and k exogenous or predetermined variables. Then the model can be written in matrix form as

$$\boldsymbol{Y\Gamma} = \boldsymbol{XB} + \boldsymbol{U}. \tag{18.01}$$

Here $\boldsymbol{Y}$ denotes an $n \times g$ matrix of endogenous variables, $\boldsymbol{X}$ denotes an $n \times k$ matrix of exogenous or predetermined variables, $\boldsymbol{\Gamma}$ denotes a $g \times g$ matrix of coefficients, $\boldsymbol{B}$ denotes a $k \times g$ matrix of coefficients, and $\boldsymbol{U}$ denotes an $n \times g$ matrix of error terms.

It is at once clear that the model (18.01) contains too many coefficients to estimate. A typical observation for the l^{th} equation can be written as

$$\sum_{i=1}^{g} \Gamma_{il} Y_{ti} = \sum_{j=1}^{k} B_{jl} X_{tj} + u_{tl}.$$

Multiplying all of the Γ_{il}'s and B_{jl}'s by any nonzero constant would simply have the effect of multiplying u_{tl} by that same constant for all t, but would not change the pattern of the error terms across observations at all. Thus it is necessary to impose some sort of normalization on each of the equations of the model. The obvious one is to set $\Gamma_{ii} = 1$ for all i; each endogenous variable, y_1 through y_g, would then have a coefficient of unity in one and only one equation. However, as we saw in Section 7.3, many other normalizations could be used. We could, for example, set $\Gamma_{1l} = 1$ for all l; the coefficient on the first endogenous variable would then be unity in every equation.

The model (18.01) makes no sense if the matrix $\boldsymbol{\Gamma}$ cannot be inverted, since otherwise it would be impossible to determine $\boldsymbol{Y}$ uniquely as a function of $\boldsymbol{X}$ and $\boldsymbol{U}$. We may therefore postmultiply both sides of (18.01) by $\boldsymbol{\Gamma}^{-1}$ to obtain

$$\boldsymbol{Y} = \boldsymbol{XB\Gamma}^{-1} + \boldsymbol{U\Gamma}^{-1} \tag{18.02}$$

$$= \boldsymbol{X\Pi} + \boldsymbol{V}. \tag{18.03}$$

Expression (18.02) is the **restricted reduced form**, or **RRF**, and expression (18.03) is the **unrestricted reduced form**, or **URF**. The restrictions are that $\boldsymbol{\Pi} = \boldsymbol{B\Gamma}^{-1}$. Notice that, even in the unlikely event that the columns of $\boldsymbol{U}$ were independent, the columns of $\boldsymbol{V}$ would not be. Thus the various equations of the reduced form are almost certain to have correlated errors.

The imposition of normalization restrictions is necessary but not sufficient to obtain estimates of $\boldsymbol{\Gamma}$ and $\boldsymbol{B}$. The problem is that, unless we impose some restrictions on it, the model (18.01) has too many coefficients to estimate. The matrix $\boldsymbol{\Gamma}$ contains $g^2 - g$ coefficients, because of the g normalization restrictions, while the matrix $\boldsymbol{B}$ contains gk. There are thus $g^2 + gk - g$ structural coefficients in total. But the matrix $\boldsymbol{\Pi}$ in the unrestricted reduced form contains only gk coefficients. It is obviously impossible to determine the $g^2 + gk - g$ structural coefficients uniquely from the gk coefficients of the