to the one for testing against AR(q) errors. Perhaps more surprisingly, the same artificial regression also turns out to be appropriate for testing against ARMA(p, q) errors, with $\max(p, q)$ lags of $\tilde{\boldsymbol{u}}$ now being included in the regression. For more details, see Godfrey (1978b, 1988).

Using something very like the Gauss-Newton regression to test for serial correlation was first suggested by Durbin (1970) in a paper that also introduced what has become known as **Durbin's $\boldsymbol{h}$ test**. The latter procedure, which we will not discuss in detail, is an asymptotic test for AR(1) errors that can be used when the null hypothesis is a linear regression model which includes the dependent variable lagged once, and possibly more than once as well, among the regressors. The h test can be calculated with a hand calculator from the output for the original regression printed by most regression packages, although in some cases it cannot be calculated at all because it would be necessary to compute the square root of a negative number. For reasons that today seem hard to understand (but are presumably related to the primitive state of computer hardware and econometric software in the early 1970s), Durbin's h test became widely used, while his so-called **alternative procedure**, a t test based on the modified GNR (10.77), was all but ignored for quite some time.[8] It was finally rediscovered and extended by Breusch (1978) and Godfrey (1978a, 1978b). All of these papers assumed that the error terms ε_t were normally distributed, and they developed tests based on the GNR as Lagrange multiplier tests based on maximum likelihood estimation. The normality assumption is of course completely unnecessary.

Equally unnecessary is any assumption about the presence or absence of lagged dependent variables in the regression function $x_t(\boldsymbol{\beta})$. All we require is that this function satisfy the regularity conditions of Chapter 5, in order that nonlinear least squares estimates will be consistent and asymptotically normal under both the null and alternative hypotheses. As the above history implies, and as we will discuss below, many tests for serial correlation require that $x_t(\boldsymbol{\beta})$ not depend on lagged dependent variables, and all of the literature cited in the previous paragraph was written with the specific aim of handling the case in which $x_t(\boldsymbol{\beta})$ is linear and depends on one or more lagged values of the dependent variable.

The problem with tests based on the GNR is that they are valid only asymptotically. This is true whether or not $x_t(\boldsymbol{\beta})$ is linear, because $\tilde{\boldsymbol{u}}_{-1}$ is only an estimate of $\boldsymbol{u}_{-1}$. Indeed, as we saw in Section 5.6, $\tilde{\boldsymbol{u}} \overset{a}{=} \boldsymbol{M}_0\boldsymbol{u}$, where $\boldsymbol{M}_0 \equiv \mathbf{I} - \boldsymbol{X}_0(\boldsymbol{X}_0^\top\boldsymbol{X}_0)^{-1}\boldsymbol{X}_0^\top$ and $\boldsymbol{X}_0 \equiv \boldsymbol{X}(\boldsymbol{\beta}_0)$. This is just the asymptotic equality (5.57). The asymptotic equality is replaced by an exact equality if $\boldsymbol{x}(\boldsymbol{\beta}) = \boldsymbol{X}\boldsymbol{\beta}$.

[8] Maddala and Rao (1973), Spencer (1975), and Inder (1984), among others, have provided Monte Carlo evidence on Durbin's h test as compared with the test based on the GNR. This evidence does not suggest any strong reason to prefer one test over the other. Thus the greater convenience and more general applicability of the test based on the GNR are probably the main factors in its favor.