they are related to y_t^* and z_t^* as follows:

$$y_t = y_t^* \text{ if } z_t^* > 0; \;\; y_t = 0 \text{ otherwise};$$
$$z_t = 1 \text{ if } z_t^* > 0; \;\; z_t = 0 \text{ otherwise}.$$

There are two types of observations: ones for which both y_t and z_t are observed to be zero and ones for which $z_t = 1$ and y_t is equal to y_t^*. The loglikelihood function for this model is thus

$$\sum_{z_t=0} \log\bigl(\Pr(z_t = 0)\bigr) + \sum_{z_t=1} \log\bigl(\Pr(z_t = 1) f(y_t^* \,|\, z_t = 1)\bigr), \tag{15.54}$$

where $f(y_t^* \,|\, z_t = 1)$ denotes the density of y_t^* conditional on $z_t = 1$. The first term of (15.54) is the summation over all observations for which $z_t = 0$ of the logarithms of the probability that $z_t = 0$. It is exactly the same as the corresponding term in a probit model for z_t by itself. The second term is the summation over all observations for which $z_t = 1$ of the probability that $z_t = 1$ times the density of y_t conditional on $z_t = 1$. Using the fact that we can factor a joint density any way we like, this second term can also be written as

$$\sum_{z_t=1} \log\bigl(\Pr(z_t = 1 \,|\, y_t^*) f(y_t^*)\bigr),$$

where $f(y_t^*)$ is the unconditional density of y_t^*, which is just a normal density with conditional mean $\boldsymbol{X}_t\boldsymbol{\beta}$ and variance σ^2.

The only difficulty in writing out the loglikelihood function (15.54) explicitly is to calculate $\Pr(z_t = 1 \,|\, y_t^*)$. Since u_t and v_t are bivariate normal, we can write

$$z_t^* = \boldsymbol{W}_t\boldsymbol{\gamma} + \rho\Bigl(\frac{1}{\sigma}\bigl(y_t^* - \boldsymbol{X}_t\boldsymbol{\beta}\bigr)\Bigr) + \varepsilon_t, \quad \varepsilon_t \sim \mathrm{NID}\bigl(0, (1-\rho^2)\bigr).$$

It follows that

$$\Pr(z_t = 1 \,|\, y_t^*) = \Phi\left(\frac{\boldsymbol{W}_t\boldsymbol{\gamma} + \rho\bigl((y_t - \boldsymbol{X}_t\boldsymbol{\beta})/\sigma\bigr)}{(1-\rho^2)^{1/2}}\right),$$

since $y_t = y_t^*$ when $z_t = 1$. Thus the loglikelihood function (15.54) becomes

$$\begin{aligned} &\sum_{z_t=0} \log\bigl(\Phi(-\boldsymbol{W}_t\boldsymbol{\gamma})\bigr) + \sum_{z_t=1} \log\Bigl(\frac{1}{\sigma}\phi\bigl((y_t - \boldsymbol{X}_t\boldsymbol{\beta})/\sigma\bigr)\Bigr) \\ &+ \sum_{z_t=1} \log\left(\Phi\left(\frac{\boldsymbol{W}_t\boldsymbol{\gamma} + \rho\bigl((y_t - \boldsymbol{X}_t\boldsymbol{\beta})/\sigma\bigr)}{(1-\rho^2)^{1/2}}\right)\right). \end{aligned} \tag{15.55}$$

The first term looks like the corresponding term for a probit model. The