where $\hat{\boldsymbol{\beta}}$ denotes the NLS estimates of $\boldsymbol{\beta}$ for the whole sample. The GNR (11.04) may be written more compactly as

$$\hat{\boldsymbol{u}} = \hat{\boldsymbol{X}}\boldsymbol{b} + \boldsymbol{\delta}*\hat{\boldsymbol{X}}\boldsymbol{c} + \text{residuals}, \tag{11.05}$$

where $\hat{\boldsymbol{u}}$ has typical element $y_t - x_t(\hat{\boldsymbol{\beta}})$, and $\hat{\boldsymbol{X}}$ has typical element $\boldsymbol{X}_t(\hat{\boldsymbol{\beta}})$. Here $*$ denotes the **direct product** of two matrices. Since $\delta_t X_{ti}(\hat{\boldsymbol{\beta}})$ is a typical element of $\boldsymbol{\delta}*\hat{\boldsymbol{X}}$, $\delta_t * \hat{\boldsymbol{X}}_t = \hat{\boldsymbol{X}}_t$ when $\delta_t = 1$ and $\delta_t * \hat{\boldsymbol{X}}_t = \mathbf{0}$ when $\delta_t = 0$. To perform the test, we simply have to estimate the model using the entire sample and regress the residuals from that estimation on the matrix of derivatives $\hat{\boldsymbol{X}}$ and on that matrix with the rows which correspond to group 1 observations set to zero. We do not have to reorder the data. As usual, there are several asymptotically valid test statistics, the best probably being the ordinary F statistic for the null hypothesis that $\boldsymbol{c} = \mathbf{0}$. In the usual case with k less than $\min(n_1, n_2)$, that test statistic will have k degrees of freedom in the numerator and $n - 2k$ degrees of freedom in the denominator.

Notice that the sum of squared residuals from regression (11.05) is equal to the SSR from the GNR

$$\hat{\boldsymbol{u}} = \hat{\boldsymbol{X}}\boldsymbol{b} + \text{residuals} \tag{11.06}$$

run over observations 1 to n_1 plus the SSR from the same GNR run over observations $n_1 + 1$ to n. This is the unrestricted sum of squared residuals for the F test of $\boldsymbol{c} = \mathbf{0}$ in (11.05). The restricted sum of squared residuals for that test is simply the SSR from (11.06) run over all n observations, which is the same as the SSR from nonlinear estimation of the null hypothesis H_0. Thus the ordinary Chow test for the GNR (11.06) will be numerically identical to the F test of $\boldsymbol{c} = \mathbf{0}$ in (11.05). This provides the easiest way to calculate the test statistic.

As we mentioned above, the ordinary Chow test (11.03) is not applicable if $\min(n_1, n_2) < k$. Using the GNR framework, it is easy to see why this is so. Suppose that $n_2 < k$ and $n_1 > k$, without loss of generality, since the numbering of the two groups of observations is arbitrary. Then the matrix $\boldsymbol{\delta}*\hat{\boldsymbol{X}}$, which has k columns, will have $n_2 < k$ rows that are not just rows of zeros and hence will have rank at most n_2. Thus, when equation (11.05) is estimated, at most n_2 elements of $\boldsymbol{c}$ will be identifiable, and the residuals corresponding to all observations that belong to group 2 will be zero. The number of degrees of freedom for the numerator of the F statistic must therefore be at most n_2. In fact, it will be equal to the rank of $[\hat{\boldsymbol{X}} \quad \boldsymbol{\delta}*\hat{\boldsymbol{X}}]$ minus the rank of $\hat{\boldsymbol{X}}$, which might be less than n_2 in some cases. The number of degrees of freedom for the denominator will be the number of observations for which (11.05) has nonzero residuals, which will normally be n_1, minus the number of regressors that affect those observations, which will be k, for a total of $n_1 - k$. Thus we can use the GNR whether or not $\min(n_1, n_2) < k$, provided that we use the appropriate numbers of degrees of freedom for the numerator and denominator of the F test.