where $\boldsymbol{\Psi}^2 = \boldsymbol{\Phi}^{-1}$, and $\boldsymbol{M}_{\Psi D}$ is the $l \times l$ orthogonal projection matrix onto the orthogonal complement of the k columns of $\boldsymbol{\Psi D}$. By construction, the l–vector $n^{-1/2}\boldsymbol{\Psi F}_0^{\top}\boldsymbol{\iota}$ has the $N(\mathbf{0}, \mathbf{I})$ distribution asymptotically. It follows, then, that (17.68) is asymptotically distributed as chi-squared with number of degrees of freedom equal to the rank of $\boldsymbol{M}_{\Psi D}$, that is, $l - k$, the number of overidentifying restrictions.

Hansen's test of overidentifying restrictions is completely analogous, in the present more general context, to the one for IV estimation discussed in Section 7.8, based on the criterion function (7.56). It is a good exercise to work through the derivation given above for the simple case of a linear regression model with homoskedastic, serially uncorrelated errors, in order to see how closely the general case mimics the simple one.[2]

Hansen's test of overidentifying restrictions is perhaps as close as one can come in econometrics to a portmanteau specification test. Because models estimated by GMM are subject to so few restrictions, their "specification" is not very demanding. In particular, if nothing more is required than the existence of the moments used to identify the parameters, then only two things are left to test. One is the set of any overidentifying restrictions used, and the other is parameter constancy.[3] Because Hansen's test of overidentifying restrictions has as many degrees of freedom as there are overidentifying restrictions, it may be possible to achieve more power by reducing the number of degrees of freedom. However, if Hansen's test statistic is small enough numerically, no such test can reject, for the simple reason that Hansen's statistic provides an upper bound for all possible test statistics for which the null hypothesis is the estimated model. This last fact follows from the observation that no criterion function of the form (17.67) can be less than zero.

Tests for which the null hypothesis is not the estimated model are not subject to the bound provided by Hansen's statistic. This is just as well, of course, since otherwise it would be impossible to reject a just identified model at all. A test for parameter constancy is not subject to the bound either, although at first glance the null hypothesis would appear to be precisely the estimated model. The reason was discussed in Section 11.2 in connection with tests for parameter constancy in nonlinear regression models estimated by means of instrumental variables. Essentially, in order to avoid problems of identification, it is necessary to double the number of instruments used, by splitting the original ones up as in (11.09). Exactly the same considerations apply for GMM models, of course, especially those that are just identified or have few overidentifying restrictions. But if one uses twice as many instruments, the null model has effectively been changed, and for that reason,

[2] Hansen's test statistic, (17.68), is sometimes referred to as the J statistic. For obvious reasons (see Chapter 11) we prefer not to give it that name.

[3] Tests of parameter constancy in models estimated by GMM are discussed by Hoffman and Pagan (1989) and Ghysels and Hall (1990).