the reported covariance matrix from the Cochrane-Orcutt and Hildreth-Lu procedures will thus be invalid in many cases. One either has to calculate the GNR (10.29) oneself or use nonlinear least squares from the beginning so that the regression package will do so.

When the conditional covariance matrix estimate is invalid, reported standard errors are always too small (asymptotically). In fact, the covariance matrix estimate produced by the GNR (10.29) for the estimates of $\boldsymbol{\beta}$ differs from that produced by (10.27) by a positive definite matrix, if we ignore the fact that the degrees of freedom are different. To see this, notice that the Gauss-Newton regression (10.29) has the same regressors as (10.27), plus one additional regressor, $\hat{\boldsymbol{u}}_{-1}$. If we apply the FWL Theorem to (10.29), we see that the covariance matrix estimate from it is the same as that from a regression in which all the variables are projected onto the orthogonal complement of $\hat{\boldsymbol{u}}_{-1}$. The residuals are unchanged by the projection and so are identical to those of (10.27), as we saw above. The difference between the covariance matrix estimates for $\hat{\boldsymbol{\beta}}$ from (10.29) and (10.27) is therefore proportional to

$$\big(\boldsymbol{X}^{*\top}(\hat{\rho})\boldsymbol{M}_{\hat{u}_{-1}}\boldsymbol{X}^{*}(\hat{\rho})\big)^{-1} - \big(\boldsymbol{X}^{*\top}(\hat{\rho})\boldsymbol{X}^{*}(\hat{\rho})\big)^{-1}, \tag{10.32}$$

except for an asymptotically negligible effect due to the different degrees-of-freedom factors. If we subtract the inverses of the two matrices in (10.32) in the opposite order, we obtain

$$\boldsymbol{X}^{*\top}(\hat{\rho})\boldsymbol{P}_{\hat{u}_{-1}}\boldsymbol{X}^{*}(\hat{\rho}),$$

which is evidently positive semidefinite. It then follows from a result proved in Appendix A that (10.32) is itself positive semidefinite. If $\hat{\boldsymbol{u}}_{-1}$ is substantially correlated with the columns of $\boldsymbol{X}^{*}(\hat{\rho})$, the incorrect variance estimate from regression (10.27) may be much smaller than the correct variance estimate from the GNR (10.29).

The Gauss-Newton regressions (10.26) and (10.29) yield estimated standard errors for $\hat{\rho}$ as well as for $\hat{\boldsymbol{\beta}}$. If the covariance matrix is asymptotically block-diagonal between ρ and $\boldsymbol{\beta}$, we see from (10.31) that the asymptotic variance of $n^{1/2}(\hat{\rho} - \rho_0)$ will be equal to

$$\omega^2 \operatorname*{plim}_{n\to\infty}\left(\frac{\hat{\boldsymbol{u}}_{-1}^{\top}\hat{\boldsymbol{u}}_{-1}}{n-1}\right)^{-1} = \omega^2\left(\frac{1-\rho_0^2}{\omega^2}\right) = 1-\rho_0^2. \tag{10.33}$$

Thus, in this special case, the variance of $\hat{\rho}$ can be estimated by

$$\frac{1-\hat{\rho}^2}{n-1}. \tag{10.34}$$

It may seem puzzling that neither the asymptotic variance $1-\rho_0^2$ nor the estimate (10.34) depends on ω^2. After all, we normally expect the variance