One simply has to interpret the test columns in the regression as empirical moments.

An interesting variant of the test regression (16.62) was suggested by Tauchen (1985). In effect, he interchanged the regressand $\boldsymbol{\iota}$ and the test regressor $\hat{\boldsymbol{m}}$ so as to obtain the regression

$$\hat{\boldsymbol{m}} = \hat{\boldsymbol{G}}\boldsymbol{c}^* + b^*\boldsymbol{\iota} + \text{residuals}. \tag{16.63}$$

The test statistic is the ordinary t statistic for $b^* = 0$. It is numerically identical to the t statistic on b in (16.62). This fact follows from a result we obtained in section 12.7, of which we now give a different, geometrical, proof. Apply the FWL Theorem to both (16.62) and (16.63) so as to obtain the two regressions

$$\begin{aligned} \hat{\boldsymbol{M}}_{\boldsymbol{G}}\boldsymbol{\iota} &= b(\hat{\boldsymbol{M}}_{\boldsymbol{G}}\hat{\boldsymbol{m}}) + \text{residuals} \quad \text{and} \\ \hat{\boldsymbol{M}}_{\boldsymbol{G}}\hat{\boldsymbol{m}} &= b^*(\hat{\boldsymbol{M}}_{\boldsymbol{G}}\boldsymbol{\iota}) + \text{residuals}. \end{aligned} \tag{16.64}$$

These are both univariate regressions with n observations. The single t statistic from each of them is given by the product of the same scalar factor, $(n-1)^{1/2}$, and the cotangent of the angle between the regressand and the regressor (see Appendix A). Since this angle is unchanged when the regressor and regressand are interchanged, so is the t statistic. The FWL Theorem implies that the t statistics from the first and second rows of (16.64) are equal to those from the OPG regression (16.62) and Tauchen's regression (16.63), respectively, times the same degrees of freedom correction. Thus we conclude that the t statistics based on the latter two regressions are numerically identical.

Since the first-order conditions for $\hat{\boldsymbol{\theta}}$ imply that $\boldsymbol{\iota}$ is orthogonal to all of the columns of $\hat{\boldsymbol{G}}$, the OLS estimate of b^* in (16.63) will be equal to the sample mean of the elements of $\hat{\boldsymbol{m}}$. This would be so even if the regressors $\hat{\boldsymbol{G}}$ were omitted from the regression. However, because $\boldsymbol{\theta}$ has been estimated, those regressors must be included if we are to obtain a valid estimate of the variance of the sample mean. As is the case with all the other artificial regressions we have studied, omitting the regressors that correspond to parameters estimated under the null hypothesis results in a test statistic that is too small, asymptotically.

Let us reiterate our earlier warnings about the OPG regression. As we stressed when we introduced it in Section 13.7, test statistics based on it often have poor finite-sample properties. They tend to reject the null hypothesis too often when it is true. This is just as true for CM tests as for LM tests or $C(\alpha)$ tests. If possible, one should therefore use alternative tests that have better finite-sample properties, such as tests based on the GNR, the HRGNR, the DLR (Section 14.4), or the BRMR (Section 15.4), when these procedures are applicable. Of course, they will be applicable in general only if the CM test can be reformulated as an ordinary test, with an explicit alternative