A comparison of (4.17) and (4.18) reveals that the behavior of the estimator $\hat{\alpha}$ is quite different under the two different rules for sample-size extension.

There is not always a simple resolution to the sort of problem posed in the above example. It is *usually* unrealistic to assume that linear time trends of the form of $\boldsymbol{\tau}$ will continue to increase forever, but it suffices to look at price series in the twentieth century (and many other centuries) to realize that some economic variables do not seem to have natural upper bounds. Even quantity series such as real GNP or personal consumption are sometimes fruitfully considered as being unbounded. Nevertheless, although the asymptotic theories resulting from different kinds of rules for extending DGPs to arbitrarily large samples can be very different, it is important to be clear that deciding among competing asymptotic theories of this sort is *not* an empirical issue. For any given empirical investigation, the sample size is what it is, even if the *possibility* of collecting further relevant data exists. The issue is always one of selecting a suitable *model*, not only for the data that exist, but for a set of economic *phenomena*, of which the data are supposed to be a manifestation. There is always an infinity of models (not all plausible of course) that are compatible with any finite data set. As a consequence, the issue of model selection among a set of such models can be decided only on the basis of such criteria as the explanatory power of the concepts used in the model, simplicity of expression, or ease of interpretation, but not on the basis of the information contained in the data themselves.

Although, in the model (4.14), the assumption that the time trend variable goes to infinity with the sample size may seem more plausible than the fixed-in-repeated-samples assumption, we will throughout most of this book assume that the DGP is of the latter rather than the former type. The problem with allowing τ_t to go to infinity with the sample size is that each additional observation gives us more information about the value of α than any of the preceding observations. That is why $\mathrm{Var}(\hat{\alpha})$ turned out to be $O(n^{-3})$ when we made that assumption about the DGP. It seems much more plausible in most cases that each additional observation should, on average, give us the same amount of information as the preceding observations. This implies that the variance of parameter estimates will be $O(n^{-1})$, as was $\mathrm{Var}(\hat{\alpha})$ when we assumed that the DGP was of the fixed-in-repeated-samples type. Our general assumptions about DGPs will likewise lead to the conclusion that the variance of parameter estimates is $O(n^{-1})$, although we will consider DGPs that do not lead to this conclusion in Chapter 20, which deals with dynamic models.

4.5 Consistency and Laws of Large Numbers

We begin this section by introducing the notion of **consistency**, one of the most basic ideas of asymptotic theory. When one is interested in estimating parameters from data, it is desirable that the parameter estimates should have certain properties. In Chapters 2 and 3, we saw that, under certain regularity