where the matrix $\boldsymbol{M}_D$ is simply the matrix that takes deviations from the group means $\bar{\boldsymbol{X}}_{.i}$ for $i = 1, \ldots, n$. Thus a typical element of $\boldsymbol{M}_D\boldsymbol{X}$ is

$$(\boldsymbol{M}_D\boldsymbol{X})_{ti} = \boldsymbol{X}_{ti} - \bar{\boldsymbol{X}}_{.i}.$$

This makes it easy to compute $\acute{\boldsymbol{\beta}}$ even when n is so large that it would be infeasible to run regression (9.74). One simply has to compute the group means $y_{.i}$ and $\boldsymbol{X}_{.i}$ for all i and then regress $y_{ti} - \bar{y}_{.i}$ on $\boldsymbol{X}_{ti} - \bar{\boldsymbol{X}}_{.i}$ for all t and i. The estimated covariance matrix should then be adjusted to reflect the fact that the number of degrees of freedom used in estimation is actually $n + k$ rather than k.

Because the fixed-effects estimator (9.75) depends only on the deviations of the regressand and regressors from their respective group means, it is sometimes called the **within-groups estimator**. As this name implies, it makes no use of the fact that the group means are in general different for different groups. This property of the estimator can be an advantage or a disadvantage, depending on circumstances. As we mentioned above, it may well be the case that the cross-sectional effects v_i are correlated with the regressors $\boldsymbol{X}_{ti}$ and consequently also with the respective group means of the regressors. In that event the OLS estimator (without fixed effects) based on the full sample would be inconsistent, but the within-groups estimator would remain consistent. However, if on the contrary the fixed effects *are* independent of the regressors, the within-groups estimator is not fully efficient. In the extreme case in which any one of the independent variables does not vary at all within groups, but only between groups, then the coefficient corresponding to that variable will not even be identifiable by the within-groups estimator.

An alternative inefficient estimator that uses only the variation among the group means is called the **between-groups estimator**. It may be written as

$$\grave{\boldsymbol{\beta}} = \big(\boldsymbol{X}^\top\boldsymbol{P}_D\boldsymbol{X}\big)^{-1}\boldsymbol{X}^\top\boldsymbol{P}_D\boldsymbol{y}. \tag{9.76}$$

Since $\boldsymbol{P}_D\boldsymbol{X}_{ti} = \bar{\boldsymbol{X}}_{.i}$, this estimator really involves only n distinct observations rather than nT. It will clearly be inconsistent if the cross-sectional effects, the v_i's, are correlated with the group means of the regressors, the $\bar{\boldsymbol{X}}_{.i}$'s. The OLS estimator can be written as a matrix-weighted average of the within-groups and between-groups estimators:

$$\begin{aligned}
\hat{\boldsymbol{\beta}} &= \big(\boldsymbol{X}^\top\boldsymbol{X}\big)^{-1}\boldsymbol{X}^\top\boldsymbol{y}\\
&= \big(\boldsymbol{X}^\top\boldsymbol{X}\big)^{-1}\big(\boldsymbol{X}^\top\boldsymbol{M}_D\boldsymbol{y} + \boldsymbol{X}^\top\boldsymbol{P}_D\boldsymbol{y}\big)\\
&= \big(\boldsymbol{X}^\top\boldsymbol{X}\big)^{-1}\boldsymbol{X}^\top\boldsymbol{M}_D\boldsymbol{X}\acute{\boldsymbol{\beta}} + \big(\boldsymbol{X}^\top\boldsymbol{X}\big)^{-1}\boldsymbol{X}^\top\boldsymbol{P}_D\boldsymbol{X}\grave{\boldsymbol{\beta}}.
\end{aligned}$$

Thus we see immediately that OLS will be inconsistent whenever the between-groups estimator (9.76) is inconsistent.