the ML estimates $\hat{\boldsymbol{\beta}}$, the estimated error variance for it, $\hat{s}^2$, will be equal to

$$
\begin{aligned}
&\frac{1}{mn-k}\sum_{t=1}^{n}(\boldsymbol{Y}_t-\hat{\boldsymbol{\xi}}_t)\hat{\boldsymbol{\psi}}\hat{\boldsymbol{\psi}}^{\top}(\boldsymbol{Y}_t-\hat{\boldsymbol{\xi}}_t)^{\top}\\
&=\frac{1}{mn-k}\sum_{t=1}^{n}(\boldsymbol{Y}_t-\hat{\boldsymbol{\xi}}_t)\hat{\boldsymbol{\Sigma}}^{-1}(\boldsymbol{Y}_t-\hat{\boldsymbol{\xi}}_t)^{\top}=\frac{mn}{mn-k}.
\end{aligned}
\tag{9.70}
$$

The last equality here follows from an argument almost identical to the one used to establish (9.65). Since it is evident that (9.70) tends asymptotically to 1, expression (9.61), which is in this case

$$
\frac{mn}{mn-k}\left(\sum_{t=1}^{n}\hat{\boldsymbol{\Xi}}_t\hat{\boldsymbol{\Sigma}}^{-1}\hat{\boldsymbol{\Xi}}_t^{\top}\right)^{-1},
$$

provides a natural and very convenient way to estimate the covariance matrix of $\hat{\boldsymbol{\beta}}$.

We have now established all the principal results of interest concerning the estimation of multivariate nonlinear regression models. Since those results have been in terms of a rather general and abstract model, it may help to make them more concrete if we indicate precisely how our general notation relates to the case of the linear expenditure system that we discussed earlier. For concreteness, we will assume that $m = 2$, which means that there is a total of three commodities. Then we see that

$$
\begin{aligned}
\boldsymbol{Y}_t &= [s_{t1}\quad s_{t2}];\\
\boldsymbol{\beta} &= [\alpha_1 \,\vdots\, \alpha_2 \,\vdots\, \gamma_1 \,\vdots\, \gamma_2 \,\vdots\, \gamma_3];
\end{aligned}
$$

$$
\boldsymbol{\xi}_t(\boldsymbol{\beta}) = \left[\frac{\gamma_1 p_{1t}}{E_t}+\frac{\alpha_1}{E_t}\left(E_t-\sum_{j=1}^{3}p_{jt}\gamma_j\right)\qquad \frac{\gamma_2 p_{2t}}{E_t}+\frac{\alpha_2}{E_t}\left(E_t-\sum_{j=1}^{3}p_{jt}\gamma_j\right)\right];
$$

$$
\boldsymbol{\Xi}_t(\boldsymbol{\beta}) = \begin{bmatrix}
\left(E_t-\sum_{j=1}^{3}p_{jt}\gamma_j\right)/E_t & 0\\
0 & \left(E_t-\sum_{j=1}^{3}p_{jt}\gamma_j\right)/E_t\\
(1-\alpha_1)p_{1t}/E_t & -\alpha_2 p_{1t}/E_t\\
-\alpha_1 p_{2t}/E_t & (1-\alpha_2)p_{2t}/E_t\\
-\alpha_1 p_{3t}/E_t & -\alpha_2 p_{3t}/E_t
\end{bmatrix}.
$$

It may be a useful exercise to set up the GNR for testing the hypothesis that $\gamma_1 = \gamma_2 = \gamma_3 = 0$, where estimates subject to that restriction have been obtained.

Our treatment of multivariate models has been relatively brief. A much fuller treatment, but only for linear SUR models, may be found in Srivastava and Giles (1987), which is also an excellent source for references to the econometric and statistical literature on the subject.