II. Self-Selection and Signalling

II.A Roy: Self-selection (B296-299)

Elements

- Two sectors ($i = 1, 2$) with output prices π_i. A person is employed in one or the other sector—people must specialize in production.
- A given person has productivity or skill-level in sector i of S_i. So $S = (S_1, \S_2)$ describes a person's situation. We assume $S_1 \geq 0$ and $S_2 \geq 0$ These are sector-specific skills. They can also be thought of a coming from a set of underlying skills used in both sectors—such as arithmetic, manual dexterity, kissing-up to the boss, etc.—but that are used with different intensity in each sector. For example, manual dexterity is more important than blue-collar jobs, so holding all else constant a person endowed with more dexterity has a greater value of blue-collar skill than someone with less.
- A person earns what they produce. With skill S_i a person would earn $W_i = \pi_i S_i$ if employed in sector i.
- Define $\pi = \frac{\pi_1}{\pi_2}$. Then the set of skill pairs that are indifferent between sector is described by the combinations such that $W_1 = W_2$ or $S_2 = \pi S_1$.

We compare and contrast two selection rules

- Random Assignment (more accurately called independent assignment): a person is assigned to sector i with probability p_i independent of their skills (S_1, S_2). If assigned to i they are paid W_i. Obviously $p_2 = 1 - p_1$.
- Self-selection (or pursuit of comparative advantage): a person chooses or is assigned the sector in which they earn the most: $i^\star(S_1, S_2) = \arg\max_{i=\{1,2\}} \pi_i S_i$ and they are paid $W^\star = \pi_{i^\star} S_{i^\star}$

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We also compare and contrast three distributions of skills across people. In each case, skill S_1 is uniformly distributed between 0 and some number U across people. Skill S_2 is also uniformly distributed between 0 and the same U. For the statistically inclined student, we say that the *marginal density* of skill is i takes the form

$$f_i(S) = \begin{cases} 1/U & \text{if } 0 \le S \le U \\ 0 & \text{otherwise} \end{cases}.$$

This simply means that the skills within sectors is evenly spread over $[0, U]$ in the population. The difference in the three situations is how S_1 and S_2 are correlated across people. That is, how ability in the two sectors move together across individuals in the population. Again, statistically, the different worlds are different assumptions of the *joint density* of skills, denoted $f(S_1, S_2)$.

The three worlds we consider are:

- Life is Unfair (LIU): $S_2 = S_1$ for all people. There is no comparative advantage. A person who is better at sector 1 than someone else is also better than in sector 2. The joint density of skills in this case

$$f^{LIU}(S_1, S_2) = \begin{cases} 1/U & \text{if } S_2 = S_1 \text{ and } 0 \le S_1 \le U. \\ 0 & \text{otherwise.} \end{cases}$$

  The statisticians will realize that the correlation between S_1 and S_2 is $+1$.

- Every has a special talent (EHT): $S_2 = 1 - S_1$. Now there is nothing but comparative advantage. A person better in sector 1 than someone else is necessarily worse in sector 2. The joint density of skills in this case

$$f^{EHT}(S_1, S_2) = \begin{cases} 1/U & \text{if } S_2 = 1 - S_1 \text{ and } 0 \le S_1 \le U. \\ 0 & \text{otherwise.} \end{cases}$$

  The statisticians will realize that when everyone has a special talent the correlation between S_1 and S_2 is -1.

- Table top talent (TTT): S_2 and S_1 are independently distributed. The independence property is equivalent to assuming the joint density factors into the marginals:

$$f^{TTT}(S_1, S_2) = f_1(S_1)f_2(S_2) = \begin{cases} 1/U^2 & \text{if } 0 \le S_1 \le U \text{ and } 0 \le S_2 \le U \\ 0 & \text{otherwise.} \end{cases}$$

Here the correlation is 0.

The Roy model says something about how the allocation of skill and the distribution of skills maps into the distribution of wages (of earnings). We have described six situations that cover the extremes: 2 assignment rules $\times$ 3 skill correlations. (For completeness we could include the very extreme world in which people are assigned to the sector they are least able in.) Let $g^c_a(w)$ denote the distribution of wages under correlation c and assignment rule a. For example, $g^{LIU}_r(w)$ is the distribution of wages in a world where life is unfair and sectors are randomly assigned, and $g^{EHT}_s(W)$ is the distribution when everybody has talent and assignment is based on self-selection.

Wage Distributions under Random Assignment when $\pi_1 = \pi_2 = \pi = 1$.

If a person is randomly assigned to sector, then i and skills (S_1, S_2) are independently distributed. The joint density of independent events is the product of the marginal densities. But since skills are sector-specific, only the density in the assigned sector affect wages. And when $\pi = 1$ wages within a sector are the same as skills within the sector. That is, when assigned $i = 1$ we can substitute a W and S_1. Thus,

$$g^c_r(W) = \frac{1}{2}f^c_1(W) + \frac{1}{2}f^c(W) = \begin{cases} 1/U & \text{if } 0 \leq W \leq U \\ 0 & \text{otherwise} \end{cases}.$$

Since all three skill distributions have uniform skills with sector the wage distribution is also uniformly distributed over $[0, U]$. (This depends on $\pi_1 = \pi_2 = \pi = 1$).

Wage Distributions under Self-selection when $\pi_1 = \pi_2 = \pi = 1$.

Now we can't use formulas that imply that sector assignment and skills are independent of each other. People pursue comparative advantage, meaning that assignment is anything but independent of skills.

- LIU: When $S_2 = S_1$ and $\pi_1 = \pi_2$ sector doesn't matter. Everyone is indifferent between being in $i = 1$ and $i = 2$. So once again

$$g_s^{LIU}(W) = f_1^{LIU}(W) = f_2^{LIU}(W) = \begin{cases} 1/U & \text{if } 0 \leq W \leq U \\ 0 & \text{otherwise} \end{cases}.$$

- EHT: At last things get interesting. Now people chose the sector they are best in. Since $S_2 = U - S_1$ they chose based on the maximum of S_1 and $1 - S_1$. Thus people with $S_1 \geq U/2$ earn S_1 and locate in $i = 1$. For $S_1 < U/2$ they earn $S_2 = U - S_1$ and chose $i = 2$. The lowest wage in either sector is $U/2$. Within sectors skils (and thus wages) are uniformly distributed, but now wages are only distributed in the interval $[U/2, U]$.

$$\begin{aligned} g_s^{EHT}(w) &= \begin{cases} f_1^{EHT}(W) + f_2^{EHT}(W) & \text{if } U/2 \leq W \leq U \\ 0 & \text{otherwise} \end{cases} \\ &= \begin{cases} 2/U & \text{if } U/2 \leq W \leq U \\ 0 & \text{otherwise} \end{cases}. \end{aligned}$$

  Pursuit of comparative advantage makes the distribution of earnings more equal than the distribution of talent.

- TTT: It's not obvious how to show this without deriving the cumulative density of wages:

$$\begin{aligned} G_s^{TTT}(W) &= Prob(Wage \leq W) \\ &= Prob(\max S_1, S_2 \leq W) \\ &= Prob(S_1 \leq W \text{ AND } S_2 \leq W) \\ &= Prob(S_1 \leq W)Prob(S_2 \leq W) = \frac{W}{U^2} \end{aligned}.$$

  for $0 \leq W \leq U$. The second-to-last step is where the independence of the table-top distribution comes into play (joint probability of independent events is the product of the marginal probabilities). If you follow that, then just take the derivative of $G_s^{TTT}(W)$ to get the density of wages:

$$g_s^{TTT}(w) = \frac{\partial G_s^{TTT}(W)}{\partial W} = 2W/U^2.$$

  Now the wage distribution doesn't look like the marginal skill distributions at all. It turns out that wages are less equal than under random assignment, but we leave it to Econ 361 to define the measures of inequality that lead to this result.

Wage Distributions under Self-selection when $\pi_1 = \pi < \pi_2 = 1$.

Now we look at how a difference in relative prices leads to different wage distributions under self-selection.

- LIU: When $S_2 = S_1$ and $\pi < 1$ sector matters. Everyone migrates to sector 2. But it has no affect on wages, because skills in sector 2 are uniformly distributed. So $g_s^{LIU}(W) = 1/U$ still.

- EHT: With equal prices there is one type of person indifferent between sector, those with $S_1 = S_2 = U - S_1 = U/2$. When $\pi < 1$ the indifference point is $\pi S_1 = S_2 = U - S_1$, so $S_1 = U/(1+\pi) > 1/2$. That person earns $\pi U/(1+\pi)$ in either sector, so the lower bound on wages is $\pi U/(1+\pi) < U/2$. Spreading out from that point there are equal uniform people at each wage. However, it takes a little statistics to work out that the density is $(1+\pi)/(\pi U)$. (Try working at $G_s^{EHT}(W)$ and then take its derivative.) The maximum wage in sector 1 is now πU, while the maximum wage in sector 2 is still U. So in the range $[\pi U, U]$ the density falls back to $1/U$. Thus for $\pi < 1$ we get

$$g_s^{LIU}(W) = \begin{cases} \frac{(1+\pi)}{\pi U} & \text{if } \frac{\pi U}{1+\pi} \leq W \leq \pi U \\ 1/U & \text{if } \pi U < W \leq U. \end{cases}$$

  As π falls the wage distribution approaches the uniform case, which means it *increases* inequality when there is no absolute advantage.

- TTT:

$$\begin{aligned} G_s^{TTT}(W) = Prob(Wage \leq W) &= Prob(\max \pi S_1, S_2 \leq W) \\ &= Prob(\pi S_1 \leq W \text{ AND } S_2 \leq W) \\ &= Prob(S_1 \leq W/\pi) Prob(S_2 \leq W) \\ &= \begin{cases} \frac{W^2}{\pi U^2} & \text{if } 0 \leq W \leq \pi U \\ \frac{W}{U} & \text{if } \pi U < W \leq U \end{cases} \end{aligned}$$

  Taking the derivative:

$$g_s^{TTT}(w) = \frac{\partial G_s^{TTT}(W)}{\partial W} = \begin{cases} \frac{2W}{\pi U^2}. & \text{if } 0 \leq W \leq \pi U \\ \frac{1}{U} & \text{if } \pi U < W \leq U \end{cases}$$

As π falls the wage distribution approaches the uniform case, which means it *decreases* inequality.

The conclusion: relative prices, assignment mechanisms, and skill distributions are all tangled up together. The same distribution of wages may be associated with quite different responses to price changes.

II.B Self-selection bias in the HCEF (B238-240)

II.C Griliches: Ability Bias Estimating Returns to Education

Correcting Ability Bias The early attempts to correct for ability bias used measures of ability, such as scores on IQ or aptitude tests in the regression as well:

$$ln(w) = \beta_0 + \beta_1 S + \beta_2 X + \beta_3 X^2 + \beta_4 AbilityMeasure + u \tag{16}$$

The typical result was that estimates of this kind of equation lead to *lower* estimated values β_1 because schooling (S) was partly picking up the effect of ability when ability was omitted from the regression equation. There are some serious statistical problems with these types of estimates. In particular, if the ability measure has a lot of noise in it, then the estimates will be biased by that as well.

But for many years the concensus was that β_1 is slightly over-estimated using linear regression.

The next attempts were to try to control for ability in some other way. For example, consider two people (person A and person B) who we somehow know ahead of time have the same ability, call it F (for reasons explained below). This term F must capture all aspects of ability that help determine education levels that also explain wages. There can other post-education factors that help explain wages as well (such as luck finding a job). Now write Mincer's equation out for those two people:

$$ln(w_A) = \beta_0 + \beta_1 S_A + \beta_2 X_A + \beta_3 X_A^2 + F + e_A \tag{17}$$