Chapter 3

Inference in Nonlinear Regression Models

3.1 Introduction

Suppose that one is given a vector $\boldsymbol{y}$ of observations on some dependent variable, a vector $\boldsymbol{x}(\boldsymbol{\beta})$ of, in general nonlinear, regression functions, which may and normally will depend on independent variables, and the data needed to evaluate $\boldsymbol{x}(\boldsymbol{\beta})$. Then, assuming that these data allow one to identify all elements of the parameter vector $\boldsymbol{\beta}$ and that one has access to a suitable computer program for nonlinear least squares and enough computer time, one can always obtain NLS estimates $\hat{\boldsymbol{\beta}}$. In order to interpret these estimates, one generally makes the heroic assumption that the model is "correct," which means that $\boldsymbol{y}$ is in fact generated by a DGP from the family

$$\boldsymbol{y} = \boldsymbol{x}(\boldsymbol{\beta}) + \boldsymbol{u}, \quad \boldsymbol{u} \sim \mathrm{IID}(\mathbf{0}, \sigma^2\mathbf{I}). \tag{3.01}$$

Without this assumption, or some less restrictive variant, it would be very difficult to say anything about the properties of $\hat{\boldsymbol{\beta}}$, although in certain special cases one can do so.

It is clear that $\hat{\boldsymbol{\beta}}$ must be a vector of random variables, since it will depend on $\boldsymbol{y}$ and hence on the vector of error terms $\boldsymbol{u}$. Thus, if we are to make inferences about $\boldsymbol{\beta}$, we must recognize that $\hat{\boldsymbol{\beta}}$ is random and quantify its randomness. In Chapter 5, we will demonstrate that it is reasonable, when the sample size is large enough, to treat $\hat{\boldsymbol{\beta}}$ as being normally distributed around the true value of $\boldsymbol{\beta}$, which we may call $\boldsymbol{\beta}_0$. Thus the only thing we need to know if we are to make asymptotically valid inferences about $\boldsymbol{\beta}$ is the **covariance matrix** of $\hat{\boldsymbol{\beta}}$, say $\boldsymbol{V}(\hat{\boldsymbol{\beta}})$. In the next section, we discuss how this covariance matrix may be estimated for linear and nonlinear regression models. In Section 3.3, we show how the resulting estimates may be used to make inferences about $\boldsymbol{\beta}$. In Section 3.4, we discuss the basic ideas that underlie all types of hypothesis testing. In Section 3.5, we then discuss procedures for testing hypotheses in linear regression models. In Section 3.6, we discuss similar procedures for testing hypotheses in nonlinear regression models. The latter section provides an opportunity to introduce the three