sense that, apart from any public good issue, married people can derive a higher degree of well-being from the same level of consumption (there is a utility of being married), and (ii) that marriage may change individual preferences, in the sense that one person's trade-off between, say, leisure and consumption typically depend on whether the person is married. The surprising result, in this context, is that the collective framework implies restrictions on household labor supply, even when male labor supply is discrete. Modelling household formation and dissolution and identifying the way that marriage affects preferences is of course another issue of critical importance. It is beyond the scope of this paper, although we view the present contribution as a step in this direction.

In this paper we do not allow for public goods or for household production. Individuals can care about each other's welfare, but not by the way in which this welfare is generated. These restrictions would be particularly stringent in the presence of children, where the decisions on how many resources should be devoted to them is of central importance and is reflected in day to day flows in consumption. For this reason we apply our model to couples without children. However, in our final section we discuss how the model would be generalized to take into account of household production and public goods. We draw on work we have been developing to discuss some new identification results as well as data requirements for this more general problem; in particular, we show that the "sharing rule" approach adopted in the present paper remains essentially valid, once public goods are adequately conditioned on. Of course, there are other issues that we do not address; these include intertemporal considerations. Some work has been carried out by Mazzocco (2001). More is left for future research in this important field, which can build on our framework.

From an econometric point of view we recognise the importance of unobserved heterogeneity, which in itself creates further difficult identification questions. We discuss estimation within the context of parametric preference structures and we show in the working paper version that given the functional form assumptions the restrictions originating from the collective framework overidentify the model.[1]

There are relatively few empirical studies of family labor supply outside the unitary model. A number of more recent studies have used micro data to evaluate the pooling hypothesis or to recover collective preferences using exclusive goods, but these studies typically look at private consumption rather than labor supply. For example, Thomas (1990) finds evidence against the pooling hypothesis by carefully examining household data from Brazil. Browning et al. (1996) use Canadian household expenditure data to examine the pooling hypothesis and to recover the derivatives of the sharing rule. Clothing in this analysis is the exclusive good providing identification, rather than labor supply which is problematical for a sample of couples without children who both work full-time.

Recent empirical studies concerning family labor supply include Lundberg (1988), Apps and Rees (1996), Kapteyn and Kooreman (1992) and Fortin and Lacroix (1997). Each of these aim to provide a test of the unitary model and to recover

[1]Working paper with proofs and detailed discussions of many of the issues is available at http://www.ifs.org.uk/workingpapers/wp0119.pdf

some parameters of collective preferences. Lundberg attempts to see which types of households, distinguished by demographic composition, come close to satisfying the hypotheses implied by the unitary model. The other three studies take this a step further by directly specifying and estimating labor supply equations from a collective specification. Apps and Rees (1996) specify a model to account for household production. Kooreman and Kapteyn (1993) use data on preferred hours of work to separately identify individual from collective preferences and, consequently, to identify the utility weight. Fortin and Lacroix (1997) follow closely the Chiappori framework and allow the utility weight to be a function of individual wages and unearned incomes. They use a functional form that nests both the unitary and the collective model as particular cases, and find that the restrictions implied by the unitary setting are strongly rejected, while the collective ones are not. In a more recent paper, Chiappori, Fortin and Lacroix (1998) extend the collective model to allow for 'distribution factors', defined as any variable that is exogenous with respect to preferences but may influence the decision process. Using PSID data and choosing the sex ratio as a distribution factor, they find that the restrictions implied by the collective model are not rejected; furthermore, they identify the intra-household sharing rule as a function of wages, non labor income and the sex ratio. It is important to note that the latter works assume that both male and female labor supplies vary continuously. The case of discrete male labor supply, which raises particular difficulties, is a specific contribution of the current paper.

In our paper we use UK survey data (FES) for the years 1978 to 1993 on couples without children. By using such long time series of cross sections we are able to exploit the large changes in the wage structure to provide identification of the labour supply model without relying on arbitrary exclusion restrictions or on cross sectional variation in wages. This in itself distinguishes us from many other papers and follows closely the approach of Blundell, Duncan and Meghir (1998). Our results show that the unitary model is rejected; however the collective model is not rejected. In the collective model, the estimated female labor supply wage (respectively income) elasticity at mean wages is 0.24 (resp. -0.1) and these results conform well with previous results where the analysis is conditioned on the male working. The estimation of the sharing rule implies that bargaining power is strongly affected by wages. One pound increase in his earnings when he is working increases his consumption by significantly more than one pound.

2. Theoretical Framework

We now present the collective model for male and female labour supply, with discrete male labour supply.[2] We then show how, given wages, other income and possible other exogenous variables, we can recover individual preferences and the sharing rule, from observations on labour supply of each individual. Our analysis is based on the assumption that within household allocations are efficient. This implies, among other things, that side-payments are possible. We view this as quite a natural assumption to make when modeling relationships of married individuals. It turns out that, when preferences are egoistic or caring, this assumption is an identifying one [3]. Preferences are defined over goods and non-market time. In the basic model, we assume that both these goods are private, and that there is no household production. The extension to account for household production is discussed in the concluding section of the paper where we also consider the case of public consumption. Although some of the assumptions underlying the basic model are restrictive, we think of it as best applying to the population of married couples with no children; children are likely to be the most important source of preference interdependence, which we choose to exclude for the moment.

The original Chiappori (1988) theorem relied on the idea that when allocations are efficient (as assumed) the marginal rates of substitution between members in

[2] The analysis assumes that unemployment is a labour supply decision.

[3] Browning and Chiappori (1998) and Chiappori and Ekeland (1997) show that efficiency alone (with general preferences) cannot provide testable restrictions upon behavior unless the number of commodities is at least 5; in addition, even with more than four commodities preferences are not identifiable.

a household are equalized. In our case, in which there is censoring and where one of the individuals faces discrete choice for one of the goods, the derivation of the implications of the collective setting has to follow a different logic. In what follows below we present the model and its assumptions formally and derive restrictions on labor supply functions. We finish the section by deriving similar restrictions of the unitary model.

2.1. The General Collective Labor Supply Model

2.1.1. Preferences and decision process

We consider a labor supply model within a two-member household; let h^i and C^i denote member i's labor supply (with $i = m, f$ and $0 \leq h^i \leq 1$) and consumption of a private Hicksian commodity C (with $C^f + C^m = C$) respectively. The price of the consumption good is set to one. We assume preferences to be 'egoistic' type; i.e., member i's utility can be written $U^i(1 - h^i, C^i)$, where U^i is continuously differentiable, strictly monotone and strongly quasi-concave.[4] Also, let w_f, w_m and y denote wages and the household's non labor income respectively.

A common assumption in previous works on collective labor supply (Chiappori, 1988, 1992; Fortin and Lacroix, 1997; Chiappori, Fortin and Lacroix, 1998) was that both labor supplies could vary continuously in response to fluctuations in wages and non labor income. If h^m and h^f are twice differentiable functions of wages and non labor income, then generically, the observation of h^m and h^f allows to test the collective setting and to recover individual preferences and individual consumptions of the private good up to an additive constant (Chiappori, 1988). Empirically, however, the continuity assumption is difficult to maintain. As shown in the empirical section, while female labor supply varies in a fairly continuous manner, male labor supply is essentially dichotomous. A first purpose of this paper is precisely to show that the collective model implies restrictions in the

[4] The utility functions can be of the caring type: Each individual may care about the overall welfare of their partner, so long as they do not care about how it comes about.

case where one labor supply is constrained to take only two values[5]. Hence we assume throughout the paper that member f can freely choose her working hours, while member m can only decide to participate (then $h^m = 1$) or not ($h^m = 0$). Let P denote the participation set, i.e., the set of wage-income bundles such that m does participate. Similarly, N denotes the non-participation set, and L is the participation frontier between P and N.

As the household is assumed to take Pareto-efficient decisions, there exists for any (w_f, w_m, y), some $\bar{u}^m(w_f, w_m, y)$ such that (h^i, C^i) is a solution to the program:

$$\max_{h^f, h^m, C^f, C^m} U^f\left[1 - h^f, C^f\right] \tag{2.1}$$

$$U^m\left[1 - h^m, C^m\right] \geq \bar{u}^m(w_f, w_m, y)$$

$$C = w_f.h^f + w_m.h^m + y$$

$$0 \leq h^f \leq 1, \quad h_m \in \{0, 1\}$$

The function $\bar{u}^m(w_f, w_m, y)$ defines the level of utility that member m can command when the relevant exogenous variables take the values w_f, w_m, y.[6] Underlying the determination of $\bar{u}^m$ is some allocation mechanism (such as a bargaining model) that leads to Pareto efficient allocations. We do not need to be explicit about such a mechanism; hence the collective model does not rely on specific assumptions about the precise way that couples share resources.

Also, note that, in general, we allow $\bar{u}^m$ to depend on the husband's wage even when the latter does not work. The idea, here, is that within a bargaining context, his threat point may well depend on the wage he would receive if he chose to work. If so, most cooperative equilibrium concepts will imply that $\bar{u}^m$ is a function of both wages and non labor income; in each case, indeed, a change in one of the threat points does modify the outcome. Regarding non-cooperative models of bargaining,

[5] The analysis could easily be extended to any discrete labor supply function; for instance, the choice might be between non activity, part-time or full-time work. In our data, however, male part-time work is negligeable.

[6] The analysis can be conditioned on any other exogenous variable.

various situations are possible. In some cases, for instance, the outcome does not depend on the threat points, which rules out any dependence of this kind. More interesting is the suggestion of MacLeod and Malcomson(1993), where the outcome of the relationship remains constant when the threat points are modified, unless one individual rationality constraint becomes binding; then the agreement is modified so that the resulting outcome "follows" the member's reservation utility along the Pareto frontier.[7] In our context, this implies that among all households where m is not working, only some will exhibit the dependence on m's wage. Finally, note that preferences (and the Pareto weights) are allowed to depend taste shifter variables, such as age etc.

2.1.2. The participation decision: who gains, who looses?

In the standard, unitary framework, the participation decision is modeled in terms of a reservation wage. At this wage, the agent is exactly indifferent between working and not working. Generalizing this property to our setting is however tricky, since now two people are involved. The most natural generalization of the standard model is to define the reservation wage by the fact that *one* member (say, the husband) is indifferent between working and not working. An important remark is that, in this case, *Pareto efficiency requires that both members are indifferent.* To see why, assume that the wife is not indifferent;- say she experiences a strict loss if the husband does not participate. Take any wage infinitesimally below the reservation wage, and consider the following change in the decision process: the husband does work, and receives ε more (of the consumption good) than previously planned. The husband is better off, since he was indifferent and he receives the additional ε; and if ε is small enough, the wife is better off too, since the ε loss in consumption is more than compensated by the discrete gain due to his participation.

[7] This will typically be the case for a generalization of the Nash bargaining concept to the case in which the relationship is non binding, in the sense that each member may at each period choose to leave.

In the remainder, we shall use the 'double indifference' assumption, that can be formally stated as follows:

Definition and Lemma DI ('double indifference'): *The participation frontier L is such that member m is indifferent between participating or not. Pareto efficiency then implies that f is indifferent as well.*

Technically, this amounts to assuming that in the program (2.1) above, $\bar{u}^m$ is a *continuous function* of both wages and non labor income. Natural as it may seem, this continuity assumption still restricts the set of possible behavior (and, as such, plays a key role for deriving restrictions).

A possible, quite general interpretation is that the household first agrees on some general 'rule' that defines, for each possible price-income bundle, the particular (efficient) allocation of welfare across members that will prevail. Then this rule is "implemented" through specific choices, including m's decision to participate. Although the latter is assumed discrete, it cannot, by assumption, lead to discontinuous changes in each member's welfare, in the neighbourhood of the participation frontier; on the contrary, the participation frontier will be defined precisely as the locus of the price-income bundles such that m's drop in leisure, when participating, can be compensated exactly by a discontinuous increase in consumption that preserves smoothness of each member's well-being.

The double indifference assumption can be justified from an individualistic point of view. That both members should be indifferent sounds like a natural requirement, especially in a context where compensations are easy to achieve via transfers of the consumption good. Conversely, a participation decision entailing a strict loss for one member is likely to be very difficult to implement; all the more when the loss is experienced by the member who is supposed to start working.

2.1.3. The sharing rules

It is well known that Pareto optima can be decentralized in an economy of this kind. Just as in Chiappori (1992), this property defines the central concept of the sharing rule. The important distinction here is that the decision of one of the members is discrete: the male can only decide to work or not.

Participation: Let us first consider the case when m does participate. His utility is thus $U^m(C^m, 0)$, and we have that :

$$U^m(C^m, 0) = \bar{u}^m(w_f, w_m, y) \tag{2.2}$$

Solving for consumption c^m we obtain

$$C^m = V^m[\bar{u}^m(w_f, w_m, y)] = \Psi(w_f, w_m, y)$$

where V^m is the inverse of the mapping $U^m(., 0)$. Function $\Psi(w_f, w_m, y)$ is called the *sharing rule.* Now, Pareto efficiency is equivalent to f's behavior being a solution of the program:

$$\max_{h^f, C^f} U^f[1 - h^f, C^f] \tag{2.3}$$

$$C^f = w_f.h^f + y + w_m - \Psi(w_f, w_m, y)$$

$$0 \leq h^f \leq 1$$

This generates a labor supply of the form :

$$h^f(w_f, w_m, y) = H^f[w_f, y + w_m - \Psi(w_f, w_m, y)] \tag{2.4}$$

where H^f is the Marshallian labor supply function associated to U^f.

A first consequence is that, for any $(w_f, w_m, y) \in P$ such that $h_f(w_f, w_m, y) > 0$:

$$\frac{1 - \Psi_{w_m}}{1 - \Psi_y} = \frac{h^f_{w_m}}{h^f_y} \doteq A(w_f, w_m, y) \tag{2.5}$$

Note that, in the absence of unobserved heterogeneity in preferences, the function h^f, and hence the ratio A, are empirically observable. Hence (2.5) provides a first restriction of Ψ.

Non participation: We now consider the non participation case. Then 2's utility is $U^m(C^m, 1)$, and we have that :

$$U^m(C^m, 1) = \bar{u}^m(w_f, w_m, y) = V_m^{-1}(\Psi(w_f, w_m, y)) \tag{2.6}$$

which can be inverted in :

$$C^m = W^m\left[V_m^{-1}(\Psi(w_f, w_m, y))\right] = F(\Psi(w_f, w_m, y))$$

where W^m is the inverse of the mapping $U^m(., 1)$ and where $F = W^m \circ (V^m)^{-1}$ is increasing because both V^m and W^m are increasing.

As before, f's decision program leads to a labor supply of the form :

$$h^f(w_f, w_m, y) = H^f\left[w_f, y - F\left(\Psi(w_f, w_m, y)\right)\right] \tag{2.7}$$

and, for any $(w_f, w_m, y) \in N$ such that $h_f(w_f, w_m, y) > 0$:

$$\frac{-F'\Psi_{w_m}}{1 - F'\Psi_y} = \frac{h^f_{w_m}}{h^f_y} \doteq B(w_f, w_m, y) \tag{2.8}$$

Note that in contrast to the unitary model f's labor supply will depend on m's (potential) wage even when m is not working, because the decision process will vary with w_m.[8]

It should finally be stressed that the function A (respectively B) is defined only on P (respectively N), i.e. for the set of wages and non-labor incomes for which the male works (does not work). Moreover functions A and B are only defined when the female works.

2.1.4. The participation decision

The participation frontier L is defined by the set of wages and non-labor income bundles $(w_f, w_m, y) \in L$, for which m is indifferent between participating or not:

[8] see Neary and Roberts (1980) on shadow prices when a good is at a corner.

Lemma 1. The participation frontier L is characterized by

$$\forall(w_f, w_m, y) \in L, \quad \Psi(w_f, w_m, y) - F(\Psi(w_f, w_m, y)) = w_m \tag{2.9}$$

Proof. Since, on L, f is also indifferent between m participating or not participating, it must be the case that f's income does not change discontinuously in the neighborhood of the frontier. Since total income does change in a discontinuous way (net increase of w_m when m participates), it must be the case that the whole gain goes to m.

■

The Lemma shows that at the participation frontier all additional income from participation goes to m to compensate him for the discrete increase in his labour supply. This is a property that depends on all goods being private and may not hold in the presence of public goods as we discuss in the last section -(Extensions)

To parameterize L, we choose to use a shadow wage condition; i.e., m participates if and only if

$$w_m > \gamma(w_f, y)$$

for some γ, that describes the frontier. Note that this reservation wage property does not stem from the theoretical set-up as in standard labor supply models, but has to be postulated. This will be true if (2.9) has a unique solution for w_m, a sufficient condition for which is that it is a contraction mapping:[9]

Assumption R : *The sharing rules are such that*

$$\forall(w_f, w_m, y), \quad |[1 - F'(\Psi(w_f, w_m, y)] \,.\Psi_{w_m}(w_f, w_m, y)| < 1 \tag{2.10}$$

[9] In words consider the increase in m's consumption resulting from an infinitesimal increase dw_m in m's wage . When m is participating, dw_m increases both the household income and m's bargaining power, while the first effect does not operate when m does not participate. Let dc^m denote the consumption change in the former case, and dc^{m*} in the latter. Then (2.10) states that the difference $dc^m - dc^{m*}$ cannot be more than the initial increase dw_m.

In this case, γ is characterized by the following equation :

$$\forall(w_f, y), \qquad \Psi(w_f, \gamma(w_f, y), y)) - F(\Psi(w_f, \gamma(w_f, y), y)) = \gamma(w_f, y) \tag{2.11}$$

which implies:

$$\begin{aligned}(\Psi_y + \gamma_y \Psi_{w_m}) &= \frac{\gamma_y}{(1-F')} \\ \Psi_{w_f} &= \frac{\gamma_{w_f}}{\gamma_y}\Psi_y\end{aligned} \tag{2.12}$$

2.1.5. Restrictions from the Collective Model

What are the restrictions implied by the collective setting just described with private consumption? And is it possible to recover the structural model - i.e., preferences and the sharing rules - from observed behavior ?

Proposition 1. Under the conditions listed in the Appendix

(i) the collective model with private commodities leads to restrictions on household behavior. In particular on the frontier (i.e. for the set of w_f, w_m and y such that $w_m = \gamma(w_f, y)$) we have that

$$\begin{aligned}-\Psi_{w_m} + A\Psi_y &= A - 1 \\ -\Psi_{w_m} + B\Psi_y &= \frac{B}{F'} \\ \gamma_y \Psi_{w_m} + \Psi_y &= \frac{\gamma_y}{(1-F^{\theta})} \\ \Psi_{w_f} &= \frac{\gamma_{w_f}}{\gamma_y}\Psi_y\end{aligned} \tag{2.13}$$

(ii) the preferences and the sharing rules can be recovered up to an additive constant everywhere where $h_f > 0$.

Proof: (i) We have assumed that $\bar{u}^m(w_f, w_m, y)$ is continuously differentiable everywhere. It follows that both (2.5) and (2.8) are valid on the frontier as well. Hence, *on the frontier*, using (2.5), (2.8) and (2.12) the sharing rule is determined by (2.13).

(ii) The proof follows in stages. First we consider the restrictions which recover the sharing rule and F' on the participation frontier. This is followed by a proof of identification outside the frontier. Identification of preferences then follows. See Appendix. Below we use these restrictions to derive direct tests of the collective model.

2.2. Unitary Model Restrictions

In the previous sections, we have derived the conditions that the labor supply functions of the female f and the participation frontier of the male m must satisfy to be compatible with the collective setting, when all goods are private. We now contrast this result to the unitary framework and discuss the extent to which the two models provide different predictions and testable implications that would allow us to discriminate between the two hypotheses. Here, the household, as a whole, is assumed to maximize some unique utility function U^H, subject to the standard budget constraint :

$$\max_{h^f, h^m, C} U^H \left[1 - h^m, 1 - h^f, C\right] \tag{2.14}$$
$$C = w_f . h^f + w_m . h^m + y$$
$$0 \leq h^f \leq 1, \quad h^m \in \{0, 1\}$$

Two points are worth mentioning here;

- We do not impose separability. This means that the household's preferences for f's leisure and total consumption may in general depend on whether m is working or not. Let V^P and V^N (respectively. h_P^f and h_N^f) denote the corresponding indirect utility functions when he works and when he does not (respectively female labor supply).

- Preferences, here, only depend on total consumption C ; we do not introduce C^f and C^m independently. This is a direct consequence of Hicks composite commodity theorem: since C^f and C^m have identical prices, they cannot be identified in this general setting.

One can immediately derive two restrictions, namely :

$$\frac{\partial h_P^f}{\partial w_m} = \frac{\partial h_P^f}{\partial y} \quad \text{Male Works} \tag{2.15}$$

and

$$\frac{\partial h_N^f}{\partial w_m} = 0 \quad \text{Male does not Work} \tag{2.16}$$

These are standard restrictions in the unitary context. Equation (2.15) is the "income pooling" property : when m's number of hours (conditional on participation) are constrained, then a change in w_m can only have an income effect upon f's labor supply. Equation (2.16), on the other hand, reflects the fact that the income effect of m's wage must be zero when he is not working.

Finally, m's participation decision depends on the difference between the household's (indirect) utility when he is working and when he is not :

$$h^m = 1 \Leftrightarrow V^W(w_f, y + w_m) \geq V^N(w_f, y)$$

In particular, the participation frontier is characterized by :

$$V^W(w_f, y + \gamma(w_f, y)) = V^N(w_f, y)$$

Differentiating and using Roy's identity gives that, on the frontier :

$$\frac{\partial \gamma}{\partial w_f} = h_N^f - h_W^f + h_N^f \frac{\partial \gamma}{\partial y} \tag{2.17}$$

The last term on the right hand side corresponds to a standard income effect: a marginal increase dw_f of female wage has the same first order effect upon participation as an increase of household non labor income equal to $h_N^f dw_f$. In addition, it also affects the cost of male participation due to the reduction of female working time; this corresponds to the term in $h_N^f - h_W^f$.

We can summarize these findings as follows :

Proposition 2. The functions γ, h_P^f and h_N^f are compatible with the unitary model if and only if conditions (2.15), (2.16) and (2.17) are satisfied.

2.3. The separable unitary model

To conclude this discussion we ask what happens when, within the unitary setting, we introduce the same separability assumption as in the collective case? Formally, this amounts to assuming that

$$U^H\left[1-h^m, 1-h^f, C^m, C^f\right] = U^H\left[U^m\left(1-h^m, C^m\right), U^f\left(1-h^f, C^f\right)\right]$$

where U^m and U^f are interpreted as individual utility functions. Note that, in this case, one can introduce C^m and C^f (instead of their sum), since the Hicksian composite good theorem no longer applies (see Chiappori (1988) for a precise statement).

In principle, this is a particular case of both the unitary model (since it corresponds to the maximization of a unique utility) and the collective model (since maximizing U^H under budget constraint obviously generates Pareto efficient outcomes). The problem, however, is that the form is now very strongly constrained. To see how, consider the assumption made above that, on the Pareto frontier, *both* members are indifferent between participation and non participation. This need not be the case here; Interpreting the utility function from the perspective of the collective model, the maximization of U^H may, and will in general, lead to participation decisions where one member is a strict loser, this loss being compensated (at the household level, as summarized by U^H) by a strict gain for the spouse.[10] The key intuition is that, within the unitary setting, the marginal utility of income, *as evaluated at the household level,* is equated across members. This by no means implies that utility levels are compensated in any sense. One can expect that the additional income generated by the husband's participation will be partially distributed to the wife, who, because of the standard income effect, will both work less and consume more. This need not always be the case, though, because the husband's marginal utility of consumption is modified when he participates

[10] Of course within the unitary model it does not make sense to talk about gainers and loosers within the household, since the unit is the household and not its members.

(unless, of course, his preferences are separable in leisure and consumption). But, in any case, there is no reason to expect bilateral indifference.

This provides an interesting illustration of the restrictive nature of the unitary model. From an individualistic point of view, that both members should be indifferent sounds like a natural requirement, especially in a context where compensations are easy to achieve via transfers of the consumption good. Conversely, a participation decision entailing a strict loss for one member is likely to be very difficult to implement. As it turns out, however, assuming a constant utility function essentially forbids an assumption of this kind; the model is not flexible enough with respect to the decision process to allow for such extensions.

3. Estimating Family Labor Supply

3.1. Data

The data we use is drawn from the UK Family Expenditure Surveys from 1978 to 1993 inclusive. It comprises married couples with no children in the household. The male is between the ages of 22 and 60. We only concentrate on households with no children so as to minimize problems relating to household production, which is excluded from our analysis.

3.2. Specification and Identifying assumptions

The discussion up to now set up the model without unobserved heterogeneity. Allowing for unobserved heterogeneity together with nonparticipation complicates matters and raises the issue of identifiability of the model from available data. The complications are compounded by the fact that preference heterogeneity will also reflect itself in the sharing rule. In the working paper version[11] we derive semiparametric identification conditions based on the restrictions of the Collective model for the case where heterogeneity enters additively, allowing in addition for wages to be endogenous. For the empirical analysis we specify the following parametric

[11] http://www.ifs.org.uk/workingpapers/wp0119.pdf

forms:

3.2.1. Female hours of work:

Following earlier work female hours of work is assumed to take the form[12]

$$\begin{aligned} h_{it}^{f} \quad = \quad & A_{0t}^{f} + A_m w_{it}^{m} + A_f \log w_{it}^{f} + A_y y_{it} + \\ & A_4 educ_{it}^{f} + A_5 age_{it}^{f} + A_6 educ_{it}^{m} + A_7 age_{it}^{m} + u_{1it} \end{aligned} \tag{3.1}$$

where f denotes female and m denotes male, w_{it}^{f} denotes the hourly wage rate for the female and w_{it}^{m} denotes the weekly earning for the male, y_{it} denotes other household (non-labor) income.[13] The variable $educ^{i}$ denotes education of member i, measured as the age that the person left full time education. Note that preferences are allowed to depend on the age and education of both partners as well as on cohort (or equivalently time effects as expressed by the inclusion of A_{0t}^{f}). These factors may affect preferences for work directly, or indirectly through the sharing rule. We also allow for differences in preferences across cohorts.

The parameters of the labor supply function will be different, depending on whether the male is working or not. Let (3.1) represent the labor supply function when the male is working. When he is not, the labor supply function is given by:

$$\begin{aligned} h_{it}^{f} \quad = \quad & a_{0t}^{f} + a_m w_{it}^{m} + a_f \log w_{it}^{f} + a_y y_{it} + \\ & d_t^{0} + a_4 educ_{it}^{f} + a_5 age_{it}^{f} + a_6 educ_{it}^{m} + a_7 age_{it}^{m} + u_{0it} \end{aligned} \tag{3.2}$$

3.2.2. Male participation:

$$\begin{aligned} p_{it}^{m} \quad = \quad & b_{pt}^{m} + b_m^{m} w_{it}^{m} + b_f^{m} \log w_{it}^{f} + b_y^{m} y_{it} + \\ & \zeta_4 educ_{it}^{f} + \zeta_5 age_{it}^{f} + \zeta_6 educ_{it}^{m} + \zeta_7 age_{it}^{m} + u_{it}^{m} \end{aligned} \tag{3.3}$$

where p_{it}^{m} is positive for male participants and negative (or zero) otherwise.

[12] This is a popular form to use on British data, see Blundell, Duncan and Meghir (1998), for example. This semi log specification is never rejected by our data.

[13] We use the level of male earnings, rather than the log, since this allows us to nest the income pooling hypothesis, where $A_m = A_y$

One can solve simply for w_{it}^m when $p_{it}^m = 0$ to derive the male reservation earnings and the parameters of the frontier of participation. These are:

$$\gamma_f = -\frac{b_f^m}{b_m^m} \qquad \gamma_y = -\frac{b_y^m}{b_m^m}. \tag{3.4}$$

Because of the sharing rule the male participation equation and the two female labor supply equations will depend, in general, on the same set of variables.

3.2.3. Wage equations:

We take a standard human capital approach to wages. However, we do not restrict the relative prices of the various components of human capital to remain constant over time. Hence

$$\begin{aligned} w_{it}^m &= \alpha_{0t}^m + \alpha_{1t}^m educ_{it}^m + \alpha_{2t}^m age_{it}^m + u_{wit}^m \\ \log w_{it}^f &= \alpha_{0t}^f + \alpha_{1t}^f educ_{it}^f + \alpha_{2t}^f age_{it}^f + u_{wit}^f \end{aligned} \tag{3.5}$$

Note that wages do not depend on the characteristics of the partner. All coefficients are time varying reflecting changes in the aggregate price of each component of human capital.

3.2.4. Stochastic specification and exclusion restrictions

We assume that all error terms (u_{1it}^f, u_{0it}^f, u_{it}^m, u_{wit}^m, u_{wit}^f) are jointly conditionally normal with constant variance (and independent of education, age, other income and time).[14] The basic exclusion restriction implied by the labour supply equations above is that education-time interactions and age-time interactions are excluded from these equations, implying that differences in the preferences and the sharing rule across education groups remain constant over time. Hence, identification of labour supply of both partners does not rely on excluding education. It relies on the way that the returns to education have changed (see, Blundell, Duncan and Meghir, 1998). The rank condition for identification is that wages have changed differentially across education groups over time. That they have done so in the UK is a well established fact (for men see Gosling, Machin and Meghir, 2000).

[14] As proved in the working paper, normality is not an identifying assumption. However this assumption substantially simplifies the estimation problem.

3.2.5. Restrictions from the Collective and Unitary Models

Using the family labor supply specification (3.1), (3.2) and (3.4), the restrictions on the collective model derived from Proposition 1 may be written (see working paper)

$$\begin{aligned} &\tfrac{A_m - a_m}{A_y - a_y} = -\tfrac{1}{\gamma_y} \quad I \\ &\tfrac{A_f - a_f}{A_y - a_y} = \tfrac{\gamma_f}{\gamma_y} \quad II \end{aligned} \tag{3.6}$$

It is interesting to contrast these restrictions with the structural restrictions that would be derived in the unitary case. Using the same notation (upper and lower cases for the two regimes), we have to impose (2.15), (2.16) on the whole space (w_m, w_f, y) and (2.17) on the frontier $(w_m = \gamma_f \log w_f + \gamma_y y + z\gamma_z)$. The first two yield:

$$\begin{aligned} A_m = A_y \quad I \\ a_m = 0 \quad II \end{aligned} \tag{3.7}$$

Using (2.17), the definition of the frontier and these two restrictions give:

$$\gamma_f = (1+\gamma_y)(a_f \log w_f + a_y y) - (A_f \log w_f + A_y(y + w_m))$$

As this equation is valid only on the frontier for any w_f and y, it is also an equation of the frontier. Therefore, the following vectors are colinear:

$$\begin{pmatrix} 1 \\ -\gamma_f \\ -\gamma_y \end{pmatrix}, \begin{pmatrix} -A_y \\ (1+\gamma_y)a_f - A_f \\ (1+\gamma_y)a_y - A_y \end{pmatrix}$$

Hence in the unitary model we have two additional restrictions, namely:

$$\begin{aligned} (1+\gamma_y)(a_y - A_y) = 0 \quad I \\ A_y\gamma_f = (1+\gamma_y)a_f - A_f. \quad II \end{aligned} \tag{3.8}$$

3.3. Estimation

First we estimate two reduced form participation equations: One for men and one for women. These are obtained by substituting out the wages from the structural participation equations. The resulting equations have the form

$$p_{it}^k = \beta_{0t}^k + \beta_{1t}^k educ_{it}^f + \beta_{2t}^k age_{it}^f + \beta_{3t}^k educ_{it}^m + \beta_{4t}^k age_{it}^m + \beta_{5t}^k y_{it} + v_{it}^k, \qquad k = m, f \tag{3.9}$$

Thus we include all variables that determine wages of men and women as well as other income y_{it}, and we allow the coefficients to change over time. The changes in the coefficients for variables that determine wages reflect the changing coefficients in the wage equations.[15]

We then estimate the female log hourly wage equation and the male weekly earnings equations including the inverse Mill's ratio obtained from the estimated participation equations (3.9). In this parametric approach the wage equation is identified from the exclusion of other income and the spouse's characteristics as well as from the normality assumption.[16]

Using the estimated wage equations we impute offered wages for all individuals in the data set. The participation frontier is then estimated using a probit, which includes other income, the imputed wages, time effects, age and education.

The likelihood function for female labor supply when the man is working is and there are n_P such observations

$$\begin{aligned} LogL^P \quad = \quad & \textstyle\sum_{i=1}^{n_P}\{1(h_{it}^f < 0)logPr(p_{it}^m > 0, h_{it}^f < 0)+ \\ & 1(h_{it}^f > 0)\left[logPr(p_{it}^m > 0) + logf(h_{it}^f|p_{it}^m > 0)\right]\} \end{aligned} \tag{3.10}$$

When the man is not working (n_N observations) this becomes

$$\begin{aligned} LogL^N \quad = \quad & \textstyle\sum_{i=1}^{n_N}\{1(h_{it}^f < 0)logPr(p_{it}^m < 0, h_{it}^f < 0)+ \\ & 1(h_{it}^f > 0)\left[logPr(p_{it}^m < 0) + logf(h_{it}^f|p_{it}^m < 0)\right]\} \end{aligned} \tag{3.11}$$

In the above $f(\cdot)$ represents the conditional normal density function and $1(a)$ is the indicator function which is equal to one when a is true and zero otherwise.

The labor supply estimates that are obtained from this procedure do not satisfy exactly the assumptions of the collective (or the unitary) model. We can then carry

[15] The changing coefficient on other income is not implied directly by the structure of the model (and it is not necessary for identification purposes). We allow this as an extra degree of flexibility.

[16] We tested for zero skewness in both the male earnings equation and in the female log hourly wage equation, taking into account the selection. The t-statistics were 1.93 and 1.38.respectively Hence the hypothesis is accepted in both cases.

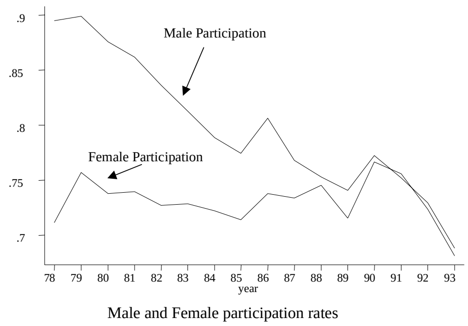

Figure 3.1:

Figure 3.2: Actual and offered male weekly wages in levels

out a test of the null hypothesis that the coefficients can be rationalized using the unitary model and/or the collective model and derive the structural estimates.

3.4. Basic facts in the data and the wage equations

For this group of households there have been momentous changes in the male participation rates. This is shown in Figure 3.1. From the late 1980s the average participation rates across all age groups have been basically the same for men and women. This rapid drop in male participation relates to all age group, but is most dramatic for those over the age of 50, which is generally interpreted as an increase in early retirement. On the other hand, participation has been relatively steady

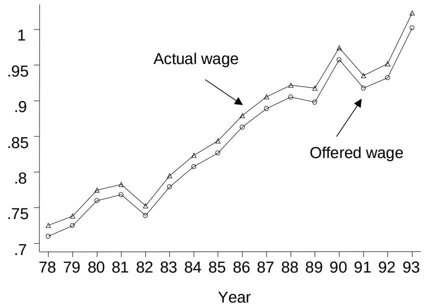

Figure 3.3: Actual and offered female hourly log wages

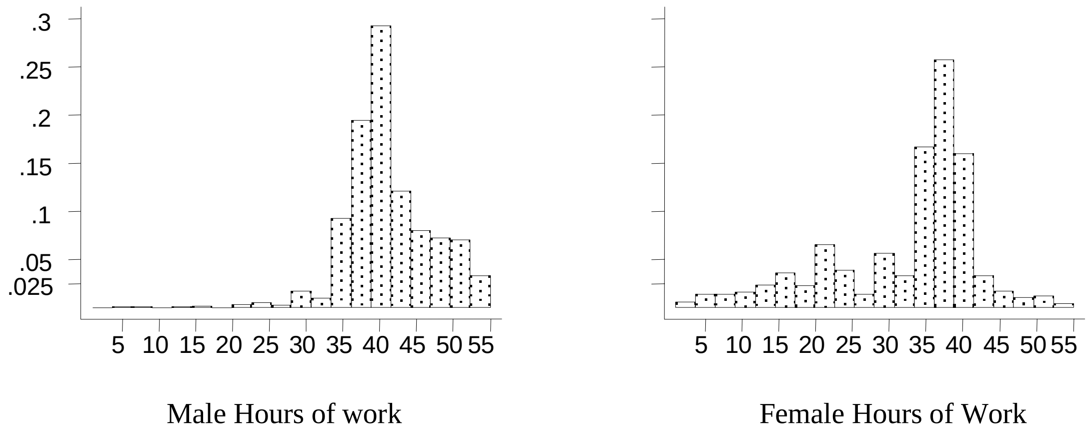

Figure 3.4: Histogram of hours of work

over these years for married women without children.

The wage equations include age and years of education interacted with time. The participation equation contains in addition the education of both partners and other household income, all interacted with time. The p-values for the female education-time interactions in the male participation equation is 2.5% and for the male education-time interactions in the female participation equation is 4.1%. The p-value for other income-time interactions is 0 in both cases. Hence the wage equations, which exclude all these variables are well identified. The education-time interactions (not including the baseline education level) are highly significant in both wage equations implying that the returns to education have been changing for both men and women, even after controlling for changes in participation.

The effects of correcting for selection on aggregate predicted wages over time can be seen in Figures 3.2 for males (real wage levels divided by 40), and for females (real log wages) in Figure 3.3. The selectivity corrected wages are labelled "offered wages".

In figure 3.4 we present the histogram of hours of work for working men and women. While women can be found working any number of hours from 1-55, practically no men work for less than 35 hours a week.[17] This fact motivated us to concentrate on the work - no work decision for males. Although, it would be interesting to explain the variation among the full time hours, we do not think that this is the most important dimension of male work decisions. Finally, Average hours for workers fluctuate over time between 32 and 34 hour, but there is no apparent trend up or down.

[17] In the empirical section we illustrate this for weekly hours of work in the UK. We have found in one other data information on weeks employed during the year. Of those working 90% work 52 weeks per year. This measure unfortunately ignores holidays, but this is unlikely to add much variability since these are quite standard. Moreover in the UK temporary layoffs are non-existent. Thus even in an annual dimension the choice by men seems to be work full time or not at all.

3.5. Testing the Unitary and Collective Models

The unrestricted estimates for male participation and female labor supply are presented in Table B.1 in Appendix B. We impose a zero correlation between the participation equation and the labor supply equations in the two regimes (male works and male does not work). This was done following earlier estimates where these correlations were shown to be very close to zero. To summarise the implications: The male wage effect translates to a participation elasticity of 0.20 ($\partial logP/\partial logw$) evaluated at the the sample means.[18] The male participation elasticity with respect to the female wage is 0.05 and with respect to other income -0.088.

The female labor supply function when he works (respectively when he does not) implies an own wage elasticity of 0.34 (resp. 0.5). The implied income elasticity is -0.1 (resp. -0.125). Finally, the male wage elasticity is approximately 0.1 (0.3). Taken on its own, the female labor supply function when he works can be rationalized as deriving from a standard unitary framework and the results are in line with earlier UK results (e.g. Blundell and Walker, 1986, Arellano and Meghir, 1992 and Blundell, Duncan and Meghir, 1998).

Using these results we can test the unitary model. A key restriction of the unitary model is income pooling which here is reflected in the restriction 3.7 I. This is unambiguously rejected with a p-value of 0.[19]. The restriction that the male wage should not affect female labor supply when he does not work (3.7 II) is easily accepted with a t-statistic of 1. This is probably due to the lack of precision, whose root cause is the relatively little variation in education among those who do not work (they are mainly low skill individuals). In addition the unitary model also implies the cross equation restrictions 3.8. The joint test for these restrictions has a p-value of 5.9%; the test of (3.8- I) has a p-value of 13% while the test of (3.8- II) has a p-value of 3.6%. Hence overall the unitary model is rejected both due to

[18] 80% participation rate and male wage of \$1.25 i.e. \$50 per week

[19] The $\chi^2(1)$ test statistic is 37.7.

Female Labour Supply - Restricted Estimates				
	Male Works		Male out of work	
male wage	1.983	*0.777*	3.194	*0.838*
Fem. log wage	8.072	*0.504*	9.114	*0.597*
other inc	-9.190	*0.368*	−11.559	*0.631*
Criterion $\chi^2(2)$	1.22			
Asymptotic standard errors in italics				

Table 3.1: Restricted labour Supply estimates

the strong rejection of income pooling and due to the cross equation restrictions (3.8- II) .

We now turn to imposing the restrictions from the collective model (3.6) and to test them. The restrictions are imposed using minimum distance. The criterion we minimize has an asymptotic χ^2 distribution with degrees of freedom equal to the number of restrictions we impose under the null. The unrestricted values of the γ_y and γ_f coefficients are minus the other income coefficient and the female wage coefficient in the male participation equation respectively, divided by the coefficient on the male wage. Note that the unrestricted value of γ_y is 2.12 and the unrestricted value of γ_f is -0.35 based on the participation equation in Table B.1. Thus the sign pattern of these expressions conforms to the predictions of the collective model. The two degree of freedom χ^2 test statistic for the hypothesis that the restrictions are acceptable is 1.22 which has a p-value of 0.54.

The restricted estimates are presented in Table 3.1. These are estimates that satisfy the restrictions from the collective model, and from which it is possible to derive the sharing rule and the structural female labor supply. The latter will be a function of her wage and the other income she has access to. Finally we can also estimate the male participation frontier.

3.6. The Estimates of the Collective Model

After imposing the restrictions above we check if a solution for the sharing rule exists. This depends on whether the quadratic equation

$$\phi^2 + (-a_y - A_m + a_m)\,\phi + A_m a_y - a_m A_y = 0 \tag{3.12}$$

has a solution for ϕ.(*see* Proposition 1) One two or no solution may exist to 3.12.[20] In our case the estimates imply two solutions. Typically only at most one of the solutions implies an integrable well behaved female labor supply; i.e. an individual labor supply satisfying Slutsky negativity. A critical parameter for understanding this is (given the value of all other coefficients) is the size of the male wage effect when he does not work: When he does not work the only reason for an impact of his wage on her labor supply behavior is the shift in the bargaining power. However when he works a shift in the wage generates in addition an income effect (since household resources grow). Hence we would expect a more positive (or less negative) effect of the male wage on her labor supply when he does not work than when he does work, which is precisely the implications of our point estimates in Table 3.1.

The female labor supply implied by her utility function is

$$\begin{array}{ccccccc} h_f & = & c_f & + & 9.415 \log w_f & - & 12.24 y^f \\ & & & & (1.038) & & (1.31) \end{array} \tag{3.13}$$

where y^f is the other income allocated to the female member of the household, after the male has been allocated his consumption. The implied wage elasticity at mean wages is 0.24 while the income elasticity is -0.1. This labor supply satisfies the integrability conditions of individual utility maximization, which of course is a requirement of the theory. The elasticities are very similar to the elasticities reported in Blundell, Duncan and Meghir (1998) which are conditional on the male working.

We now turn to the implied sharing rule. This is of course a unique element of our approach, since it directly relates to the distribution of resources within

[20] If no solution exists then we need to impose one further restriction which implies existence.

the household. The sharing rules implied by the restricted estimates for working husbands are[21]

$$\Psi = \underset{}{\kappa_1} + \underset{(0.064)}{1.16 w_m} - \underset{(0.036)}{0.11 \log w_f} + \underset{(0.088)}{0.25 y} \qquad (3.14)$$

The estimate of ϕ is 0.23 with a standard error of 0.041. Thus when the husband does not work the sharing rule becomes

$$F(\Psi) = \kappa_0 + 0.23(1.16 w_m - 0.11 \log w_f + 0.25 y) \qquad (3.15)$$

Thus when he works, he gets to keep all of a wage increase plus 0.16 for every extra pound earned. The hypothesis that he just keeps all his extra earnings and just that on the margin ($\psi_m = 1$) is formally rejected (t-statistic is 2.5) but is a plausible interpretation of the result. When her wage increases by one pound her consumption (at the means) goes up by \$1.13 and his falls by \$0.13. This of course implies that the bargaining power of women increases with their own wage. In interpreting the result one must note that the estimated impacts of wages and other income are marginal. The level of the sharing rule is not identified. If she happens to obtain a large baseline consumption it is not implausible that he gets to keep marginal increases in his wage. In some sense this is corroborated by the fact that she gets to keep the lion's share of increase in "other income". Interestingly, Chiappori, Fortin and Lacroix (2001) find a very similar result using PSID data for the US and with quite a different approach. Although the result is surprising it is theoretically consistent and originates in very reasonable labour supply estimates. It implies that the recent increase in female wages relative to male ones would have caused some improvements in women's relative standard of living.

Increases in other income are shared between husband and wife: However she gets a larger proportion: For every \$ increase in other income his consumption goes up by \$0.25 and hers by \$0.75. Thus the woman's relative position improves with unearned income but both consumptions increase. Finally, when he does not

[21] In the equations that follow asymptotic standard errors are reported in brackets below the estimated coefficients.

work the derivatives of the sharing rule with respect to the economic variables are multiplied by 0.23, implying a substantially smaller effect of changing economic conditions on his consumption. When he does not work her consumption increases more or less with her wage, with no substantial reduction in his consumption. However, she gets to keep a larger proportion of increases in non-labor income; moreover increases in male market opportunities improve male consumption when he is out of work but only by small amounts.

The implied participation frontier from these estimates is

$$\begin{array}{ccccccc} w_m^r & = & c_m & + & \underset{(0.16)}{2.95y} & - & \underset{(0.29)}{0.86 \log w_f} \end{array} \tag{3.16}$$

Since his consumption grows with other income, increases in the latter reduce his reservation wage. However, increases in her wage reduce his consumption and hence make it more likely that he works.

4. The Extension to Public Goods and Household Production

4.1. The Extension to Public goods

Recent results show that an extension of the model to public consumption is feasible, although it may require additional information and/or particular assumptions. Not only are the main conclusions of the private good setting (i.e., identification and testability) preserved in the extended framework, but the current model can be interpreted, in this perspective, as the reduced form of the general problem, the emphasis being put here on private consumptions only. Although these developments are outside of the scope of the present paper, and will be the topic of future empirical investigations, one can indicate the general flavor of these extensions. We summarize the state of knowledge in this area which can be found in references made below and in Chiappori, Blundell and Meghir (2001).

There are several ways of introducing public consumption within the collective model. The simplest manner, and perhaps the most natural one in the absence of price variation, is to assume that the Hicksian good C is collectively consumed - i.e., individual utilities are of the form $U^i(1-h^i, C)$. In this context, one can show (Chiappori and Ekeland, 2001; Donni, 2001) that the knowledge of individual labor supply functions generically allows *exact identification* of the structural model, i.e. preferences and Pareto weights, at least when the number of hours is continuous (the extension to discrete participation, in the spirit of the present paper, is left for further research).

In a more general setting, public and private consumptions can be simultaneously considered. Then difficult identification problems arise. While preferences over the private goods can readily be identified *conditional* on the quantities consumed of the public goods, general identification typically requires additional information or more structure. A natural solution is to assume that private consumption is separable, with member i' utility of the form $W^i\left[u^i\left(1-h^i, C^i\right), K\right]$ (here K denotes public consumption, assumed observable).[22] Even in the absence of price variation (i.e., assuming that the price of both the public and the private good are normalized to one), this model is identifiable: the observation of labor supply and demand for public good as (continuous) functions of wages and non labor income allows to uniquely recover the underlying structural model. Specifically, once the demand for public good is known, then one can also recover the utility indices W^i and the decision process, as summarized by the corresponding, individual Pareto weights.[23] This first model can be extended in different ways to include the case where the production of the public good also requires leisure.

Finally, an interesting perspective is provided in a recent contribution by Zhang

[22]Note, however, that given the collective structure of the model, at the household level there will be no separability property between members' leisure and the demand for public good.

[23]In practice, efficiency requires that the household demand and labor supplies solve a Pareto program of the form:

$$\max \lambda W^1 + (1-\lambda) W^2$$

under budget constraint. The outcome of the decision process (i.e., the location of the final choice on the pareto frontier) is fully summarized by the Pareto weight λ. In addition, if the W^i are such that private and public consumptions are normal goods, then there exists an increasing, one-to-one correspondance between the Pareto weight λ and the sharing rule ρ (as functions of wages and non labor income).

and Fong (2000). In their model, leisure is partly private and partly public, in the sense that member i's leisure ($i = m, f$) can be written as $L^i = L^i_p + L$ where L^i_p represents i's private leisure and L is the common leisure of the couple. While individual labor supplies (hence total leisures) are observable, the allocation of time between private and public leisure is not. Under mild separability assumptions, Zhang and Fong show the following result: if there exist a private good (at least), the husband's and the wife's consumptions of which are independently observable, then the structural model (including the allocation of leisure between private and public time) can be fully recovered. Again, this result suggest that data on private consumptions can help achieve identification of more general models, entailing private and public consumptions. As an example of such private consumption, one may think of clothing, as in Browning et al. (1994).

4.2. The Extension to Household production

In Blundell, Chiappori, Magnac and Meghir (2000) we discuss the extension to household production. The generalization we consider is the case where the produced good is privately consumed. The framework follows that proposed in Chiappori (1997) in which there are two leisure and two private consumption goods: One market good c, the price of which is normalized to one, and a domestic good x, that can be produced within the household. In the production function of the domestic good, we allow for differences in marginal productivity of labor between members, which can account for partial specialization (time input being non zero for each member). Also, given that quantities of the x good are not observable, it is natural to assume that the technology exhibits constant returns to scale. A standard issue in household production models is whether there exists a market for good x.[24] When the domestic good is marketable and when the quantity actually purchased on the market, denoted x^M, is positive, then in the decision process it is valued at its market price which is *exogenous* for the household. Otherwise, we

[24]For instance, meals can be taken at home or at restaurant; one can either clean one's house or pay a cleaning lady to perform the job; etc..

can still define a shadow price, π, for the domestic good, that is some endogenous, household-specific function of wages and non labor income.

In this generalized set up, the sharing rule is shown to exist. A generalized version of the double indifference result also holds but identification is still an open issue. However we can show identification in parametric cases where there is no market and where the shadow value is affected by individual preferences.

However, identification with household production requires time use data; this is because the model is informative about the woman's leisure rather than her work time. With no household production leisure is simply the complement of hours of work. This is no longer the case with household production where non-market time is shared between leisure and productive activities.

5. Summary and Conclusions

In this paper we have specified a model of family labor supply based on the collective framework. In doing so, we have introduced two important theoretical innovations: First we allow the possibility that one or both partners do not work. Second we allow for the possibility that one of the partners makes just a discrete work choice, i.e. to work or not. We show that knowledge of the male participation rule and knowledge of the female labor supply schedule, allows us to test the collective restrictions and to recover the individual preferences as well as the rule governing sharing of household resources, as a function of market wages and other incomes.

Finally we use this framework to analyze family labor supply using data from 1978 to 1993. The data is not at odds with the collective model, but the unitary model is rejected. Once we estimate the collective model we find that female wage elasticity is about 0.25 which is very close to the figure that earlier UK studies have found, including work on single parents. Moreover we find that the level of male consumption is sensitive to wages and other income. Although he gets to consume all the increase in his earnings, increases in the female wage and other income lead to substantial increases in her consumption. The implication is that the improvement in the labor market conditions for women over the last 20 years would have translated to significant improvements in their relative welfare.

To conclude the paper we briefly considered two possible extensions of the basic framework: household public goods and household production.

Appendices

A. Proof of Proposition 1

A.1. Identification on the participation frontier

On any point on the frontier, (2.13) is a non-linear system of equations in the unknowns $(\Psi_{w_f}, \Psi_{w_m}, \Psi_y, F')$. The first three equations characterize the three unknowns Ψ_{w_m}, Ψ_y and F'. Specifically, from the first two, one gets that

$$\begin{aligned} \Psi_y &= \tfrac{1}{(a-b)} \left(a - 1 - \frac{b}{F'}\right) \\ \Psi_{w_m} &= \tfrac{b}{(a-b)} \left(a - 1 - \frac{a}{F'}\right) \end{aligned}$$

where

$$\begin{aligned} a &= a(w_f, y) = A\left[w_f, \gamma(w_f, y), y\right] \\ b &= b(w_f, y) = B\left[w_f, \gamma(w_f, y), y\right]. \end{aligned} \tag{A.1}$$

Replacing in the third equation in (2.13) gives the following equation in F' :

$$(\gamma_y ba - 1 + a - \gamma_y b)\left(F'\right)^2 + (-b + 1 - 2\gamma_y ba + \gamma_y a - a) F' + b + \gamma_y ba = 0$$

This equation must have a solution, hence the standard discriminant condition:

$$(-2\gamma_y ab + \gamma_y a - b - a + 1)^2 \geq 4\left(\gamma_y ab + b\right)\left(a - 1 + \gamma_y ab - \gamma_y b\right)$$

Conversely, assume this condition is satisfied. Let $\phi\left(w_f, y\right)$ be a solution of the quadratic equation above (note that there are at most two such solutions). We know that if this function corresponds to a solution, then it is such that:

$$F'\left[\Psi\left(w_f, \gamma(w_f, y), y\right)\right] = \phi\left(w_f, y\right)$$

Then the partials Ψ_{w_m}, Ψ_{w_f} and Ψ_y are identified - although, of course, on the frontier only. Specifically, one can define three functions K, L and M such that:

$$\begin{aligned}
\Psi_{w_m}\left[w_f, \gamma(w_f, y), y\right] &= K\left(w_f, y\right) = \frac{b}{(a-b)}\left(a - 1 - \frac{a}{\phi\left(w_f, y\right)}\right) \\
\Psi_{w_f}\left[w_f, \gamma(w_f, y), y\right] &= L\left(w_f, y\right) = \frac{\gamma_{w_f}}{(a-b)\,\gamma_y}\left(a - 1 - \frac{b}{\phi\left(w_f, y\right)}\right) \\
\Psi_{y}\left[w_f, \gamma(w_f, y), y\right] &= M\left(w_f, y\right) = \frac{1}{(a-b)}\left(a - 1 - \frac{b}{\phi\left(w_f, y\right)}\right)
\end{aligned}$$

Now, let us consider the testable restrictions implied by these results. First, that F' can be written as a function of Ψ only has a consequence, namely that

$$\frac{\phi_{w_f}}{\phi_y} = \frac{\Psi_{w_f} + \Psi_{w_m}\gamma_{w_f}}{\Psi_y + \Psi_{w_m}\gamma_y} = \frac{L + K\gamma_{w_f}}{M + K\gamma_y} = \frac{\gamma_{w_f}}{\gamma_y}$$

which implies that the function ϕ can be written as some function of γ.

Moreover, Ψ_{w_m}, Ψ_{w_f} and Ψ_y are the partials of the same function. This implies the following condition:

$$L_y - M_{w_f} = \gamma_y K_{w_f} - \gamma_{w_f} K_y$$

Since the functions K, L, M and ϕ are exactly identified from the functions a, b and γ, these two equations are testable restrictions upon the latter functions. Finally, the quadratic equation defining ϕ may have two solutions. But, generically, one (at most) will satisfy the two conditions above. In addition, the function F is identified up to an additive constant.

In summary: the function Ψ and function F are identified up to an additive constant on the male participation frontier.

A.2. Identification outside the frontier

We now consider the general problem of identifying the sharing rule off the participation frontier. Let us start by the participation set P. We will assume that for all (w_f, y): $a(w_f, y).\gamma_y(w_f, y) + 1 \neq 0$. Under this assumption we have:

Lemma 2. On P, Ψ is identified up to an additive constant.

Proof. We know that Ψ must satisfy the partial differential equation (2.5), i.e.:

$$-\Psi_{w_m} + A\Psi_y = A - 1 \tag{A.2}$$

In addition, the values of the partials on the frontier have been identified above. The basic idea, now, is that *the latter provide boundary conditions for the partial differential equation.* From standard theorems in partial differential equation theory, this defines Ψ (up to an additive constant) provided the following condition is fulfilled. First, remark that, at any point on the frontier, (A.2) can be written as :

$$\nabla\Psi.\vec{u} = A - 1$$

where $\nabla\Psi$ denotes the gradient of Ψ, and $\vec{u}$ is the vector $(0, -1, A)'$. Now, the condition is that $\vec{u}$ is not tangent to the frontier L. Since the equation of L is :

$$w_m - \gamma(w_f, y) = 0$$

and given that, on the frontier, A coincides with a, this condition states that, for all (w_f, y) :

$$a(w_f, y).\gamma_y(w_f, y) + 1 \neq 0 \tag{A.3}$$

If this relation is fulfilled on the frontier, then the PDE (2.5), together with the boundary condition, defines Ψ up to an additive constant.

Practically, there are cases where the PDE can be solved analytically. Then the solution is defined up to a function of 2 variables; and this function is identified by its values upon the frontier. The next section provides an example on a specific functional form. Even when the PDE cannot be solved analytically, it is always possible to numerically compute Ψ using the PDE and the boundary condition on the frontier. See Appendix A for the detail of the algorithm. ■

In the non-participation set (N), the approach is exactly the same :

Lemma 3. Assume that, for all (w_f, y) :

$$b(w_f, y).\gamma_y(w_f, y) + 1 \neq 0 \tag{A.4}$$

Then on N, Ψ is identified up to an additive constant.

Proof. As above, using the PDE

$$-\Psi_{w_m} + B\Psi_y = \frac{B}{F'} \tag{A.5}$$

(remember that F' has been exactly identified above). ■

Note, incidentally, that generically both (A.2) and (A.4) are fulfilled almost everywhere on the frontier.

For any (arbitrary) value of the constant, the equations (2.4) and (2.7) allow to recover the Marshallian demand H; then preferences can be identified in the usual way. Finally, note that integration requires at that stage additional restrictions. While Slutsky symmetry is not binding with only two goods, the sign of compensated own price elasticity still has to be positive, a constraint that can readily be verified.

To see the intuition underlying the restrictions of the collective model note that they reflect the fact that wages have three effects. Two are the familiar income and substitution effects of price (or wage) changes. The third effect, which is specific to the collective model, is that any wage (or income) variation may affect the sharing rule (say, through its impact on bargaining power). The nature of the collective approach is that this latter effect is not restricted. However, any given change in the sharing rule must impact on a member's labor supply in the same way whatever the origin of the change. This, together with the fact that the sharing rule affects the disposable income of both agents, generates the restrictions implied by the collective setting.

B. Unrestricted Estimates

References

[1] Apps, P.F. and Rees, R. (1988),'Taxation and the Household', *Journal of Public Economics*, 35, 199-219.

	Male Participation		Female Labor Supply			
			Male Works		Male does not Work	
male wage	0.507	*0.134*	1.806	*1.607*	5.886	*5.945*
Fem. log wage	0.175	*0.083*	7.823	*1.047*	12.241	*3.644*
other income	-1.075	*0.038*	-9.209	*0.737*	-11.514	*1.269*
male age	-0.134	*0.031*	1.752	*0.432*	0.211	*1.332*
fem. age	0.038	*0.028*	-7.943	*0.392*	-8.053	*1.156*
male educ	0.004	*0.017*	-0.411	*0.195*	-1.106	*0.788*
Sample	13760		11018		2742	

Year and Year^2 included. Standard errors in italics. The Female wage is measured in \$ per hour other income and male weekly earnings are divided by 40.Means: Female wage 0.82, male wage 1, other income 0.25

Table B.1: Unrestricted labour Supply estimates - No Selection correction

[2] Apps, P.F. and Rees, R. (1996),'Labor Supply, Household Production and Intra-Family Welfare Distribution", *Journal of Public Economics*, 60, 199-219.

[3] Arellano, M. and Meghir, C. (1992),"Female Labor Supply and On-the-Job Search: An Empirical Model Estimated using Complementary Data Sets", *Review of Economic Studies*, 59, 537-59.

[4] Becker, Gary S. (1965), 'A Theory of the Allocation of Time', *Economic Journal*, 75, 493-517.

[5] Becker, Gary S. (1974), 'A Theory of Marriage', *Journal of Political Economy*, 82, 511-26.

[6] Blundell, R. W., P.A. Chiappori , T. Magnac and C. Meghir (2000) "Collective Labor Supply and Household Production", mimeo November

[7] Blundell, R. W. and Walker, I. (1986), 'A Life Cycle Consistent Empirical Model of Labor Supply using Cross Section Data', *Review of Economic Studies*, 53, 539-58.

[8] Blundell, R.W., Duncan, A. and Meghir, C. (1998), 'Estimating Labor Supply Using Tax Policy Reforms', *Econometrica, July Vol 66.4 pp 827-61.*

[9] Boskin, Michael J and Sheshinski, Eytan (1983), 'Optimal Tax Treatment of the Family', *Journal of Pubic Economics*, 20, 281-297.

[10] Browning. M., Bourguignon, F., Chiappori, P.-A. and Lechene, V.,'Incomes and Outcomes: A Structural Model and Some Evidence from French Data', *Journal of Political Economy*, 102, 1067-96.

[11] Browning M. and P.-A. Chiappori (1997), " Efficient Intra-Household Allocations: a General Characterization and Empirical Tests", *Econometrica,*

[12] Browning, M. and C. Meghir (1991), Testing for Separability between Goods and Leisure using conditional Demand Systems, *Econometrica*, 59, July, 925-52.

[13] Chiappori, P.-A. (1988), 'Rational Household Labor Supply', *Econometrica*, 56, 63-89.

[14] Chiappori, P.-A. (1992), 'Collective Labor Supply and Welfare', *Journal of Political Economy*, 100, 437-67.

[15] Chiappori, P.-A. (1997), "Introducing Household Production in Collective Models of Labor Supply, *Journal of Political Economy*, 105, 191–209.

[16] Chiappori, P.A., R., Blundell and C., Meghir (2001), "Collective Labor Supply with Children", mimeo.

[17] Chiappori, P.-A. , and I. Ekeland (1998): "Collective models of household behavior: a mathematical characterization', Mimeo, University of Chicago

[18] Chiappori, P.A., B. Fortin and G. Lacroix (2002), "Household Labor Supply, the Sharing Rule and the Marriage Market", *Journal of Political Economy,*

[19] Donni, O. (2000), *Essais sur les modèles collectifs de comportement du ménage*, PhD dissertation, EHESS, Paris.

[20] Fortin, Bernard and Lacroix, Guy (1997), A Test of Neoclassical and Collective Models of Household Labor Supply, *Economic Journal*, 107, 933-955.

[21] Gorman, W.M. (1968),The Structure of Utility Functions, *Review of Economic Studies*, 32, 369-390.

[22] Gosling, A, S. Machin and C. Meghir, (2000).'The changing distribution of male wages in the UK', *Review of Economic Studies*

[23] Kapteyn A. and P. Kooreman (1992) "Household Labour Supply: What kind of Data can tell us how many decision makers there are?", *European Economic Review*, 36:365-71.

[24] Kooreman, P. and Kapteyn, A. (1990),'On the Implementation of Some Game Theoretic Models of Household Labor Supply', *Journal of Human Resources*, 25, 584-98.

[25] Lundberg, S, (1988),'Labor Supply of Husbands and Wives', *Review of Economics and Statistics*, 70, 224-35.

[26] MacLeod, B., and J. Malcomson (1993), "Investments, Holdup, and the Form of Market Contracts", *American Economic Review*, 83, 811-837.

[27] Manser, M. and Brown, M, (1980),'Marriage and Household Decision Making', International Economic Review, 21, 31-44.

[28] Mazzocco, M., (2001), "Household Intertemporal Behavior: a Collective Characterization and Empirical Tests", mimeo.

[29] Neary, J.P. and Roberts, K.W.S. (1980),'The Theory of Household Behaviour under Rationing' *European Economic Review*, 13, 24-35.

[30] Thomas, D. (1990),'Intra-household Resource Allocation: An Inferential Approach', *Journal of Human Resources*, 25, 635-664.

[31] Zhang, J., and K. Fong (2001), 'The Identification of Unobservable Independent and Spousal Leisure', *Journal of Political Economy*, 109(1), 191-202.